1. SCalc8 3.2.005. (3354209)

Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers *c* that satisfy the conclusion of Rolle's Theorem.

$f(x) = 2x^2 - 4x + 5, \quad [-1, 3]$

$f(-1) = 2(-1)^2 - 4(-1) + 5$
$= 2 + 4 + 5 = 11$

$f(3) = 2(3)^2 - 4(3) + 5$
$= 18 - 12 + 5 = 11$ ✓

f is poly. ⇒ cnt^s & difbl ∀ $x \in \mathbb{R}$. So on $[-1,3]$ & $(-1,3)$, respectively.

$f'(x) = 4x - 4 \overset{SET}{=} 0 \Rightarrow$

$\Rightarrow x = \boxed{1 = c}$

2. SCalc8 3.2.007. (3354404) *randomized*

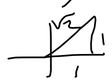
Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers *c* that satisfy the conclusion of Rolle's Theorem.

$f(x) = \sin\left(\frac{x}{2}\right), \quad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

π

sine is trig. & trigs cnt^s & difbl on the is D, so, on $[\frac{\pi}{2}, \frac{3\pi}{2}]$ & $(\frac{\pi}{2}, \frac{3\pi}{2})$, respec.

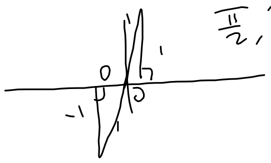
$f(\frac{\pi}{2}) = \sin\left(\frac{\frac{\pi}{2}}{2}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$



$f(\frac{3\pi}{2}) = \sin\left(\frac{\frac{3\pi}{2}}{2}\right) = \sin\frac{3\pi}{4} =$
$= \frac{1}{\sqrt{2}}$ ✓

$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right) \overset{SET}{=} 0$

$\Rightarrow \cos\left(\frac{x}{2}\right) = 0$



$\frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow$

$x = \pi, 3\pi$
$= c$

3. SCalc8 3.2.009. (3354472) slightly modified

Consider the following function.

$f(x) = 1 - x^{2/3}$

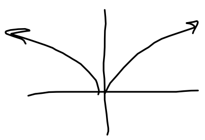
Find $f(-1)$ and $f(1)$.

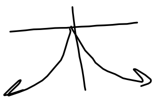
$f(1) = 1 - 1^{2/3} = 0$ ✓ $f(a) = f(b)$ ✓

$f(-1) = 1 - (-1)^{2/3} = 1 - \left((-1)^2\right)^{\frac{1}{3}} = 0$ ✓

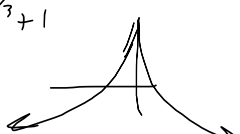
Find all values *c* in $(-1, 1)$ such that $f'(c) = 0$. (Enter your answers as a comma-separated list.)

Based off of this information, what conclusions can be made about Rolle's Theorem?





$-x^{2/3} + 1$



$f'(x) = -\frac{2}{3}x^{-\frac{1}{3}} \overset{SET}{=} 0$

$\frac{-2}{3x^{-\frac{1}{3}}} = 0$

$~~-2 = 0~~$

Says NADA about Rolle's b/c f(x) isn't difbl @ $x = 0 \in (-1, 1)$, so Rolle's doesn't apply

4. SCalc8 3.2.011. (3354228) *ntly modified*

Does the function satisfy the hypotheses of the Mean Value Theorem on the given interval?

$f(x) = 2x^2 - 3x + 1$, $[0, 2]$ f cont's & dif'ble EVERYWHERE!

If it satisfies the hypotheses, find all numbers c that satisfy the conclusion of the Mean Value Theorem. (Enter your answers as a comma-separated list.

