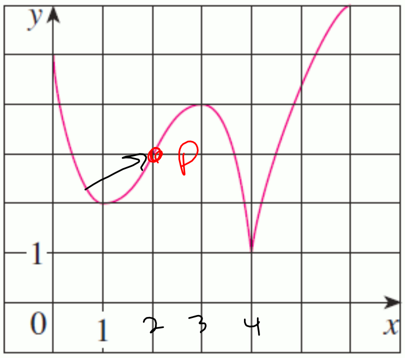
1. Question Details SCalc8 3.3.001. [3353676]

Use the given graph of f over the interval (0, 6) to find the following.



(a) The open intervals on which f is increasing. (Enter your answer using interval notation.) $(1,3) \cup (4,\infty)$

(b) The open intervals on which f is decreasing. (Enter your answer using interval notation.) $(0,1) \cup (3,4)$

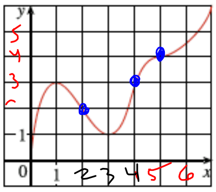
(c) The open intervals on which f is concave upward. (Enter your answer using interval notation.) $(0,2)$

(d) The open intervals on which f is concave downward. (Enter your answer using interval notation.) $(2,4) \cup (4,\infty)$

(e) The coordinates of the point of inflection. $(2,3) = P$

2. Question Details SCalc8 3.3.002. [3353623]

Use the given graph of *f* over the interval (0, 7) to find the following.



a) $(0,1) \cup (3,7)$

b) $(1,3)$

c) $(2,4) \cup (5,7)$

d) $(0,2) \cup (4,5)$

e) $(2,2), (4,3), (5,4)$

(a) The open intervals on which *f* is increasing. (Enter your answer using interval notation.)

(b) The open intervals on which *f* is decreasing. (Enter your answer using interval notation.)

(c) The open intervals on which *f* is concave upward. (Enter your answer using interval notation.)

(d) The open intervals on which *f* is concave downward. (Enter your answer using interval notation.)

(e) The coordinates of the points of inflection.

3. Question Details SCalc8 3.3.003. [3353848]

Suppose you are given a formula for a function *f*.

(a) How do you determine where *f* is increasing or decreasing?

(b) How do you determine where the graph of *f* is concave upward or concave downward?

(c) How do you locate inflection points?

(a) Find $f'(x)$, Set $f'(x)=0$, Do sign pattern on f'.

$f'>0$, $+$, increasing

$f'<0$, $-$, decreasing

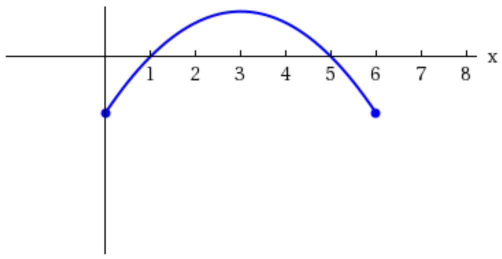
(b) Find f'', set $f''=0$, Do sign pattern.

$f''>0$, $+$, $\cup$, $f''<0$, $-$, $\cap$.

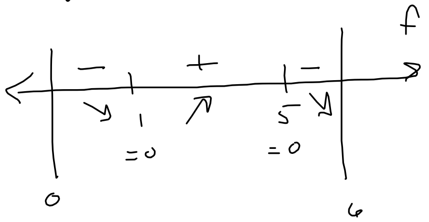
(c) Inflection points are points where f'' changes sign.

4. Question Details SCalc8 3.3.005. [3353812]

The graph of the derivative f' of a function *f* is shown.



sign pattern for f'



(a) On what interval is *f* increasing? (Enter your answer using interval notation.) $(1,5)$

On what intervals is *f* decreasing? (Enter your answer using interval notation.) $(0,1)\cup(5,6)$

(b) At what values of *x* does *f* have a local maximum or minimum? (Enter your answers as a comma-separated list.)

$x=1$ MIN

$x=5$ MAX

5. Question Details SCalc8 3.3.009. [3353668]

Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = x^3 - 9x^2 - 21x + 4$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.) $(-\infty,-1)\cup(7,\infty)$

Find the interval on which f is decreasing. (Enter your answer using interval notation.) $(-1,7)$

(b) Find the local minimum and maximum values of f.

(b) MAX @ $x=-1$

$$\begin{array}{r|rrrr} -1 & 1 & -9 & -21 & 4 \\ & & -1 & 10 & 11 \\ \hline & 1 & -10 & -11 & 15 \end{array}$$

$15 = f(-1)$ local max

(b) Min @ $x = 7$

$$\begin{array}{r|rrrr} 7 & 1 & -9 & -21 & 4 \\ & & 7 & -14 & -245 \\ \hline & 1 & -2 & -35 & -241 \end{array}$$

$f(7) = -243$ local min.

(c) Find the inflection point.

Find the interval on which f is concave up. (Enter your answer using interval notation.)

Find the interval on which f is concave down. (Enter your answer using interval notation.)

$f(x) = x^3 - 9x^2 - 21x + 4 \implies$

$f'(x) = 3x^2 - 18x - 21 \overset{\text{SET}}{=} 0$

$\implies x^2 - 6x - 7 = 0$

$\implies (x-7)(x+1) = 0$

$\implies x \in \{-1, 7\}$

f': $+$ $\nearrow$ -1 $-$ $\searrow$ 7 $+$ $\nearrow$

$f''(x) = 6x - 18 \overset{\text{SET}}{=} 0$

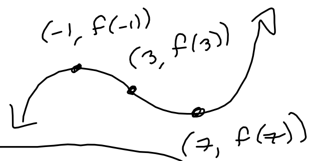
$6x = 18$

$x = 3$

f'': $-$ 3 $+$

Combine

-1 3 7



(c) Inflection

$$\begin{array}{r|rrrr} 3 & 1 & -9 & -21 & +4 \\ & & 3 & -18 & -117 \\ \hline & 1 & -6 & -39 & -113 \end{array}$$

$\boxed{\text{I.P. } (3, -113)}$

Concave up: $(3, \infty)$

c-down: $(-\infty, 3)$

$$\begin{array}{r} 13 \\ \times 9 \\ \hline 117 \end{array}$$

6. Question Details SCalc8 3.3.014. [3353783]

Consider the equation below. (If an answer does not exist, enter DNE.)

