Section 3.5: Summary of Curve Sketching

A. Domain
B. Intercepts
C. Symmetry
D. Asymptotes

Algebra (MAT 121)

If asked-for, FIND THEM

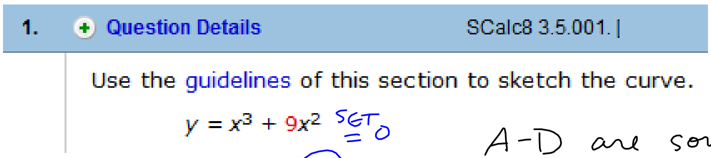
E. Intervals of Increase or Decrease
F. Local Maximum and Minimum Values
G. Concavity and Points of Inflection
H. Sketch the Curve

$f' = 0$
$f' = \nexists$ & Do sign pattern

$f'' = 0$ & Do sign pattern
$f'' \nexists$

Combine.

ON A TEST, I'm mainly looking for the calculus. Intercepts secondary.



1. Question Details SCalc8 3.5.001.

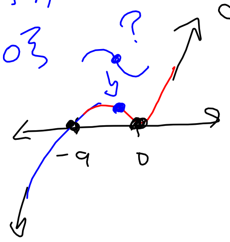
Use the guidelines of this section to sketch the curve.

$y = x^3 + 9x^2$ SET = 0

$\Rightarrow x^2(x+\overset{①}{9})=0$

$\Rightarrow x \in \{-9, 0\}$

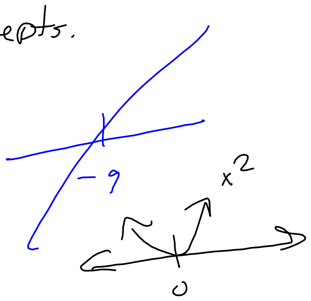
Quick Pic: ↙...↗



A-D are somewhat secondary.

$\mathcal{D} = \mathbb{R}$ always look @.

Don't spend too much time looking for x-intercepts.



$y' = 3x^2 + 18x$ SET = 0

$3x^2$ ↖...↗

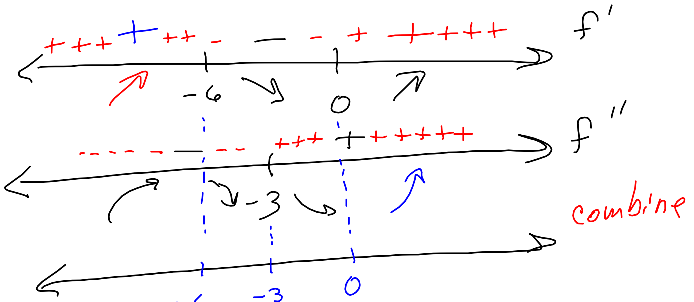
$\Rightarrow 3x^1(x+6)^1 = 0$

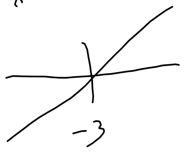
$\Rightarrow x \in \{-6, 0\}$

$y'' = 6x + 18$ SET = 0

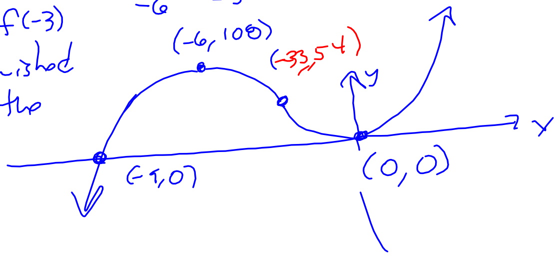
$\Rightarrow 6x = -18$

$\Rightarrow x = -3$ $(x \in \{-3\})$





Save f(-6), f(-3) 'til you've finished the rest of the whole test!

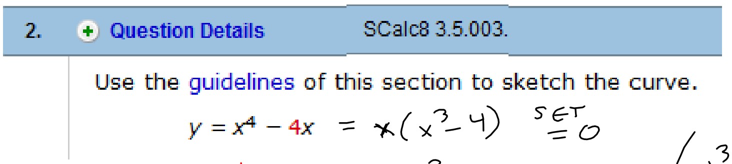


f(-6)

$$\begin{array}{r|rrrr} -6 & 1 & 9 & 0 & 0 \\ & & -6 & -18 & 108 \\ \hline & 1 & 3 & -18 & 108 = f(-6) \end{array}$$

$$\begin{array}{r} -18 \\ -6 \\ \hline 108 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 9 & 0 & 0 \\ & & -3 & -18 & 54 \\ \hline & 1 & 6 & -18 & 54 = f(-3) \end{array}$$



Use the guidelines of this section to sketch the curve.

$y = x^4 - 4x = x(x^3-4) \overset{\text{SET}}{=} 0$

$x \overset{1}{=} 0$ OR $x^3 - 4 = 0$

$x^3 - (\sqrt[3]{4})^3 = 0$

$(x-\sqrt[3]{4})(x^2 + \sqrt[3]{4}\,x + 2\sqrt[3]{2})$

No real roots, ever

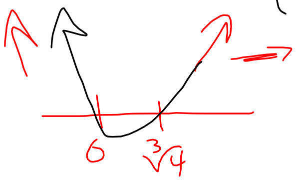
$\mathcal{D}(\text{Poly}) = \mathbb{R}$

$(\sqrt[3]{4})^2 = \sqrt[3]{16} = \sqrt[3]{2^3 \cdot 2} = 2\sqrt[3]{2}$

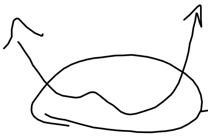
$x^3 - 4 = 0$

$x^3 = 4$

$x = \sqrt[3]{4}$



$\Rightarrow y = \sqrt[3]{4}$



maybe?

one min, so not this much wiggle.

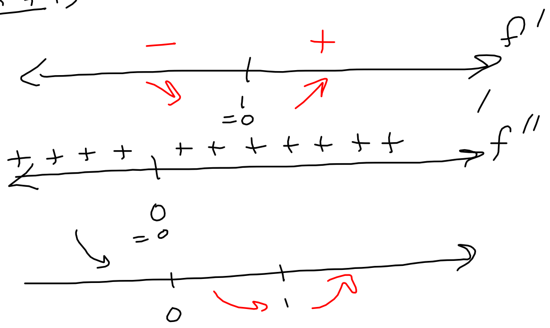
$y' = 4x^3 - 4 = 4(x^3-1) = 4(x-1)(x^2+x+1)$

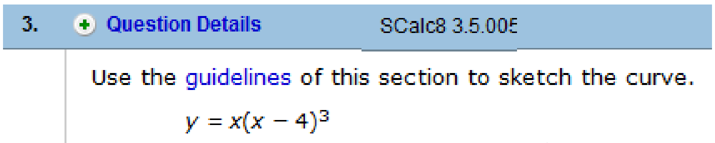
$4x^3$

$x = 1$

$y'' = 12x^2 \overset{\text{SET}}{=} 0 \Rightarrow$

$x = 0$




3. Question Details SCalc8 3.5.005

Use the guidelines of this section to sketch the curve.

