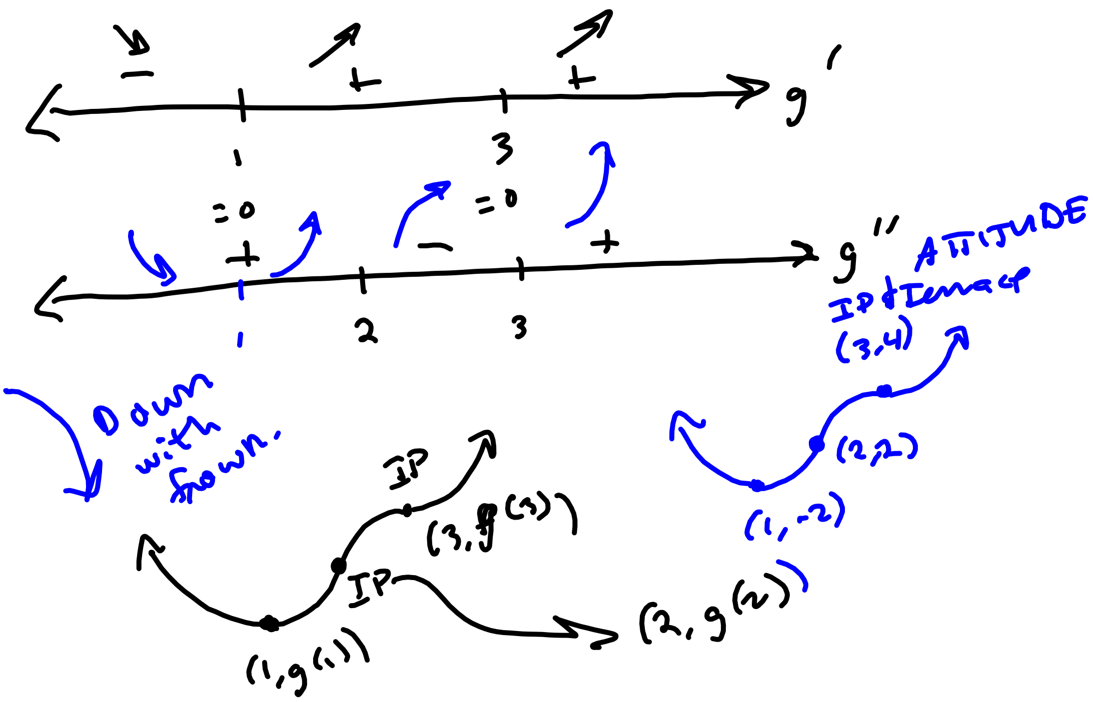
4. (10 pts) Suppose a function g satisfies all of the following properties. Sketch a graph of g that incorporates all of the following properties into it:

$g(1)=-2 \quad g(2)=2 \quad g(3)=4$

$g'(1)=0 \quad g'(3)=0$

$g'(x)>0$ on $(1,3)\cup(3,\infty)$, $\quad g'(x)<0$ on $(-\infty,1)$

$g''(x)>0$ on $(-\infty,2)\cup(3,\infty)$, $\quad g''(x)<0$ on $(2,3)$



Don't worry about x-intercepts of $f(x)$, unless I specifically ask for them.

Tangent line thru $(x_1, f(x_1)) = (x_1, y_1)$

$m = f'(x_1)$

$y = m(x - x_1) + y_1$

$= f'(x_1)(x - x_1) + f(x_1)$

§3.9 Find the most general antiderivative for $\boxed{f(x) = x^2 - 5x + 2}$ $\Rightarrow$

$F(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 2x + C$

Find $F(x)$ $\ni$ $\boxed{F(1) = 3}$

$F(1) = \frac{1}{3} - \frac{5}{2} + 2 + C = \frac{2 - 15 + 12}{6} + C$

$= \frac{-1}{6} + C = 3$

$\Rightarrow$ $C = 3 + \frac{1}{6} = \frac{19}{6}$, so

$F(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 2x + \frac{19}{6}$

Unique

$F(x)$ is the ! function $\ni$

$F'(x) = f(x)$ AND $F(1) = 3$.