$f(x) = 8\cos^2(x) - 16\sin(x), \quad 0 \le x \le 2\pi$

(a) Find the interval on which f is increasing. (Enter your answer using interval notation.)

Find the interval on which f is decreasing. (Enter your answer using interval notation.)

(b) Find the local minimum and maximum values of f.

(c) Find the inflection points.

Find the interval on which f is concave up. (Enter your answer using interval notation.)

Find the interval on which f is concave down. (Enter your answer using interval notation.)

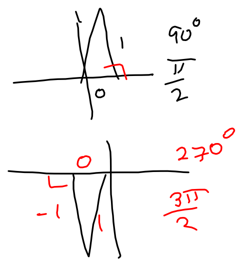
$f(x) = 8\cos^2 x - 16\sin x$ on $[0, 2\pi]$

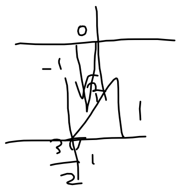
$f'(x) = (16\cos x)(-\sin x) - 16\cos x \overset{SET}{=} 0$

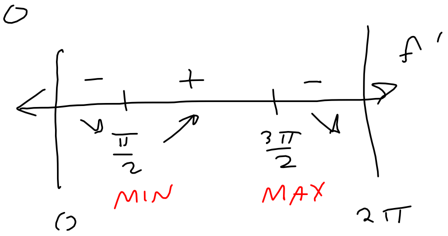
$\Rightarrow (\cos x)(\sin x) + \cos x = 0$

$\Rightarrow (\cos x)(\sin x + 1) = 0$

$\cos x = 0$ OR $\sin x = -1$







$f'\left(\frac{\pi}{4}\right) = 16\cos\left(\frac{\pi}{4}\right)\left(-\sin\left(\frac{\pi}{4}\right)\right) - 16\cos\left(\frac{\pi}{4}\right)$

$= \left(16\left(\frac{1}{\sqrt{2}}\right)\right)\left(-\frac{1}{\sqrt{2}}\right) - 16\left(\frac{1}{\sqrt{2}}\right)$

$= -8 - \frac{16}{\sqrt{2}} < 0$ —

$f'(\pi) =$ etc

$f'(\pi) > 0$

$f'\left(\frac{7\pi}{4}\right) < 0$

```
Y2(π/4)
          -19.3137085
Y2(π)
                   16
Y2(7π/4)
         -3.313708499
```

$f\left(\frac{\pi}{2}\right) = 8\cos^2\left(\frac{\pi}{2}\right) - 16\left(\frac{\pi}{2}\right)$

$= -16 \rightsquigarrow \boxed{\left(\frac{\pi}{2}, -16\right) \text{MIN}}$

$f\left(\frac{3\pi}{2}\right) = 8\cos^2\left(\frac{3\pi}{2}\right) - 16\sin\left(\frac{3\pi}{2}\right)$

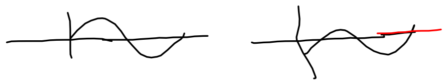
$= 0 + 16 \rightsquigarrow \boxed{\left(\frac{3\pi}{2}, 16\right) \text{MAX}}$

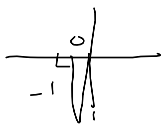
$f'(x) = (16\cos x)(-\sin x) - 16\cos x$

$= -16\sin x\cos x - 16\cos x$

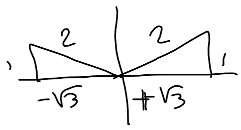
$$f''(x) = -16\cos x\cos x - (16\sin x)(-\sin x) + 16\sin x$$
$$= -16\cos^2 x + 16\sin^2 x + 16\sin x$$
$$= -16(1-\sin^2 x) + 16\sin^2 x + 16\sin x$$
$$= -16 + 16\sin^2 x + 16\sin^2 x + 16\sin x$$
$$= 32\sin^2 x + 16\sin x - 16 \overset{\text{SET}}{=} 0$$

$$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$$
$$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$$
$$\Rightarrow \sin x = \tfrac{1}{2} \quad \text{OR} \quad \sin x = -1$$



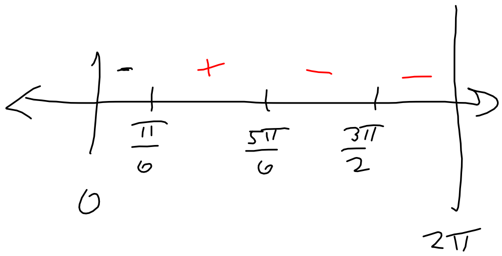


270° $\quad \frac{3\pi}{2}$



$30^\circ \quad \frac{\pi}{6}$

$180^\circ - 30^\circ = 150^\circ$ OR $\frac{5\pi}{6}$



Test: $x = 0, \frac{\pi}{2}, \pi, 2\pi$ for f'' sign pattern.