$y = x(x-4)^3$

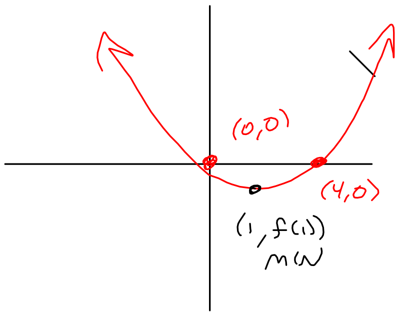
$y' = 1(x-4)^3 + x\left(3(x-4)^2\right)$

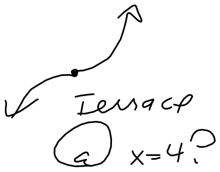
$= (x-4)^3 + 3x(x-4)^2$

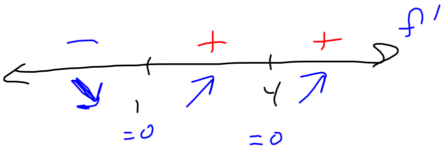
$= (x-4)^2[x-4+3x]$

$= (x-4)^2(4x-4) \overset{SET}{=} 0$

$\Rightarrow x \in \{4, 1\}$







$y'' = 2(x-4)(4x-4) + (x-4)^2(4)$

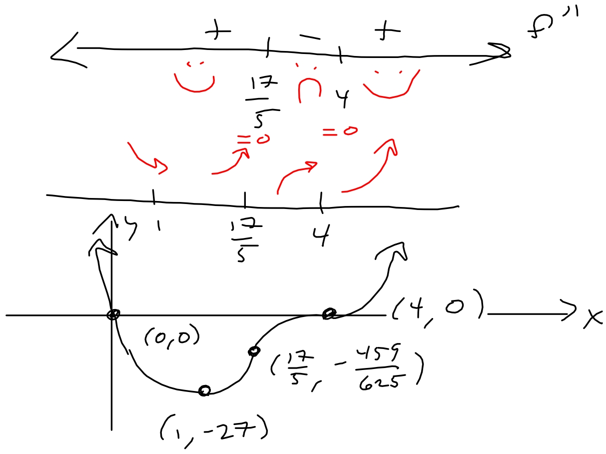
$= 8(x-4)(x-1) + 4(x-4)^2$

$= 8(x-4)[(x-1) + 4(x-4)] =$

$= 8(x-4)(x-1+4x-16)$

$= 8(x-4)(5x-17) \overset{SET}{=} 0$

$\Rightarrow x \in \{\frac{17}{5}, 4\}$



$y(1) = 1(1-4)^3 = -27$

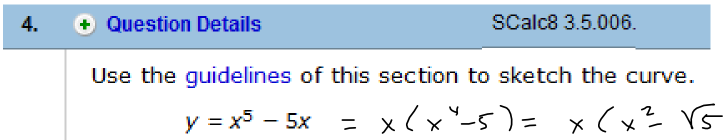
$y(4) = 0$

$y(\frac{17}{5}) = \frac{17}{5}(\frac{17}{5} - 4)^3$

$= \frac{17}{5}(\frac{-3}{5})^3$

$= \frac{17}{5}(\frac{-27}{125}) = \frac{-459}{625}$

4
27
17
189
270
459



4. ⊕ **Question Details** SCalc8 3.5.006.

Use the guidelines of this section to sketch the curve.

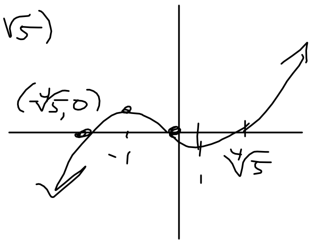
$y = x^5 - 5x = x(x^4-5) = x(x^2 - \sqrt{5})(x^2+\sqrt{5})$

$= x(x-\sqrt[4]{5})(x+\sqrt[4]{5})(x^2+\sqrt{5}) \overset{SET}{=} 0$

$x \in \{-\sqrt[4]{5}, 0, \sqrt[4]{5}\}$

$y' = 5x^4 - 5 = 5(x^4-1) = 5(x^2-1)(x^2+1)$

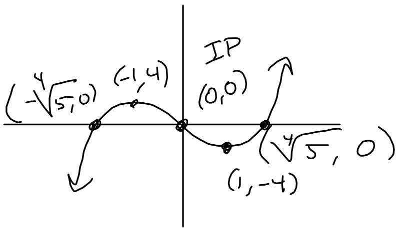
$= 5(x-1)(x+1)(x^2+1)$

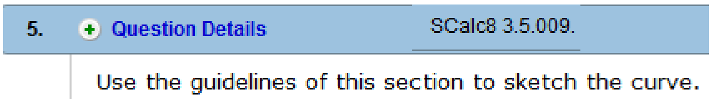


$y'' = 20x^3$ $x=0$ is I.P.

$\sqrt[4]{5} = \sqrt{\sqrt{5}} \approx \sqrt{2.m} \approx 1.5$ ish

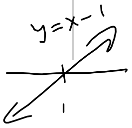
$y(1) = 1^5 - 5(1) = 1-5 = -4$





5. SCalc8 3.5.009.

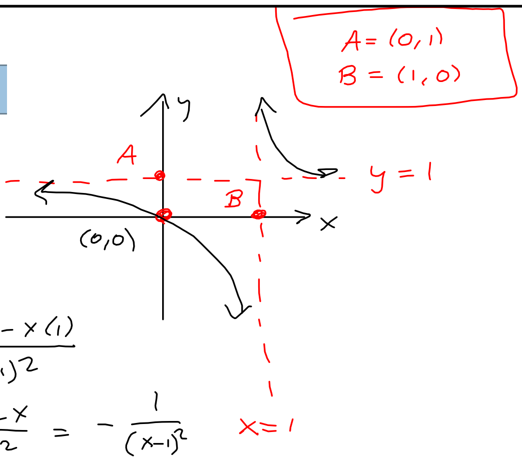
Use the guidelines of this section to sketch the curve.



$$y = \frac{x}{x-1} = \frac{x}{x(1-\frac{1}{x})} = \frac{1}{1-\frac{1}{x}}$$

$|x| \to \infty \Rightarrow 1 = y$ H.A.

$x=1$ is V.A.



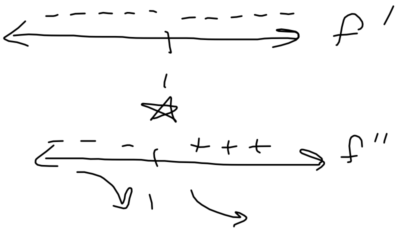
$$y' = \frac{1(x-1) - x(1)}{(x-1)^2}$$

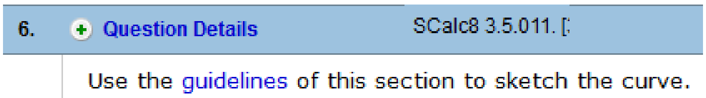
$$= \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

$$y' = -\frac{1}{(x-1)^2} = -(x-1)^{-2}$$

$$\Rightarrow y'' = 2(x-1)^{-3}(1)$$

$$= \frac{2}{(x-1)^3}$$





6. Question Details SCalc8 3.5.011. [:

Use the guidelines of this section to sketch the curve.

$R(x) = \cancel{y} = \frac{x - x^2}{6 - 7x + x^2} = \frac{-x^2+x}{x^2-7x+6} = \frac{-x\cancel{(x-1)}}{(x-6)\cancel{(x-1)}} = \frac{-x}{x-6} = R^*(x)$, $x \neq 1$

$x = 1$ is a hole.

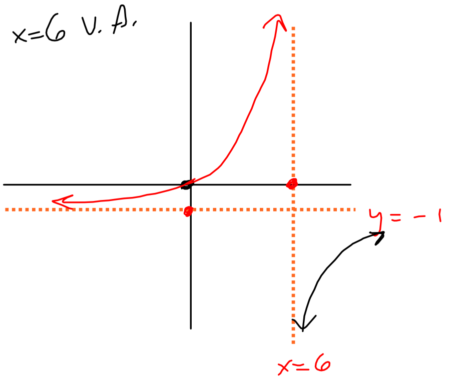
$y = \frac{-x^2+x}{x^2-7x+6} \xrightarrow{|x| \longrightarrow \infty} \frac{-x^2}{x^2} = \boxed{-1 = y = \text{H.A.}}$

$R^*(x) = 0 \Rightarrow x = 0$

$R^* \nexists$ @ $x=6$ V.A.