$$\frac{f(2)-f(1)}{2-1} = \frac{2(2)^2-3(2)+1-\left(2(1)^2-3(1)+1\right)}{1}$$

$$= 8-6+1-(2-3+1) = 3-0 = 3 = m_{AVG}$$

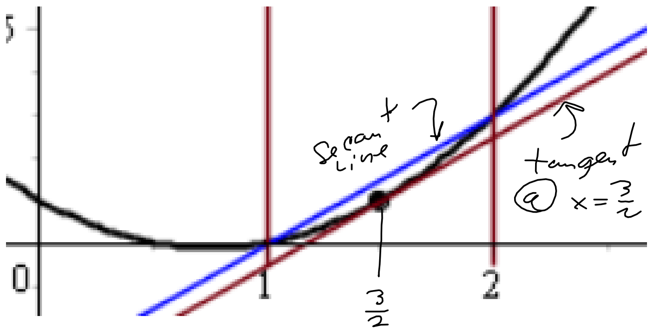
SET $4x-3 = f'(x)$

$\Rightarrow 4x-3=3$

$\Rightarrow 4x=6$

$x=\frac{6}{4}=\boxed{\frac{3}{2}=c}$

Wrong Interval: [1,2]
wanted [0,2]



5. SCalc8 3.2.015. (3354541)

Find the number c that satisfies the conclusion of the Mean Value Theorem on the given interval. (Enter your answers

$f(x) = \sqrt{x}$, $[0, 4]$ $\sqrt{x}$ cont's on $[0,\infty)$ & dif'ble on $(0,\infty)$

Graph the function, the secant line through the endpoints, and the tangent line at $(c, f(c))$.

Are the secant line and the tangent line parallel?

$y=\frac{1}{2}(x-1)+1$

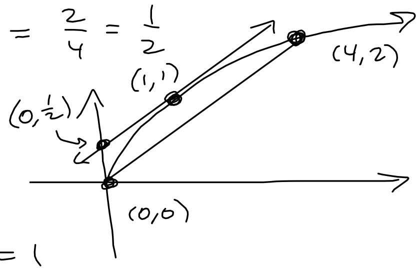
$y=\frac{1}{2}x+\frac{1}{2}$

$x^{\frac{1}{2}}=f(x)$ $\frac{f(4)-f(0)}{4-0} = \frac{\sqrt{4}-0}{4} = \frac{2}{4} = \frac{1}{2}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$ SET $\frac{1}{2}$

$\Rightarrow x^{-\frac{1}{2}}=1$

$\Rightarrow x^{\frac{1}{2}}=1 \Rightarrow x=1$



6. SCalc8 3.2.019. (3354496) ay

Show that the equation has exactly one real root.

$3x + \cos(x) = 0$

cnt's

$3(-1) + \cos(-1) \le -2$
$3(1) + \cos(1) \ge 2$
$\exists\, c \in (-1,1) \ni f(c) = 0$ by IVT.

If slope is positive, then it can't cross more than once, by continuity & dif'bl

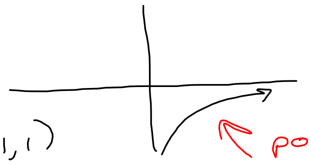
$f'(x) = 3 - \sin(x) \ge 2 > 0$

So no more than one.

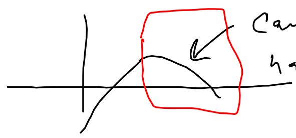
$\Rightarrow$ Exactly one!

Positive slope's not enough

$f(x) = -\frac{1}{x}$ on $(0, \infty)$



pos. slope. Never touches x-axis.



Can't happen

$f' > 0$

7. SCalc8 3.2.021. (3354368) ay

Show that the equation $x^3 - 14x + c = 0$ has at most one root in the interval $[-2, 2]$.

?

$f'(x) = 3x^2 - 14 \overset{SET}{=} 0$

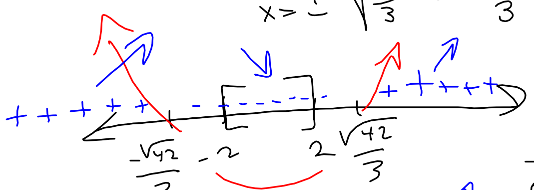
$x^2 = \frac{14}{3}$ $\pm$

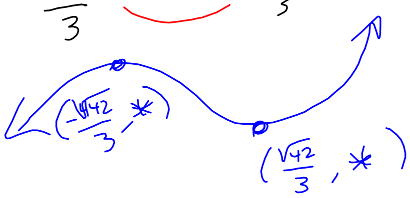
$x = \pm\sqrt{\frac{14}{3}} = \frac{\sqrt{42}}{3}$

$\frac{\sqrt{42}}{3} > 2$

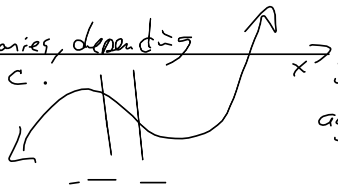
$-\frac{\sqrt{42}}{3} < -2$

So, f is decreasing on all of $[-2, 2]$





varies, depending on c.