$$f''(0) = (2\sin(0) - 1)(\sin(0) + 1)$$
$$= (-1)(1) = -1$$

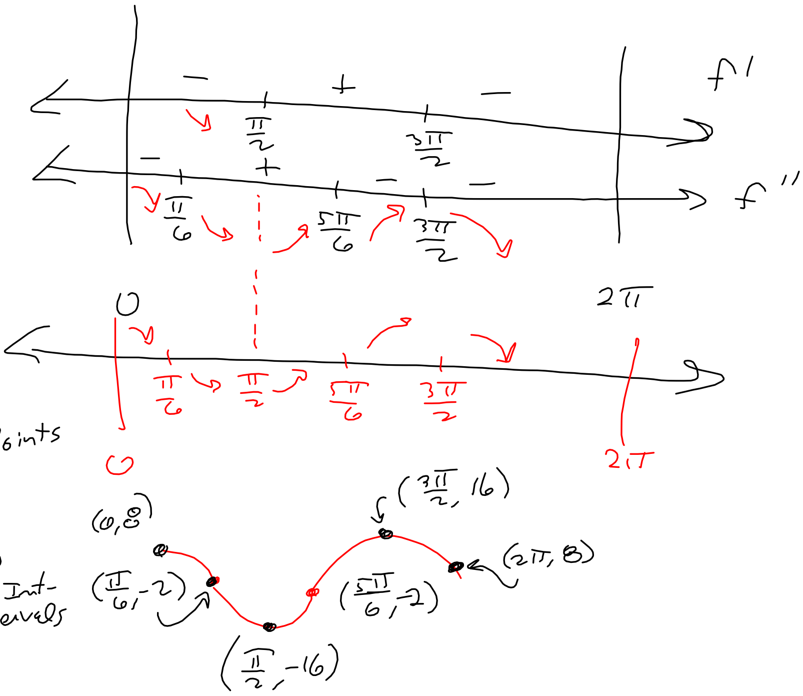
```
Y3(0π)
                -16
Y3(π/2)
                 32
Y3(π)
                -16
```

$= f''(0) = f''(2\pi)$

$f(\frac{\pi}{6}) = -2$

$f(\frac{5\pi}{6}) = -2$

$f(0) = f(2\pi) = 8$



ABS Max: $(\frac{3\pi}{2}, 16)$

ABS MIN: $(\frac{\pi}{2}, -16)$

I.P.: $(\frac{\pi}{6}, -2), (\frac{5\pi}{6}, -2)$

Points

C-up: $(\frac{\pi}{6}, \frac{5\pi}{6})$

C-Down: $(0, \frac{\pi}{6}), (\frac{5\pi}{6}, 2\pi)$

Intervals

Including endpoints is debatable.

7. Question Details SCalc8 3.3.015. [3353903]

Find the local maximum and minimum values of f using both the First and Second Derivative Tests.

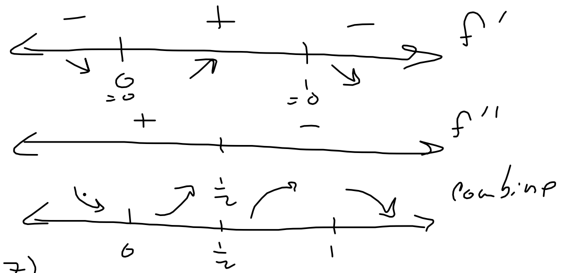
$f(x) = 3 + 12x^2 - 8x^3 = -8x^3 + 12x^2 + 3$

$f'(x) = -24x^2 + 24x \overset{\text{SET}}{=} 0 \Rightarrow$

$-24x(x-1) \Rightarrow x \in \{0, 1\}$

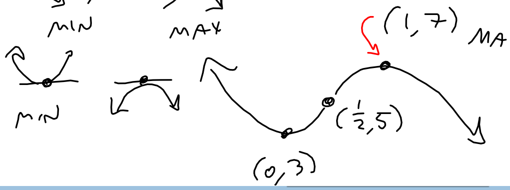
$f''(x) = -48x + 24 \overset{\text{SET}}{=} 0 \Rightarrow$

$-24(2x-1) = 0 \Rightarrow x \in \{\tfrac{1}{2}\}$



1st Deriv. MIN MAX

2nd Deriv: MIN



$$\begin{array}{c|cccc} \tfrac{1}{2} & -8 & 12 & 0 & 3 \\ & & -4 & 4 & 2 \\ \hline & -8 & 8 & 4 & 5 \end{array}$$

8. Question Details SCalc8 3.3.018. [3354064]

(a) Find the critical numbers of the function $f(x) = x^6(x - 2)^5$.

(b) What does the Second Derivative Test tell you about the behavior of f at these critical numbers?

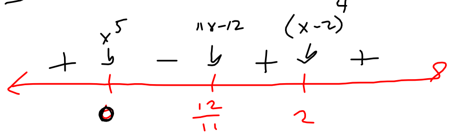
(c) What does the First Derivative Test tell you that the Second Derivative test does not? (Enter your answers from smallest to largest x value.)

a) $f(x) = x^6(x-2)^5 \Rightarrow f'(x) = 6x^5(x-2)^5 + x^6(5(x-2)^4)$

$= x^5(x-2)^4[6(x-2) + x(5)] = x^5(x-2)^4[6x - 12 + 5x]$

$= x^5(x-2)^4[11x - 12] \overset{\text{SET}}{=} 0$

$\Rightarrow x \in \{0, \tfrac{12}{11}, 2\}$ a)