HOLE: $R^*(1) = \frac{-1}{1-6} = \frac{1}{5}$

$\boxed{(1, \tfrac{1}{5}) = \text{HOLE.}}$



$\frac{-x}{x-6} = R^*(x)$, $x \neq 1$

$R^{*\prime}(x) = \frac{-1(x-6) - (-x)(1)}{(x-6)^2}$

$= \frac{-x+6+x}{(x-6)^2} = \frac{6}{(x-6)^2} > 0$ on $\mathcal{D}$

$= 6(x-6)^{-2} = R^{*\prime}(x) \Longrightarrow$

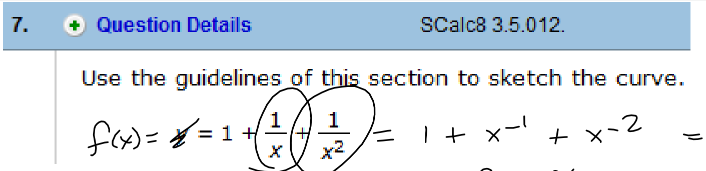
$R^{*\prime\prime}(x) = -12(x-6)^{-3}$

$= \frac{-12}{(x-6)^3}$

$\xleftarrow{\quad ++++ \quad | \quad ++++ \quad} \rightarrow$

$\xleftarrow{\quad +++ \quad | \quad - - - - \quad} \rightarrow$ f''

6



7. Question Details SCalc8 3.5.012.

Use the guidelines of this section to sketch the curve.

$f(x) = \cancel{y} = 1 + \frac{1}{x} + \frac{1}{x^2} = 1 + x^{-1} + x^{-2} = \frac{x^2+x+1}{x^2}$

$b^2-4ac = 1^2-4(1)(1) < 0$

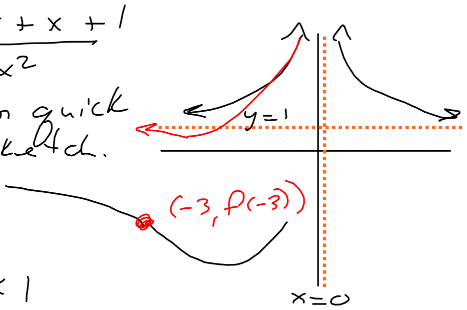
Never zero.

for f'

for quick sketch.

$f'(x) = -x^{-2} - 2x^{-3}$

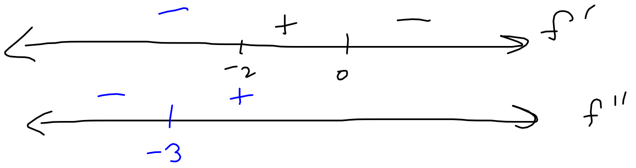
$= \frac{-1}{x^2} \cdot \frac{x}{x} - \frac{2}{x^3} = \frac{-x-2}{x^3} = -\frac{x+2}{x^3}$

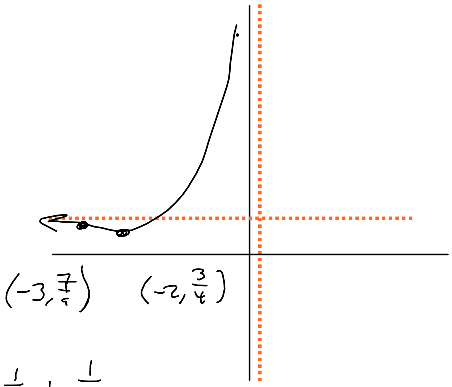


$f(-2) = 1 + \frac{1}{-2} + \frac{1}{(-2)^2} = 1 - \frac{1}{2} + \frac{1}{4} = \frac{3}{4} < 1$

$f''(x) = 2x^{-3} + 6x^{-4}$

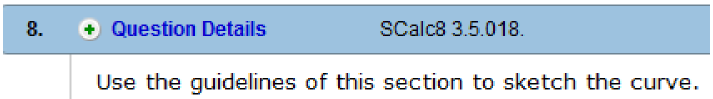
$= \frac{2}{x^3} \cdot \frac{x}{x} + \frac{6}{x^4} = \frac{2x+6}{x^4}$





$f(-3) = 1 + \frac{1}{-3} + \frac{1}{9}$

$= \frac{9-3+1}{9} = \frac{7}{9}$


8. Question Details SCalc8 3.5.018.

Use the guidelines of this section to sketch the curve.

$f(x) = \frac{x}{x^3-1} = \frac{x'}{(x-1)(x^2+x+1)} = 0$

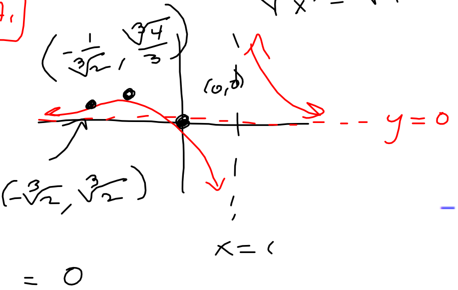
$a^3-b^3=(a-b)\underbrace{(a^2+ab+b^2)}$ ↳ complex roots.

$x^3=1 \Rightarrow$

$\sqrt[3]{x^3}=\sqrt[3]{1} =$

$\Rightarrow x=0$ PROPER : $\boxed{y=0 \quad H.A.}$

$f(x) \nexists$ @ $\boxed{x=1 \quad V.A.}$



$$-\frac{2x^3+1}{\left(x^3-1\right)^2}$$

$f'(x)=\frac{1(x^3-1)-x(3x^2)}{(x^3-1)^2}$

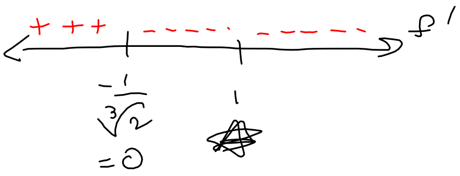
$=\frac{x^3-1-3x^3}{(\quad)^2} = \frac{-2x^3-1}{(x-1)^2(x^2+x+1)^2} = 0$

$\frac{-2x^3-1}{(x^3-1)^2} = \frac{-2x^3}{x^6+\ldots 1}$

$\Rightarrow -2x^3=1$

$x^3=-\frac{1}{2}$

$x=\sqrt[3]{-\frac{1}{2}} = \frac{-1}{\sqrt[3]{2}}$



$f''=\frac{-6x^2(x^3-1)^2 - (-2x^3-1)(2(x^3-1)'(3x^2))}{(x^3-1)^4}$

$=\frac{6x^2(x^3-1)\left[-(x^3-1)+2x^3+1\right]}{(x^3-1)^4} = \frac{6x^2(x^3-1)\left[-x^3+1+2x^3+1\right]}{(x^3-1)^4}$

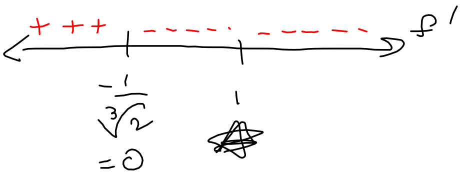
$=\frac{6x^2(x^3-1)\left[x^3+2\right]}{(x^3-1)^4}$ SET 0

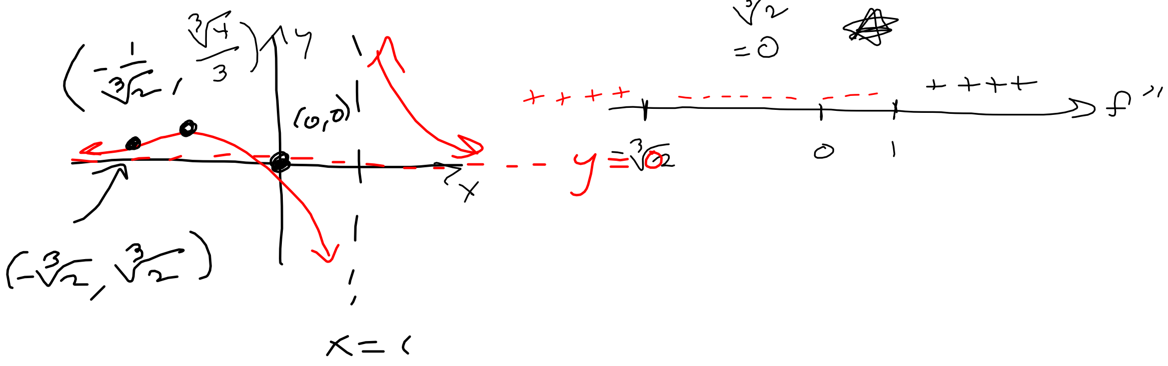
$x^3+2=0$

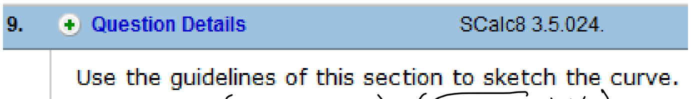
$x^3=-2$

$x=-\sqrt[3]{2}$

$\Rightarrow x \in \{0, 1, -\sqrt[3]{2}\}$







9. **Question Details** SCalc8 3.5.024.

