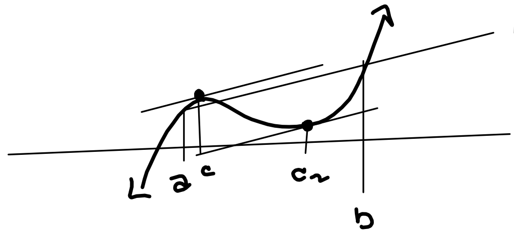
Mean Value Theorem

f cnt^s on $[a,b]$, f difb^l on (a,b)

$\Rightarrow \exists\, c \in (a,b) \ni f'(c) = \frac{f(b)-f(a)}{b-a}$



Proof

Define $g(x) = f(x) - f(a) - \frac{f(b)-f(a)}{b-a}(x-a)$

Then $g(x)$ is cnt^s on $[a,b]$ & difb^l on (a,b) because f is and b/c $x-a$ is cnt^s & difb^l everywhere.

Notice $g(a) = f(a) - f(a) - \frac{f(b)-f(a)}{b-a}(a-a) = 0$

$g(b) = f(b) - f(a) - \frac{f(b)-f(a)}{b-a}(b-a)$

$= f(b) - f(a) - f(b) + f(a) = 0$

So g satisfies Rolle's hypotheses!

$\Rightarrow \exists\, c \in (a,b) \ni g'(c) = 0 \Rightarrow$

$g'(c) = f'(c) - \frac{f(b)-f(a)}{b-a} = 0 \Rightarrow$

$f'(c) = \frac{f(b)-f(a)}{b-a}$ ■