(b) $f''(x) = 5x^4(x-2)^4(11x-12) + 4x^5(x-2)^3(11x-12) + x^5(x-2)^4(11)$

$= x^4(x-2)^3\left(5(x-2)(11x-12) + 4x(11x-12) + 11x(x-2)\right)$

$= x^4(x-2)^3\left((5x-10)(11x-12) + 44x^2 - 48x + 11x^2 - 22x\right)$

$= x^4(x-2)^3\left(55x^2 - 60x - 110x + 120 + 55x^2 - 70x\right)$

$= x^4(x-2)^3(110x^2 - 240x + 120)$

$= 10x^4(x-2)^3(11x^2 - 24x + 12)$ SET $= 0$

$a = 11, b = -24, c = 12$

$b^2 - 4ac = (-24)^2 - 4(11)(12) = 48$

$x = \dfrac{24 \pm 4\sqrt{3}}{2(11)} = \dfrac{\cancel{4}^{2}(6 \pm \sqrt{3})}{\cancel{2}(11)} = \dfrac{12 \pm 2\sqrt{3}}{11}$

$2|48$, $2|24$, $2|12$, $2|6$, 3

$\sqrt{48} = 4\sqrt{3}$

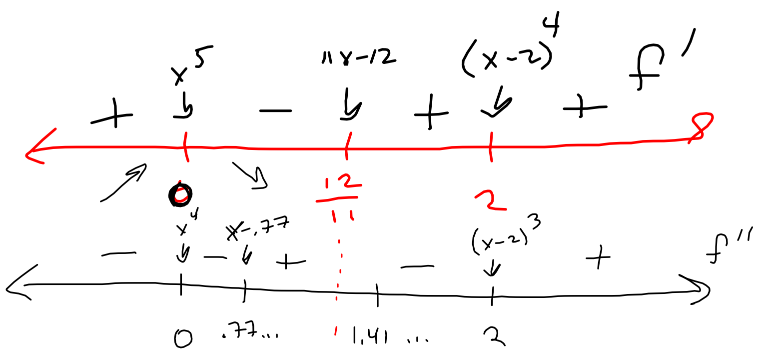
$\dfrac{12}{11} + \dfrac{2}{11}\sqrt{3},\ \dfrac{12}{11} - \dfrac{2}{11}\sqrt{3} \Rightarrow$ EXACT $\left(x - \dfrac{12+2\sqrt{3}}{11}\right)\left(x - \dfrac{12-2\sqrt{3}}{11}\right)$

1.405827420, 0.7759907623

$f'' = 0 \Rightarrow x = 0, 2, \dfrac{12 \pm 2\sqrt{3}}{11}$

Rough

$(x - .77)(x - 1.41)$



$= 10x^4(x-2)^3(11x^2 - 24x + 12)$

x^9 ...

2nd Deriv Test:

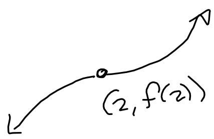
$x=0$ $f''=0$ NADA

$x=\frac{12}{11}$ $f''>0$ MIN

$x=2$ $f''=0$ NADA (Although f'' changes sign, there, so it's an inflection point

The 1st Deriv. Test gives us

max @ $x=0$, No help from f''



Terrace @ $x=2$, No help from f''

min @ $x=\frac{12}{11}$, which f'' confirmed.

Oct 10-10:21 AM

9. Question Details SCalc8 3.3.019.MI. [3354044]

Suppose f'' is continuous on $(-\infty, \infty)$.

(a) If $f'(-5) = 0$ and $f''(-5) = 3$, what can you say about f?

$f(-5)$ is a min !

$(-5, f(-5))$:

$f'(-5) = 0$

$f''(-5) = 3$

$3 > 0$

$+$

(b) If $f'(-2) = 0$ and $f''(-2) = 0$, what can you say about f?

$f''(-2) = 0$ No help.

$f'(-2) = 0$

which?

Can't say anything about f.

No way of telling

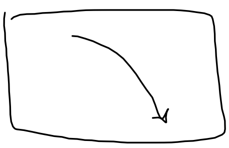
10. Question Details SCalc8 3.3.020. [3353922]

Sketch the graph of a function that satisfies all of the given conditions.

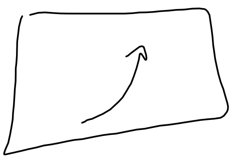
(a) $f'(x) < 0$ and $f''(x) < 0$ for all x

(b) $f'(x) > 0$ and $f''(x) > 0$ for all x

ⓐ $f'(x) < 0$ and $f''(x) < 0$ $\forall x$



ⓑ $f'(x) > 0$ and $f'' > 0$ $\forall x$



11. Question Details SCalc8 3.3.024. [3353709]

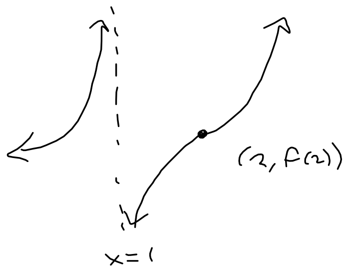
Sketch the graph of a function that satisfies all of the given conditions.

$f'(x) > 0$ for all $x \neq 1$,
vertical asymptote $x = 1$,
$f''(x) > 0$ if $x < 1$ or $x > 2$,
$f''(x) < 0$ if $1 < x < 2$

f': + ↗ | ↗ + ↗

+ | − | +

1 2



12. Question Details SCalc8 3.3.033. [3354084]

Consider the function below. (If an answer does not exist, enter DNE.)

$f(x) = x^3 - 27x + 3$

$f'(x) = 3x^2 - 27 \overset{SET}{=} 0 \implies$

$3(x^2 - 9) = 0$

$x^2 = 9$

$x = \pm 3$

f': + −3 − 3 +

(a) Find the interval of increase. (Enter your answer using interval notation.)

Inc: $(-\infty, 3) \cup (3, \infty)$

Find the interval of decrease. (Enter your answer using interval notation.)