Use the guidelines of this section to sketch the curve.

$$y = \left(\sqrt{x^2 + 2x} - x\right)\left(\frac{\sqrt{x^2+2x} + x}{\sqrt{x^2+2x} + x}\right) = \frac{x^2+2x-x^2}{\sqrt{x^2+2x} + x} =$$

$x^2(1+\frac{2}{x}$

$$= \frac{\cdots}{|x|\sqrt{1+\frac{2}{x}} + x} = \frac{2x}{x\left(\sqrt{1+\frac{2}{x}} + 1\right)} = \frac{2}{\sqrt{1+\frac{2}{x}} + 1}$$

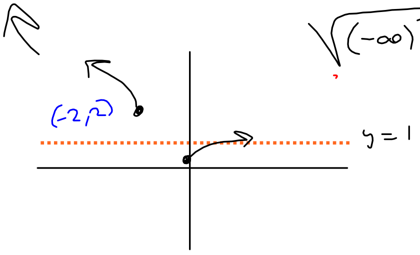
for $x \to +\infty$

$$\sqrt{x^2+2x} = \sqrt{x^2(1+\frac{2}{x})} = \sqrt{x^2}\sqrt{1+\frac{2}{x}} = |x|\sqrt{1+\frac{2}{x}}$$

$$\xrightarrow{x \to \infty} \frac{2}{2} = \boxed{1 = y = H.A.!}$$

$\xrightarrow{x \to -\infty}$ Case, this thing zooms off to ∞,

$$\sqrt{(-\infty)^2 + 2(-\infty)} - (-\infty) = \infty + \infty = \infty$$



$$\sqrt{x^2+2x} - x = f(x) = (x^2+2x)^{\frac{1}{2}} - x$$

$$\Rightarrow f'(x) = \frac{1}{2}(x^2+2x)^{-\frac{1}{2}}(2x+2) - 1$$

$$\boxed{= \frac{x+1}{(x^2+2x)^{\frac{1}{2}}} - \frac{1}{1} \cdot \frac{(x^2+2x)^{1/2}}{(x^2+2x)^{1/2}}}$$

$$= \frac{x+1 - \sqrt{x^2+2x}}{\sqrt{x^2+2x}} \quad \underline{\text{SET}}\ 0 \Rightarrow$$

$$f(-2) = \sqrt{(-2)^2 + 2(-2)} - (-2)$$

$$= \sqrt{4-4} + 2 = 2$$

$$f(0) = 0$$

$$\Rightarrow x+1 = \sqrt{x^2+2x}$$

$$x^2+2x+1 = x^2+2x$$

$1 = 0$ ✗ Nevah!

$$f'(x) = (x+1)(x^2+2x)^{-\frac{1}{2}} \Rightarrow$$

But f' ∄ when $x^2+2x = 0 \Rightarrow x \in \{-2, 0\}$ } goin' vertical.

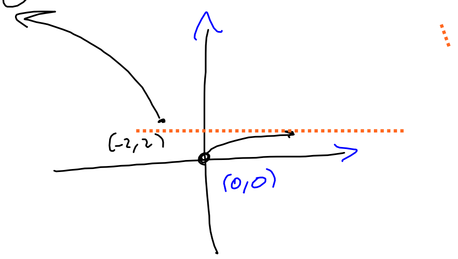
$$f''(x) = 1(x^2+2x)^{-\frac{1}{2}} + (x+1)\left(-\frac{1}{2}(x^2+2x)^{-\frac{3}{2}}(\cancel{2}x+\cancel{2})\right)$$

$$= \frac{1}{(x^2+2x)^{1/2}} \cdot \frac{x^2+2x}{x^2+2x} - \frac{x^2+2x+1}{(x^2+2x)^{3/2}}$$

$$= \frac{x^2+2x - x^2 - 2x - 1}{(x^2+2x)^{3/2}} = \frac{-1}{(x^2+2x)^{3/2}} \neq 0$$

& f'' ∄ @ $x = -2, 0$, again

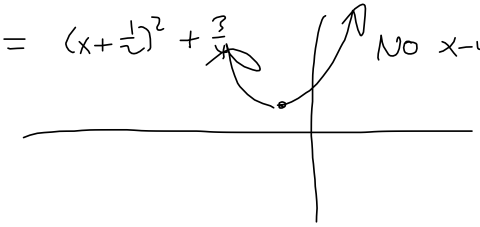
wherever it's defined, it's $\frac{-1}{\text{positive}}$ So ⌢



$x^2+x+1 =$

$x^2+x+\left(\frac{1}{2}\right)^2-\frac{1}{4}+1$

$= (x+\frac{1}{2})^2+\frac{3}{4}$

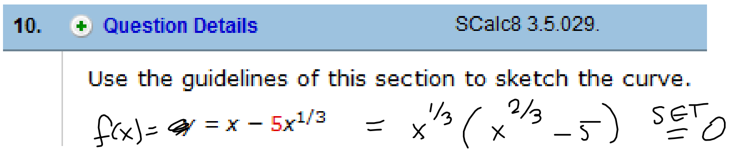


No x-int.

$a=1, b=1, c=1$

$b^2-4ac = 1^2-4(1)(1) < 0$

No x-int.



10. Question Details SCalc8 3.5.029.

Use the guidelines of this section to sketch the curve.

$f(x) = \cancel{y} = x - 5x^{1/3} = x^{1/3}(x^{2/3} - 5) \overset{SET}{=} 0 \Rightarrow x \in \{0, 5^{3/2}\}$

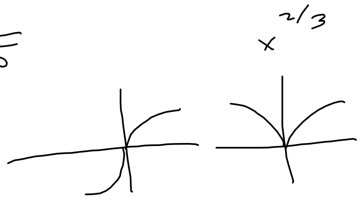
$x^{2/3} = 5$

$x = 5^{3/2} = \sqrt{125} = 5\sqrt{5}$

$\Rightarrow f'(x) = 1 - \frac{5}{3}x^{-2/3}$

$= 1 - \frac{5}{3x^{2/3}} = \frac{3x^{2/3} - 5}{3x^{2/3}} \overset{SET}{=} 0$

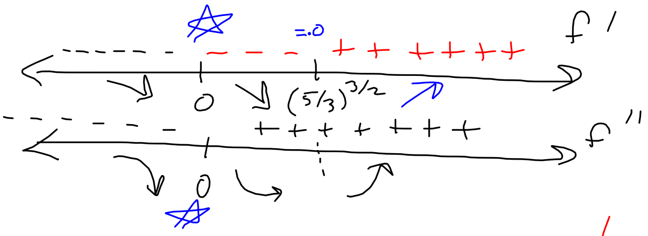
$\Rightarrow x^{2/3} = \frac{5}{3} \Rightarrow x \in \left\{\left(\frac{5}{3}\right)^{3/2}\right\}$



$f' \nexists$ when $x = 0$, so $x = 0, \left(\frac{5}{3}\right)^{3/2}$ are critical #s

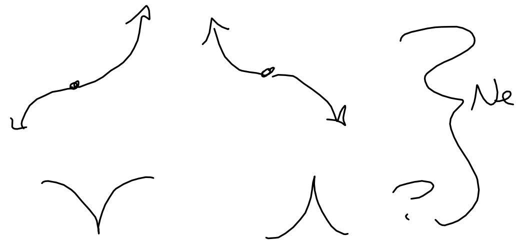
$f''(x) = \frac{10}{9}x^{-5/3} \overset{SET}{=} 0 \Rightarrow \cancel{\ }$

$f'' \nexists$ @ $x = 0$



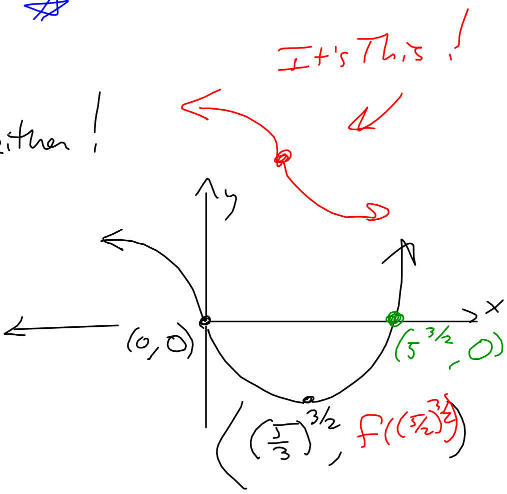
Terrace?