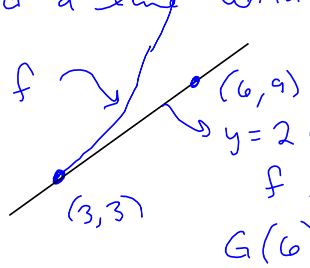


It has to increase in order to cross, again, assuming it crossed!

8. SCalc8 3.2.025. (3354550)

If $f(3) = 3$ and $f'(x) \ge 2$ for $3 \le x \le 6$, how small can $f(6)$ possibly be?

Build a line with slope $m=2$, passing thru $(3,3)$



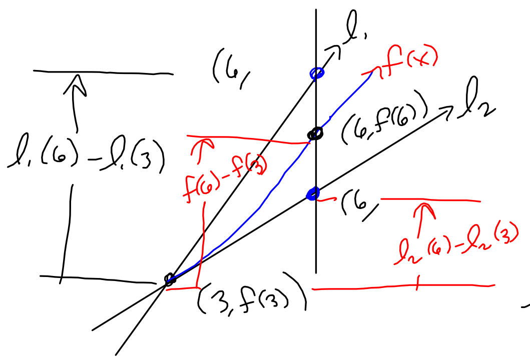
$y = 2(x-3)+3 = G(x)$

f lives ABOVE this line

$G(6) = 2(6-3)+3 = 2(3)+3 = \boxed{9 \le f(6)}$

9. SCalc8 3.2.026. (3354177)

Suppose that $2 \le f'(x) \le 4$ for all values of x. What are the minimum and maximum possible values of $f(6) - f(3)$?



$l_1(x) = 4(x-3) + f(3)$

$l_1(6) - l_1(3) =$

$4(6-3) + f(3) - \left(4(3-3) + f(3)\right)$

$= 4(3) + f(3) - 0 - f(3) = \boxed{12 \text{ MAX}}$

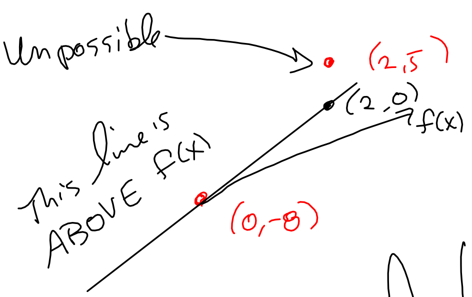
$l_2(x) = 2(x-3) + f(3)$

$l_2(6) - l_2(3) = 2(6-3) - \left(2(3-3)\right)$

$= 2(3) = \boxed{6 = \text{MIN}}$

10. SCalc8 3.2.027. (3354094)

Does there exist a function f such that $f(0) = -8$, $f(2) = 5$, and $f'(x) \le 4$ for all x?



$l(x) = 4(x-0) - 8$

$= 4x - 8$

$l(2) = 4(2) - 8 = 0$

NO!

11. SCalc8 3.2.034. (3354242)

At 2:00 PM a car's speedometer reads 30 mi/h. At 2:30 PM it reads 50 mi/h. Show that at some time between 2:00 and 2:30 the acceleration is exactly 40 mi/h^2.

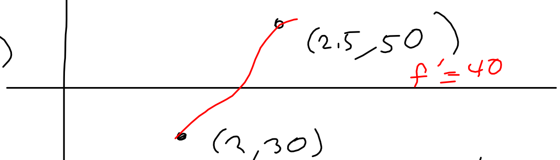
$(2, 30)$, $(2.5, 50)$

$= (2, f'(2))$, $(2.5, f'(2.5))$

$(2.5, f(2.5))$

$(2, f(2))$

Hmmm. Treat f' as "the function."



Assuming that speed is "smooth," this is an IVT question.

→ Continuous Derivative.

IVT applied to f' = velocity/speed.