Dec: $(-3, 3)$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$(3, -51)$

$f(3)$ MIN $(3, -51)$

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$(-3, 57)$

$f(-3)$ MAX $(-3, 57)$

$f''(x) = 6x = 0 \implies x = 0$

$f(3) = 3^3 - 27(3) + 3$
$(-27)(2) + 3 = -51$

$f(-3) = -27 - (27)(-3) + 3$
$(27)(2) + 3 = 57$

(c) Find the inflection point.

$(0, 3)$

f'': − 0 +

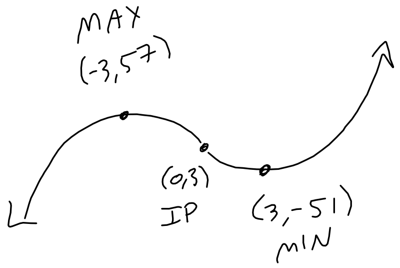
Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$(0, \infty)$

Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

$(-\infty, 0)$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.



13. Question Details SCalc8 3.3.037. [3353913]

Consider the function below. (If an answer does not exist, enter DNE.)

$h(x) = (x + 1)^9 - 9x - 3$

(a) Find the interval of increase. (Enter your answer using interval notation.)

$(-\infty,-2)\cup(0,\infty)$

Find the interval of decrease. (Enter your answer using interval notation.)

$(-2,0)$

(b) Find the local minimum value(s). (Enter your answers as a comma-separated list.)

$(0,-2)$

Find the local maximum value(s). (Enter your answers as a comma-separated list.)

$(-2,14)$

(c) Find the inflection point.

$(-1,12)$

Find the interval where the graph is concave upward. (Enter your answer using interval notation.)

$(-1,\infty)$

Find the interval where the graph is concave downward. (Enter your answer using interval notation.)

$(-\infty,-1)$

(d) Use the information from parts (a)-(c) to sketch the graph. Check your work with a graphing device if you have one.

$h'(x) = 9(x+1)^8 - 9 \overset{SET}{=} 0$

$\Rightarrow 9(x+1)^8 = 9$

$\Rightarrow (x+1)^8 = 1$

$\Rightarrow x+1 = \pm 1 \Rightarrow x = -1 \pm 1$

$\Rightarrow x \in \{0, -2\}$

$+ \quad - \quad +$ f'

$-2 \quad 0$

$f(-2) = (-1)^9 - 9(-2) - 3$
$= -1 + 18 - 3 = 14$

$f(0) = 1^9 - 9(0) - 3$
$= 1 - 3 = -2$

$h'(x) = 9(x+1)^8 - 9 \Rightarrow$

$h''(x) = 72(x+1)^7 \overset{SET}{=} 0$

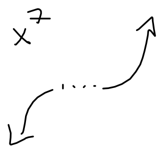
$\Rightarrow x = -1$

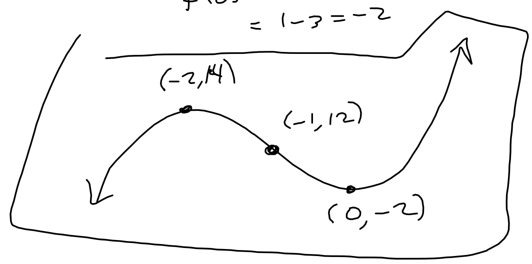
$- \quad +$ f''

-1

$f(-1) = (-1+1)^9 - 9(-1) + 3$
$= 9 + 3 = 12$

$(-1, 12)$ IP





$(x+1)^8 - 1 = u^8 - 1$

$= (u^4-1)(u^4+1)$

$= (u^2-1)(u^2+1)(u^4+1)$

$= (u-1)(u+1)(u^2+1)(u^4+1)$

$= (x+1-1)(x+1+1)((x+1)^2+1)((x+1)^4+1)$

$= (x)(x+2)(\ldots) = 0 \Rightarrow$

14. Question Details SCalc8 3.3.038. [3354076]

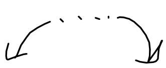
Consider the function below. (If an answer does not exist, enter DNE.)

$h(x) = 5x^3 - 3x^5$ is ODD. $f(-x) = -f(x)$

More Test-Like Instructions:

Sketch the graph of $h(x)$, showing all max's, min's, and inflection points.

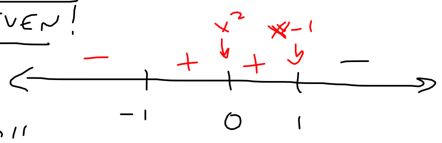
$h'(x) = 15x^2 - 15x^4 = \boxed{-15x^4 + 15x^2}$ SET $= 0 \Rightarrow$ $-15x^4$



$-15x^2(x^2-1) = 0$ is EVEN!

$\Rightarrow -15x^2(x-1)(x+1) = 0$

$\Rightarrow x \in \{-1, 0, 1\}$



ODD is f''

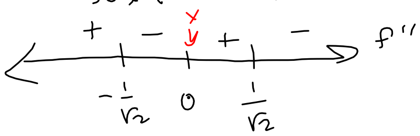
$h''(x) = -60x^3 + 30x$ SET $= 0$

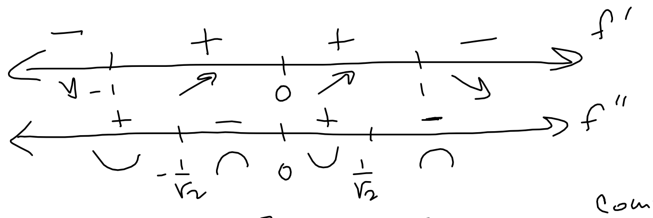
$-30x(2x^2-1) = -30x(\sqrt{2}x - 1)(\sqrt{2}x + 1)$