Cusp?



Neither!

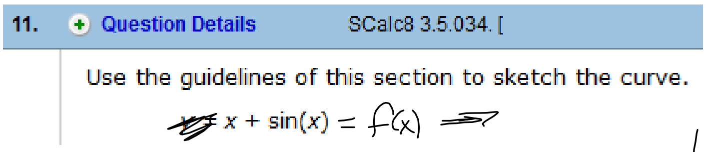
It's This!



$f\left(\left(\frac{5}{3}\right)^{3/2}\right) = \left(\frac{5}{3}\right)^{3/2} - 5\left(\left(\frac{5}{3}\right)^{3/2}\right)^{\frac{1}{3}}$

$= \left(\frac{5}{3}\right)^{3/2} - 5\left(\sqrt{\frac{5}{3}}\right)$

= Calculator Time!



11. ⊕ Question Details SCalc8 3.5.034. [

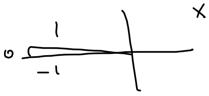
Use the guidelines of this section to sketch the curve.

~~$y =$~~ $x + \sin(x) = f(x) \Longrightarrow$

$f(x) = 0$ requires tech. or Newton's !

$f'(x) = \cos x + 1 \overset{SET}{=} 0$

$\Longrightarrow \cos x = -1$

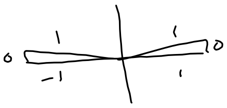


$x = \pi + 2n\pi, n \in \mathbb{Z}$

$f''(x) = -\sin x \overset{SET}{=} 0$

$x = n\pi, n \in \mathbb{Z}$

$\Longrightarrow$



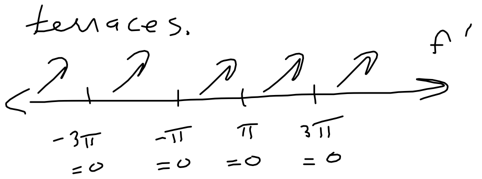
$\{0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots\}$

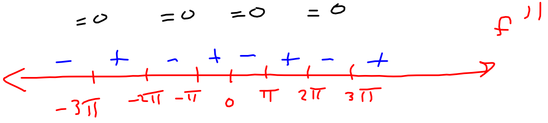
$f(\pi) = \sin\pi + \pi$

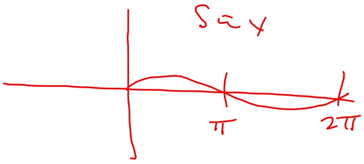
$-1 \leq \cos x \leq 1$

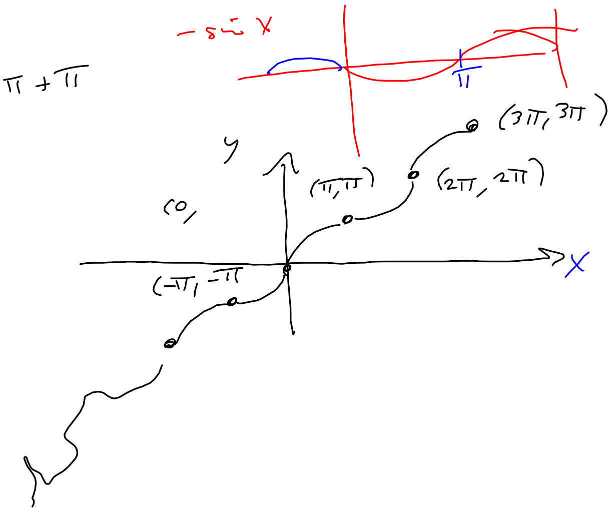
$0 \leq \cos x + 1 \leq 2$

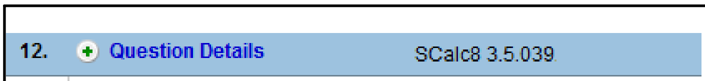
so $f'(x) \geq 0 \Longrightarrow$ No local extremes. Just a lot of terraces.











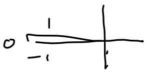
12. Question Details SCalc8 3.5.039

Use the guidelines of this section to sketch the curve.

$f(x) = \frac{\sin(x)}{1+\cos(x)} = 0 \Rightarrow \sin x = 0 \Rightarrow x = n\pi, n \in \mathbb{Z}$

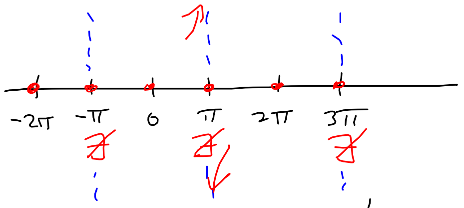
$f \nexists \quad 1 + \cos x = 0$

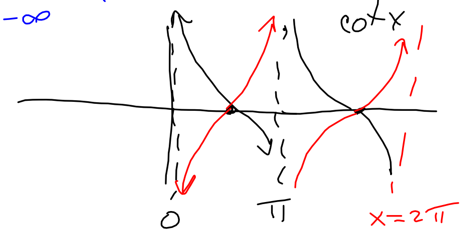
$\cos x = -1 \qquad \pi + 2n\pi, n \in \mathbb{Z}$



$\lim_{x\to\pi^+} \frac{\sin x}{1+\cos x} \overset{?}{=} \lim_{x\to\pi^+} \frac{0}{0} = \lim_{x\to\pi^+} \frac{\cos x}{-\sin x} = -\infty$

$\lim_{x\to\pi^-} (-\cot x) = +\infty$



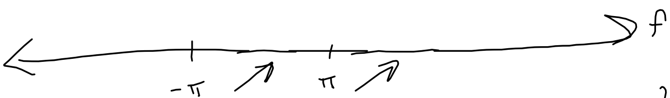


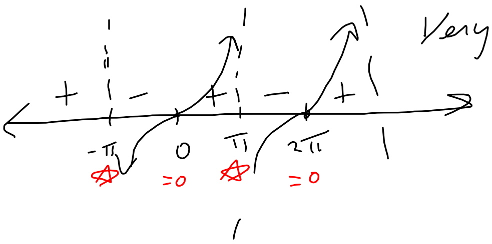
$f'(x) = \frac{(\cos x)(1+\cos x) - (\sin x)(-\sin x)}{(\cos x + 1)^2} =$

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(\cos x + 1)^2} = \frac{\cos x + 1}{(\cos x + 1)^2} = \frac{1}{\cos x + 1} \neq 0$

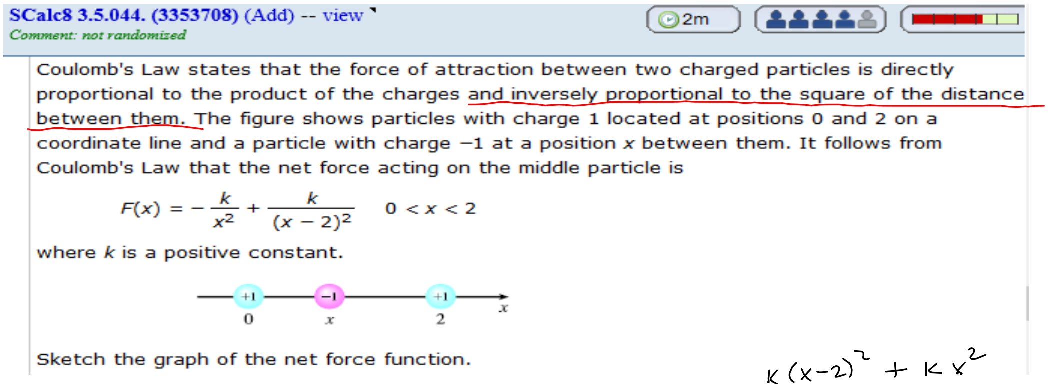
$f' \nexists \ \forall \ x = \pi + 2n\pi, n \in \mathbb{Z}. \quad \notin D(f)$

$f'(x) = (\cos x + 1)^{-1} \Rightarrow f''(x) = -(\cos x + 1)^{-2}(-\sin x) = \frac{\sin x}{(\cos x + 1)^2}$





Very rough, but f'' that's the idea.



SCalc8 3.5.044. (3353708) (Add) -- view

Comment: not randomized

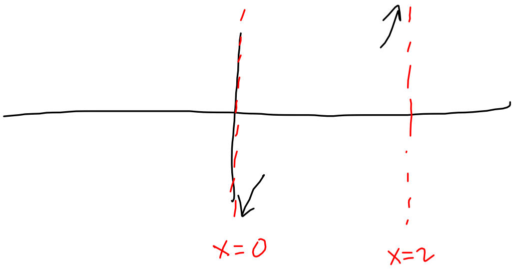
Coulomb's Law states that the force of attraction between two charged particles is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The figure shows particles with charge 1 located at positions 0 and 2 on a coordinate line and a particle with charge −1 at a position x between them. It follows from Coulomb's Law that the net force acting on the middle particle is

$$F(x) = -\frac{k}{x^2} + \frac{k}{(x-2)^2} \qquad 0 < x < 2$$

where k is a positive constant.

Sketch the graph of the net force function.

$$F = -Kx^{-2} + K(x-2)^{-2} = \frac{K(x-2)^2 + Kx^2}{x^2(x-2)^2} = K\frac{(x-2)^2+x^2}{x^2(x-2)^2}$$



$$F' = 2Kx^{-3} - 2K(x-2)^{-3}$$

$$= \frac{2K}{x^3}\cdot\frac{(x-2)^3}{(x-2)^3} - \frac{2K}{(x-2)^3}\cdot\frac{x^3}{x^3} = 2K\frac{(x-2)^3 - x^3}{x^3(x-2)^3} \quad \underline{\text{SET}}\ 0 \Longrightarrow$$

$$((x-2)-x)((x-2)^2 + x(x-2) + x^2) = 0$$

$$\Longrightarrow -2(x^2-4x+4+x^2-2x+x^2) = 0$$

$$\Longrightarrow 3x^2-6x+4 = 0$$

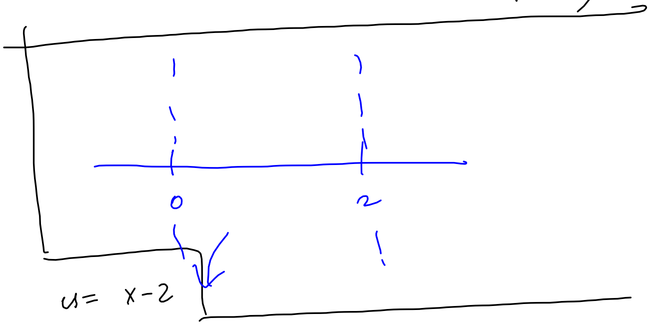
$$a=3,\ b=-6,\ c=4$$

$$(-6)^2-4(3)(4) = 36-48 < 0$$

$\Longrightarrow$ No real roots!

So, $F' \neq 0$

$F' \nexists$ @ $x = 0, 2$ (Same as F)



$$f'(x) = 2Kx^{-3} - 2K(x-2)^{-3} \Longrightarrow$$

$$f''(x) = -6Kx^{-4} + 6K(x-2)^{-4}$$

$$= -6K\left[\frac{1}{x^4}\cdot\frac{(x-2)^4}{(x-2)^4} - \frac{1}{(x-2)^4}\cdot\frac{x^4}{x^4}\right]$$

$$= -6K\left[\frac{(x-2)^4 - x^4}{x^4(x-2)^4}\right]$$

$$= -6K\left[\frac{((x-2)-x)((x-2)+x)((x-2)^2+x^2)}{x^4(x-2)^4}\right]$$

$$= -6K\left[\frac{-2(2x-2)(x^2-4x+4+x^2)}{x^4(x-2)^4}\right]$$

$u = x-2$

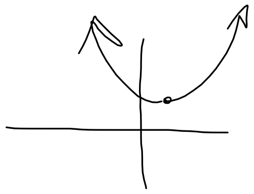
$v = x$

$$u^4 - v^4 = (u^2-v^2)(u^2+v^2)$$

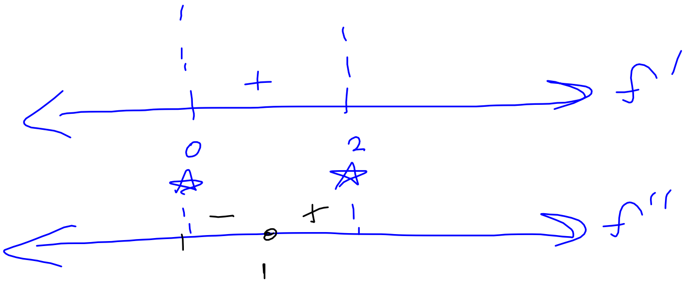
$$= (u-v)(u+v)(u^2+v^2)$$

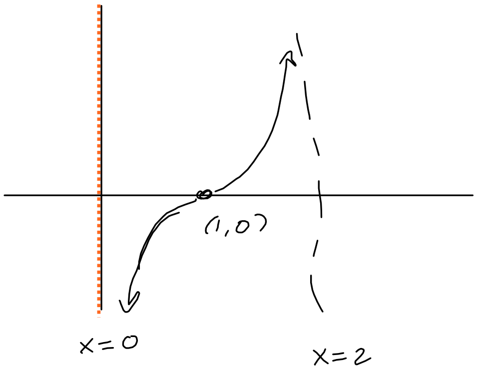
$$= 24K\left[\frac{(x-1)(2x^2-4x+4)}{x^4(x-2)^4}\right] = 48K\left[\frac{(x-1)(x^2-2x+2)}{x^4(x-2)^4}\right] \quad \underline{\underline{\text{SET}}}\ 0 \Rightarrow x=1$$

$$\Rightarrow x^2-2x+2 = x^2-2x+1^2-1+2 = (x-1)^2+1$$



No real roots



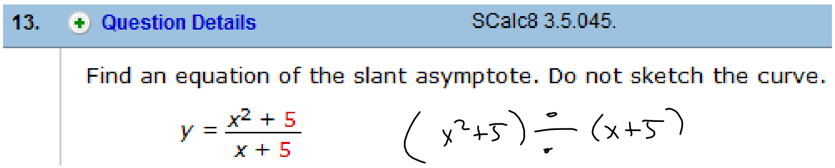


$$f(x)=0 \Rightarrow -\frac{K}{x^2}+\frac{K}{(x-2)^2}=0$$

$$\Rightarrow -\frac{1}{x^2}+\frac{1}{(x-2)^2}=0$$

$$\Rightarrow \frac{-(x^2-4x+4)+x^2}{x^2(x-2)^2} = \frac{-x^2+4x-4+x^2}{x^2(x-2)^2}=0$$

$$\Rightarrow 4x-4=0 \Rightarrow x=1$$



13. Question Details SCalc8 3.5.045.

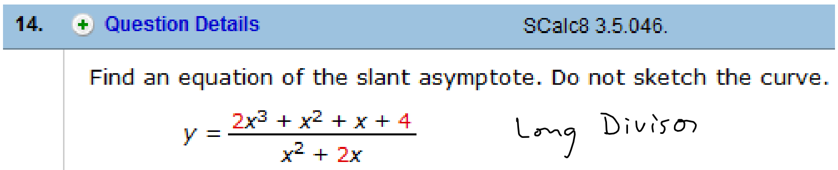
Find an equation of the slant asymptote. Do not sketch the curve.