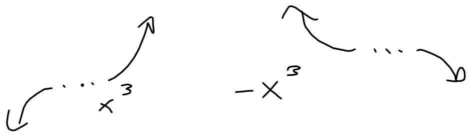
$\sqrt{2}x - 1 = 0$

$x = \frac{1}{\sqrt{2}}$

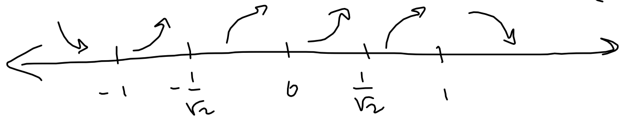
$-60x^3$

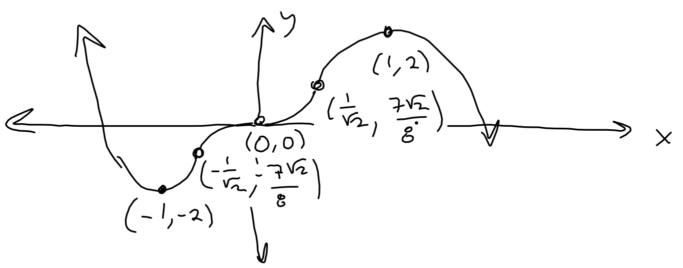






Combine





Question Details SCalc8 3.3.039. [3353955]

Consider the function below. (If an answer does not exist, enter DNE.)

$F(x) = x\sqrt{9-x}$

Need $9 - x \geq 0 \Rightarrow 9 \geq x$, i.e. $x \leq 9$

More Test-Like Instructions:

Sketch the graph of $h(x)$, showing all max's, min's, and inflection points.

$F(x) = x(9-x)^{\frac{1}{2}} \Rightarrow$

$F'(x) = (9-x)^{\frac{1}{2}} + x\left(\frac{1}{2}(9-x)^{-\frac{1}{2}}\right)(-1)$

$= \frac{(9-x)^{\frac{1}{2}}}{1} \cdot \frac{2(9-x)^{\frac{1}{2}}}{2(9-x)^{\frac{1}{2}}} - \frac{x}{2(9-x)^{\frac{1}{2}}}$

$= \frac{2(9-x) - x}{2(9-x)^{\frac{1}{2}}} = \frac{18 - 2x - x}{2(9-x)^{1/2}}$

$= \frac{18-3x}{2(9-x)^{\frac{1}{2}}}$ SET 0 $\Rightarrow x = 6$

$\{6, 9\}$

Denom $= 0 \Rightarrow x = 9$

$= (9-x)^{\frac{1}{2}} - x(9-x)^{-\frac{1}{2}} \Rightarrow$

$F''(x) = \frac{1}{2}(9-x)^{-\frac{1}{2}}(-1) - (9-x)^{-\frac{1}{2}} + x\left(-\frac{1}{2}(9-x)^{-\frac{3}{2}}\right)$

$= -\frac{1}{2(9-x)^{\frac{1}{2}}} - \frac{1}{(9-x)^{\frac{1}{2}}} \cdot \frac{2}{2} +$

$= \frac{-3}{2(9-x)^{\frac{1}{2}}} - \frac{x}{2(9-x)^{\frac{3}{2}}}$

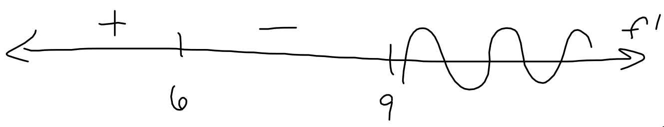
$= \frac{-3(9-x) - x}{2(9-x)^{3/2}}$

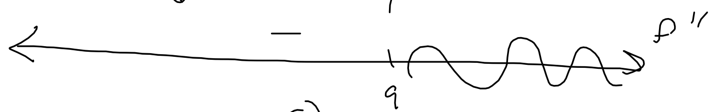
$= \frac{-27 + 3x - x}{} = \frac{2x - 27}{2(9-x)^{3/2}}$

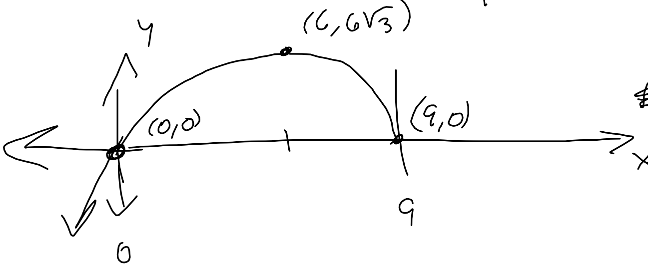
$f'' < 0$ on $D(f)$

$x = \frac{27}{2}$ 9

$\notin D$







$F(6) = 6\sqrt{9-6}$

$= 6\sqrt{3}$

16. Question Details SCalc8 3.3.041. [3353678]

Consider the function below. (If an answer does not exist, enter DNE.)

$C(x) = x^{1/3}(x + 4)$

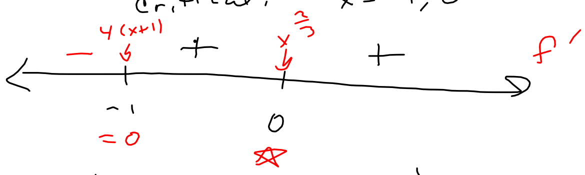
More Test-Like Instructions:

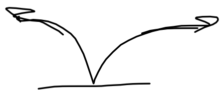
Sketch the graph of $h(x)$, showing all max's, min's, and inflection points.