$$y = \frac{x^2+5}{x+5}$$

$(x^2+5) \div (x+5)$

$$\begin{array}{r|rrr} -5 & 1 & 0 & 5 \\ & & -5 & \cancel{25} \\ \hline & 1 & -5 & \cancel{30} \\ & x & c & r \end{array}$$

So $\boxed{y = x-5 \text{ is O.A.}}$



14. Question Details SCalc8 3.5.046.

Find an equation of the slant asymptote. Do not sketch the curve.

$$y = \frac{2x^3+x^2+x+4}{x^2+2x}$$

Long Divison

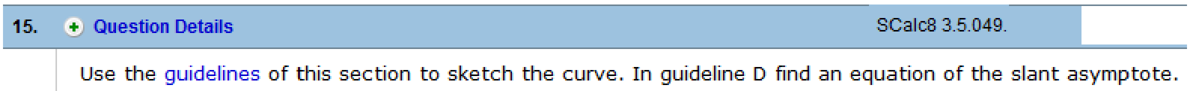
$$\begin{array}{r|l} & 2x - 3 \\ \hline x^2+2x & 2x^3 + x^2 + x + 4 \\ & -(2x^3+4x^2) \\ \hline & -3x^2+x+4 \end{array}$$

$\frac{-3x^2}{x^2} = -3$

So $y = 2x-3$ is O.A.

Oblique Asymptote

$\frac{2x^7+x^5}{x^2-5x+1}$ has $2x^5$-type O.A.



15. Question Details SCalc8 3.5.049.

Use the guidelines of this section to sketch the curve. In guideline D find an equation of the slant asymptote.

$y = \frac{x^2}{x-9} = 0 \longrightarrow x = 0 \rightsquigarrow (0,0)$

$\nexists \Longrightarrow x = 9$ is V.A.

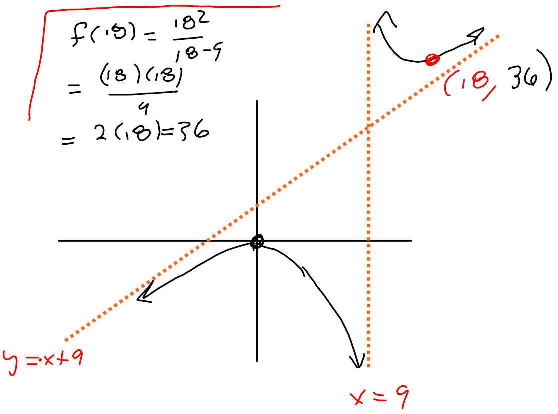
$\xleftarrow{\quad - \quad \underset{\substack{0 \\ =0}}{|} \quad - \quad \underset{\substack{9 \\ \nexists}}{|} \quad + \quad}$

No H.A.

O.A.:

$$\begin{array}{r|rrr} 9 & 1 & 0 & 0 \\ & & 9 & \cancel{81} \\ \hline & 1 & 9 & \end{array}$$

$\rightarrow y = x + 9$ is O.A.



$f(18) = \frac{18^2}{18-9}$

$= \frac{(18)(18)}{9}$

$= 2(18) = 36$

(18, 36)

y = x + 9

x = 9

$f'(x) = \frac{2x(x-9) - x^2(1)}{(x-9)^2} = \frac{2x^2 - 18x - x^2}{(x-9)^2} = \frac{x^2 - 18x}{(x-9)^2} = \frac{x(x-18)}{(x-9)^2}$

$\xleftarrow{\quad + \quad \underset{0}{|} \quad - \quad \underset{\substack{9 \\ \nexists}}{|} \quad - \quad \underset{\substack{18 \\ =0}}{|} \quad + \quad}$

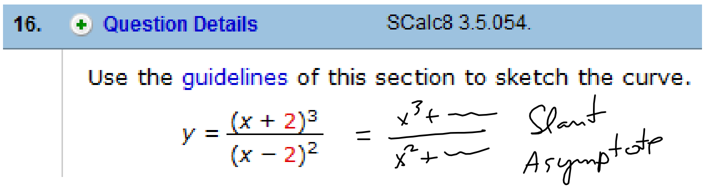
$f''(x) = \frac{(2x-18)(x-9)^2 - (x^2-18x)(2(x-9)'(1))}{(x-9)^4}$

$= \frac{\cancel{(x-9)}\left[(2x-18)(x-9) - (x^2-18x)(2)\right]}{(x-9)^{\boxed{4}} \rightarrow 3}$

$= \frac{2x^2 - 18x - 18x + 162 - 2x^2 + 36x}{(x-9)^3}$

$= \frac{162}{(x-9)^3}$ ∄ @ x = 9

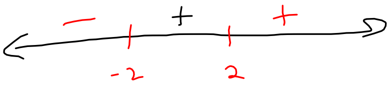
$\xleftarrow{\quad - \quad \underset{9}{|} \quad + \quad}$



16. ⊕ Question Details SCalc8 3.5.054.

Use the guidelines of this section to sketch the curve.

$$y = \frac{(x+2)^3}{(x-2)^2} = \frac{x^3 + \sim\sim}{x^2 + \sim\sim}$$ Slant Asymptote



1
1 1
1 2 1
1 3 3 1

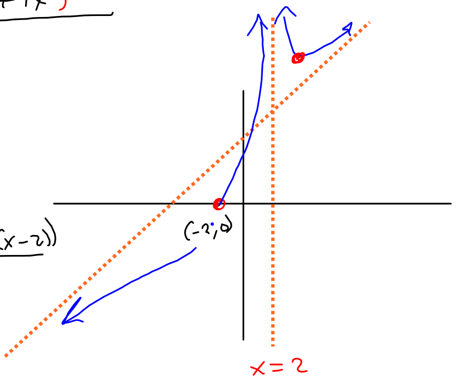
$$(x+2)^3 = x^3 + (3x^2)(2) + 3(x^1)(2)^2 + 1\,(x^0)(2^3)$$
$$= x^3 + 6x^2 + 12x + 8$$
$$(x-2)^2 = x^2 + (2)(x)^1(-2) + (1)(x)^0(-2)^2$$
$$= x^2 - 4x + 4$$

$$\begin{array}{r} x+10 \\ x^2-4x+4 \overline{\big)\, x^3+6x^2+12x+8} \\ -(\,x^3-4x^2+4x\,) \\ \hline 10x^2 \end{array}$$

$$\frac{x^3}{x^2} = x$$

$$\frac{10x^2}{x^2} = 10$$

$$\boxed{y = x+10 \text{ is O.A.}}$$



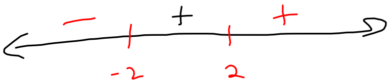
$$f(x) = \frac{(x+2)^3}{(x-2)^2} \Longrightarrow$$

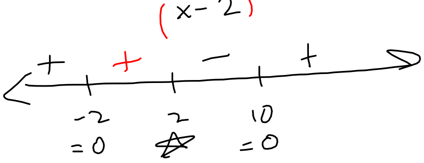
$$f'(x) = \frac{3(x+2)^2(x-2)^2 - (x+2)^3(2(x-2))}{(x-2)^4}$$

$$= \frac{(x+2)^2(x-2)[3(x-2) - (x+2)(2)]}{(x-2)^4}$$

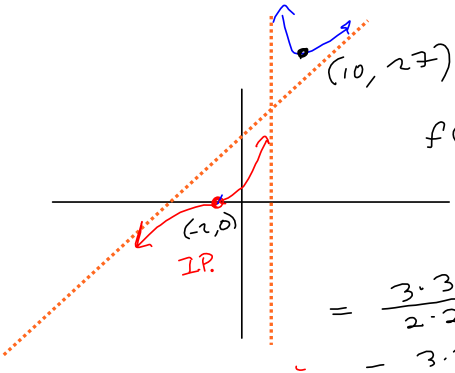
$$= \frac{(x+2)^2[3x-6-2x-4]}{(x-2)^3}$$

$$= \frac{(x+2)^2[x-10]^1}{(x-2)^3}$$





f′ OK



$$f(10) = \frac{(10+2)^3}{(10-2)^2}$$

$$= \frac{12^3}{8^2} = \frac{12 \cdot 12 \cdot 12}{8 \cdot 8}$$

$$= \frac{3 \cdot 3 \cdot 12}{2 \cdot 2}$$

$$= 3 \cdot 3 \cdot 3 = 27$$

$$f''(x) = \frac{\left(2(x+2)(x-10) + (x+2)^2\right)(x-2)^3 - (x+2)^2(x-10)\left(3(x-2)^2\right)}{(x-2)^6}$$

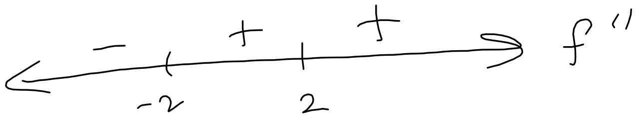
$$= \frac{(x+2)(x-2)^2\left[\left(2(x-10) + x+2\right)(x-2)^1 - (x+2)(x-10)(3)\right]}{(x-2)^6}$$

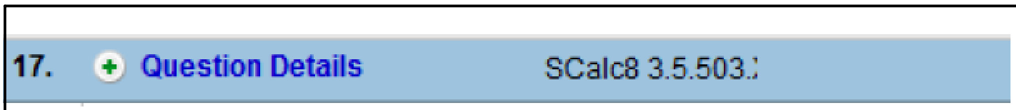
$$= \frac{(x+2)\left[\left(2x-20+x+2\right)(x-2) - (3)(x^2-8x-20)\right]}{(x-2\quad)^4}$$

$$= \frac{(x+2)\left[(3x-18)(x-2) - 3x^2+24x+60\right]}{(\quad)^4} =$$

$$= \frac{(x+2)\left[3x^2-24x+36-3x+24x+60\right]}{(\qquad)^4}$$

$$= \frac{(x+2)[96]}{(x-2)^4}$$





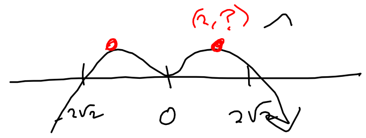
17. Question Details SCalc8 3.5.503.

Use the guidelines of this section to sketch the curve.

$2\sqrt{2}$

$f(x) = \cancel{y} = 8x^2 - x^4 = -x^4 + 8x^2 = -x^2(x^2 - 8) = -x^2(x - \sqrt{8})(x + 2\sqrt{2}) \overset{SET}{=} 0$

$\Rightarrow x \in \{0, \pm 2\sqrt{2}\}$



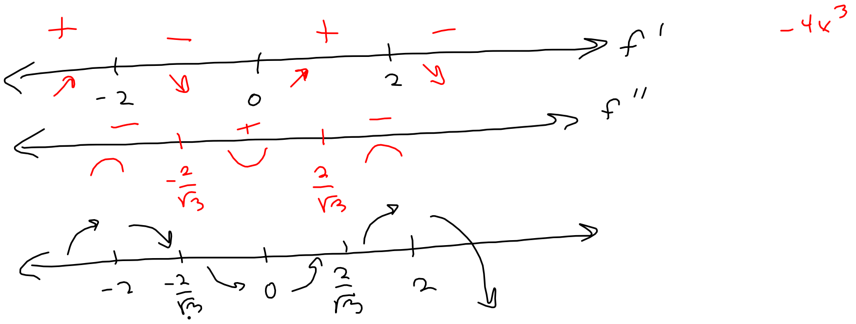
$f'(x) = -4x^3 + 16x = -4x(x^2 - 4) = -4x(x-2)(x+2) \overset{SET}{=} 0$

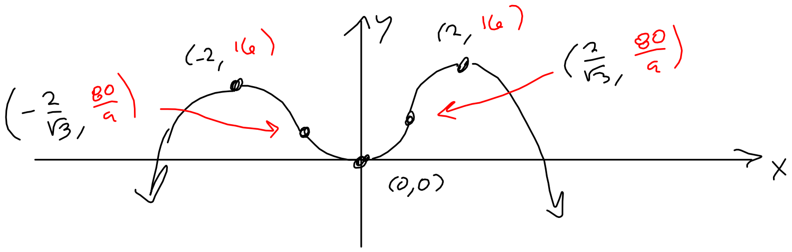
$\Rightarrow x \in \{0, \pm 2\}$

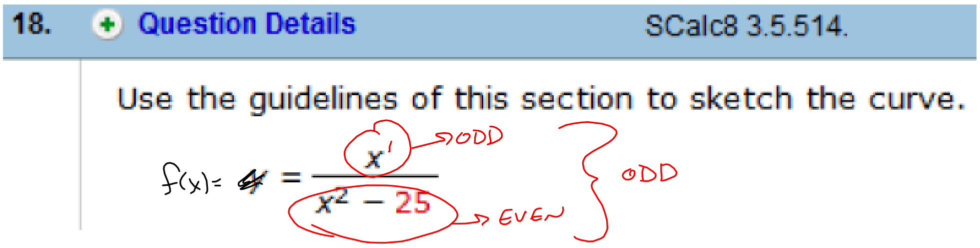
$f''(x) = -12x^2 + 16 = -4(3x^2 - 4) = -4(\sqrt{3}x - 2)(\sqrt{3}x + 2) \overset{SET}{=} 0$

$\Rightarrow x \in \{\pm \frac{2}{\sqrt{3}}\}$ $\frac{2\sqrt{3}}{3} \approx$ 1.154700539 $\approx \frac{2}{\sqrt{3}}$

$\sqrt{8} \approx$ 2.828427124







18. Question Details SCalc8 3.5.514.

Use the guidelines of this section to sketch the curve.

$$f(x)= \cancel{y} = \frac{x^{1}}{x^2 - 25}$$

x^1 → ODD

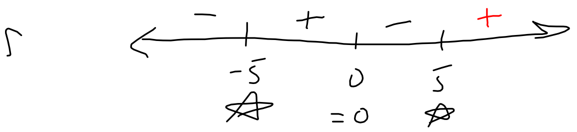
x^2-25 → EVEN

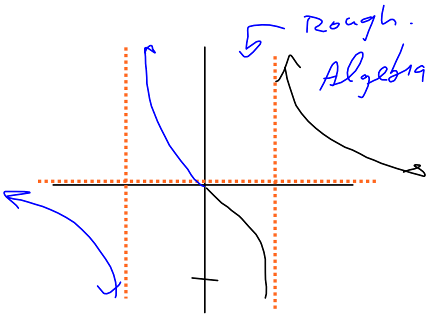
ODD

$f \cancel{\exists}: \; x^2-25 = (x-5)(x+5) = 0 \Rightarrow x \in \{\pm 5\}$

$\mathcal{D} = \mathbb{R} \setminus \{\pm 5\} \Rightarrow \boxed{V.A.: \; x = \pm 5}$

$\boxed{H.A.: \; y = 0}$ (Proper!)





Rough. Algebra

$$f'(x) = \frac{1(x^2-25) - x(2x)}{(x^2-25)^2} = \frac{-\cancel{x}^2-25}{(x^2-25)^2}$$

$$-\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$$

$f' \cancel{\exists} \; x \in \{\pm 5\}$

$$= -\frac{x^2+25}{(x^2-25)^2} \neq 0, \text{ ever!}$$

$$f''(x) = \frac{-2x(x^2-25)^2 - (-x^2-25)(2(x^2-25)(2x))}{((x^2-25)^2)^2}$$

$$= \frac{(x^2-25)[-2x(x^2-25) - (-x^2-25)(4x)]}{(x^2-25)^4} = \frac{-2x^3+50x+4x^3+100x}{(x^2-25)^3}$$

$$= \frac{2x^3 + \overbrace{50x + 100x}^{150x}}{(\quad)^3} = \frac{2x(x^2+75)}{(x^2-25)^3}$$

$(x-5)^3(x+5)^3$

$\underline{\underline{SET}}\; 0 \Rightarrow x = 0$

$$f'(x) = -\frac{x^2+25}{(x^2-25)^2}$$

$$f''(x) = \frac{2x(x^2+75)}{(x^2-25)^3}$$