$$C'(x) = \frac{1}{3}x^{-\frac{2}{3}}(x+4) + x^{\frac{1}{3}} = \frac{x+4}{3x^{2/3}} + \frac{x^{\frac{1}{3}}}{1}\cdot\frac{3x^{2/3}}{3x^{2/3}} = \frac{x+4+3x}{3x^{2/3}}$$

$$= \frac{4x+4}{3x^{2/3}} \quad \underset{=}{\text{SET}}\ 0 \Rightarrow x = -1 \qquad \text{SET} = \not{\exists} \Rightarrow x = 0$$

critical: $x = -1, 0$

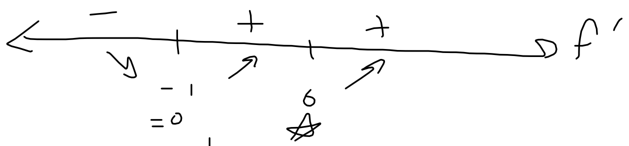


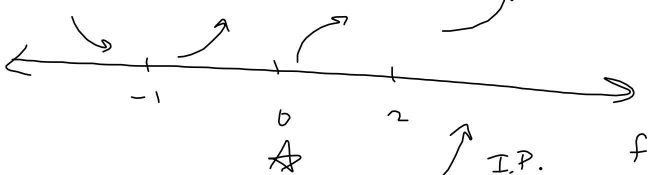


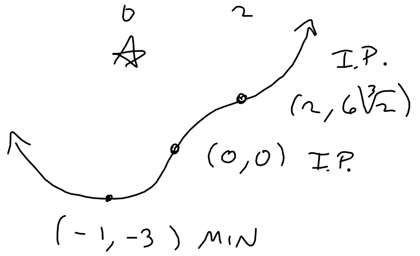
$$C'(x) = \frac{4x}{3x^{2/3}} + \frac{4}{3x^{2/3}} = \frac{4x^{\frac{1}{3}}}{3} + \frac{4x^{-\frac{2}{3}}}{3} \Rightarrow$$

$$C''(x) = \frac{4}{9}x^{-\frac{2}{3}} - \frac{8}{9}x^{-\frac{5}{3}} = \frac{4}{9x^{2/3}}\cdot\frac{x}{x} - \frac{8}{9x^{5/3}} = \frac{4x-8}{9x^{5/3}} \quad \underset{=}{\text{SET}}\ 0 \Rightarrow x = 2$$

$$\underset{=}{\text{SET}}\ \not{\exists} \Rightarrow x = 0$$







$f(x) = x^{\frac{1}{3}}(x+4)$

$f(-1) = (-1)^{\frac{1}{3}}(-1+4) = -3$

$f(0) = 0^{\frac{1}{3}}(0+4) = 0$

$f(2) = 2^{\frac{1}{3}}(2+4) = \sqrt[3]{2}\,(6)$

Inc: $(-1, \infty)$

Dec: $(-\infty, -1)$

C-up: $(-\infty, 0) \cup (2, \infty)$

C-Down $(0, 2)$

17. Question Details

Consider the function below. (If an answer does not exist, enter DNE.)

$f(\theta) = 2\cos(\theta) + \cos^2(\theta), \quad 0 \le \theta \le 2\pi$

More Test-Like Instructions:

Sketch the graph of $h(x)$, showing all max's, min's, and inflection points.

$f'(\theta) = -2\sin\theta + 2\cos\theta\sin\theta = 2\cos\theta\sin\theta - 2\sin\theta$

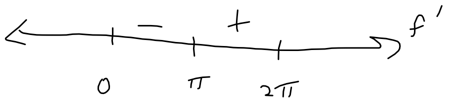
$= 2\sin\theta(\cos\theta - 1) \overset{SET}{=} 0$

$\sin\theta = 0 \qquad \cos\theta = 1$

$\theta = 0, \pi, 2\pi \qquad \theta = 0, 2\pi$

$\frac{\pi}{2} \quad (2\sin\frac{\pi}{2})(\cos\frac{\pi}{2} - 1) = -2$

$\frac{3\pi}{2} \quad (2\sin\frac{3\pi}{2})(\cos(\frac{3\pi}{2}) - 1) = 2$



$f''(x) = -2\sin^2\theta + 2\cos^2\theta - 2\cos\theta$

$= -2(1 - \cos^2\theta) + 2\cos^2\theta - 2\cos\theta$

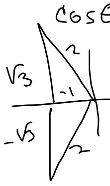
$= -2 + 2\cos^2\theta + 2\cos^2\theta - 2\cos\theta$

$= 4\cos^2\theta - 2\cos\theta - 2 \overset{SET}{=} 0$

$\Rightarrow 2\cos^2\theta - \cos\theta - 1 = 0$

$\Rightarrow (2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ OR $\cos\theta = 1$



$120°, \frac{2\pi}{3}$

$240°, \frac{4\pi}{3}$

$\theta = 0, 2\pi$

f''

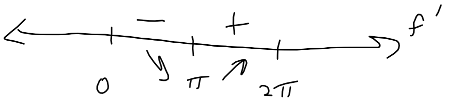
$(2\cos\pi + 1)(2\cos\pi - 1)$

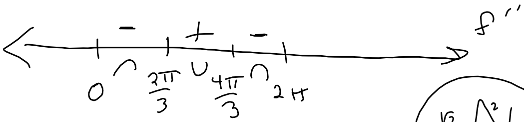
$(2(-1) + 1)(-2 - 1)$

$= (-1)(-3) = 3$

$(2\cos(\frac{3\pi}{2}) + 1)(2\cos\frac{3\pi}{2} - 1)$

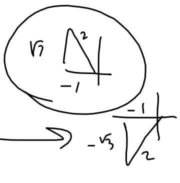
$= (1)(-1)$





$f(\theta) \quad 2\cos\theta + \cos^2\theta$

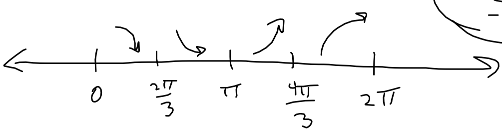
$f(0) = 2 + 1 = 3$



$f(\frac{2\pi}{3}) = 2\cos(\frac{2\pi}{3}) + \cos^2(\frac{2\pi}{3})$

$= 2(-\frac{1}{2}) + (-\frac{1}{2})^2$

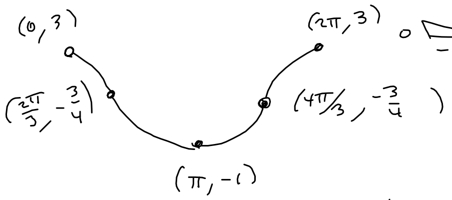
$= -1 + \frac{1}{4} = -\frac{3}{4}$



$f(\pi) = 2\cos\pi + \cos^2\pi$

$= 2(-1) + (-1)^2$

$= -2 + 1 = -1$



Dec: $(0, \pi)$

Inc: $(\pi, 2\pi)$

C-up: $(\frac{2\pi}{3}, \frac{4\pi}{3})$

C-down $(0, \frac{2\pi}{3}) \cup (\frac{4\pi}{3}, 2\pi)$

18. Question Details SCalc8 3.3.045.MI. [3353710]

Suppose the derivative of a function f is $f'(x) = (x+2)^6(x-4)^5(x-5)^8$. On what interval is f increasing? (Enter your answer in interval notation.)

This one is pretty quick and good for a test.

$\sqrt{\ (x+2)^6(x-4)^5(x-5)^8}$

$(x+2)^6(x-4)^5(x-5)^8$

$(x+2)^6 \quad (x-4)^5 \quad (x-5)^8$

f': $-$ | $-$ | $+$ | $+$ at -2, 4, 5 ($=0$, $=0$, $=0$)

Inc: $(4, \infty)$

Dec: $(-\infty, 4)$

-2 5

19. Question Details SCalc8 3.3.047. [3353988]

Consider the function below.

$$f(x) = \frac{x+1}{\sqrt{x^2+1}}$$

(a) Find the exact value of the maximum of f.

(b) Find the exact value of x at which f increases most rapidly.

(a) $$f'(x) = \frac{1(x^2+1)^{1/2} - (x+1)\left(\frac{1}{2}(x^2+1)^{-1/2}(2x)\right)}{x^2+1}$$

$$= \frac{1}{x^2+1}\left[\frac{(x^2+1)^{\frac{1}{2}}}{1}\cdot\frac{(x^2+1)^{\frac{1}{2}}}{(x^2+1)^{\frac{1}{2}}} - \frac{(x+1)(x)}{(x^2+1)^{1/2}}\right]$$

$$= \frac{1}{(x^2+1)^{3/2}}\left[x^2+1-x^2-x\right]$$

$$= \frac{1}{(\ )^{3/2}}\left[1-x\right]$$

SET $=0 \implies x=1$

set $= \nexists \implies$ Nevah!

$+$ | $-$ at 1

$$f(1) = \frac{1+1}{\sqrt{1^2+1}} = \boxed{\frac{2}{\sqrt{2}} = MAX \text{ @ } x=1}$$

(b) $$\frac{1-x}{\sqrt{x^2+1}^{\,3}} = f'(x) \implies$$

$$f''(x) = \frac{-1(x^2+1)^{\frac{3}{2}} - (1-x)\left(\frac{3}{2}(x^2+1)^{\frac{1}{2}}(2x)\right)}{\left((x^2+1)^{3/2}\right)^2}$$

$$= \frac{1}{(x^2+1)^3}\left[-(x^2+1)^{\frac{3}{2}} - (1-x)(3)(x^2+1)^{\frac{1}{2}}x\right]$$

$$= \frac{1}{(x^2+1)^3}\left[(x^2+1)^{\frac{1}{2}}\left[-x^2-1-3x(1-x)\right]\right]$$

$$= \frac{(x^2+1)^{\frac{1}{2}}}{(x^2+1)^3}\left[-x^2-1-3x+3x^2\right] = \boxed{\Sigma}\left[2x^2-3x-1\right]$$

$2x^2-3x-1=0$

$a=2,\ b=-3,\ c=-1$

$b^2-4ac = 9-4(2)(-1)$

$= 17$

$x = -\frac{1}{4},\ 1$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

$$\frac{3+\sqrt{17}}{2}$$

f': $+$ | $-$ at 1

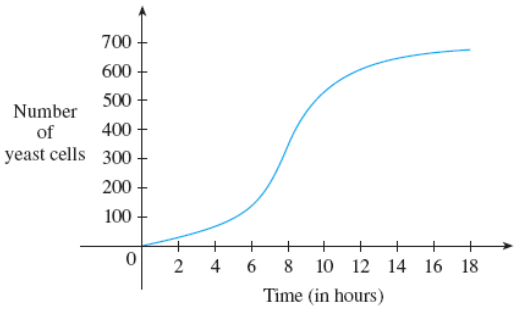
f'': $+$ | $-$ | $+$ at $\frac{3-\sqrt{17}}{2}$, $\frac{3+\sqrt{17}}{2}$

$\boxed{x = \frac{3-\sqrt{17}}{2}}$ is where $f' =$ Max

Oct 8-9:55 PM

20. Question Details SCalc8 3.3.053. [3353734]

A graph of a population of yeast cells in a new laboratory culture as a function of time from $t = 0$ to $t = 18$ is shown.



(a) Describe how the rate of population increase varies.

(b) At what point is the rate of population increase the greatest?

(c) On what interval is the population function concave upward? (Enter your answer using interval notation.)

On what interval is the population function concave downward? (Enter your answer using interval notation.)