What's the deal on all this "continuous on closed interval" and "differentiable on the open interval" stuff? How do we *know* if a function's continuous or not?

BY GETTING A HANDLE ON ALL THE DOMAIN STUFF FROM PREVIOUS CLASSES!!! BASICALLY, IF IT'S IN THE DOMAIN, THEN *F* IS CONTINUOUS THERE, FOR JUST ABOUT ANY FUNCTION WE CAN WRITE, DAGNABBIT!!!

Continuity & Differentiability

$\mathcal{D}(f)$ & $\mathcal{D}(f')$

$\mathbb{Z}$

$\mathbb{Q}$ $\frac{r}{s}$

$\mathcal{D}$ = Domain = $\{x \mid f(x) \text{ is defined}\}$

$\mathbb{R}$

Pretty much everything IS defined, except...

$\sqrt[2n]{\text{Negative}}$ $\qquad \frac{\text{Stuff}}{0}$ $\qquad \forall n \in \mathbb{Z}$

"for x such that x is an integer."

$\forall x \in \mathbb{Z}$

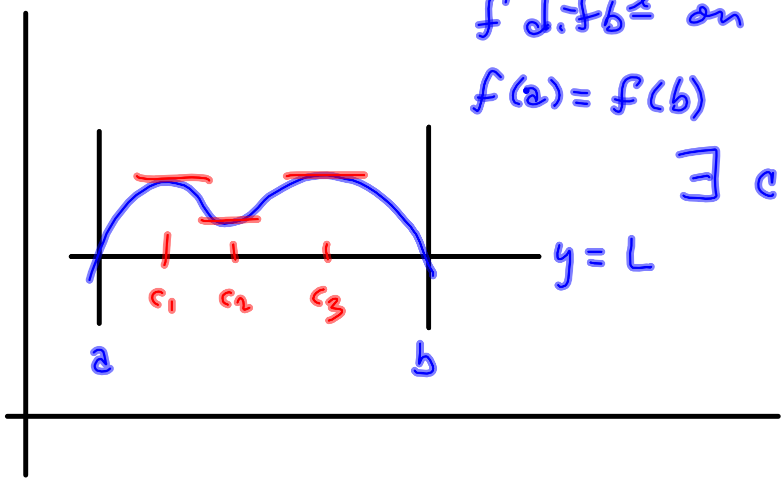
is an element/member of

§3.1 #14
I dropped a sign.

§3.1 & 3.2
Understanding the hypotheses of the theorems.

FINDING that pesky 'c.'

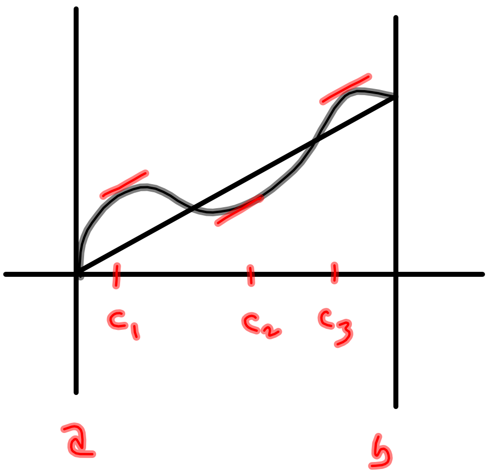
Rolle's Theorem - f cnt's on $[a,b]$,
f dif'ble on (a,b), and
$f(a) = f(b)$ $\Longrightarrow$
$\exists\, c \in (a,b) \ni f'(c) = 0.$



MVT - $\boxed{\begin{array}{l} f \text{ cnt}^s \text{ on } [a,b], \\ f \text{ difbl on } (a,b) \end{array}} \Longrightarrow$

$\exists\, c \in (a,b) \ni f'(c) = m_{avg}$ on $[a,b]$.

$$m_{avg} = \frac{f(b)-f(a)}{b-a}$$



Continuity – Domain for the most part
'Most any function we will see is continuous on its domain.

$$\lim_{x\to a} f(x) = f(a) \iff \text{cnt}^{\underline{s}} \text{ @ } a.$$

$\mathcal{D}$: Division by zero
$\sqrt[2n]{\text{negative}}$
} only trouble areas.

Differentiability – Domain of f'.
Watch out! $(g(x))^m$, where $0 < m < 1$
f is $\text{cnt}^{\underline{s}}$, but f' might not exist.

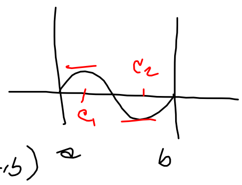
$x^{2/3}$ $(x+7)^{3/5}$

Proof of Rolle's Theorem.

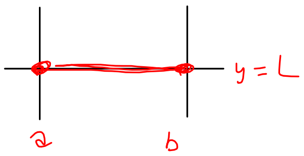
f cnt^s on $[a,b]$, difb^{le} on (a,b), and $f(a)=f(b)=L$

$$\Longrightarrow$$

$$\exists\ c\in(a,b) \ni f'(c)=0$$

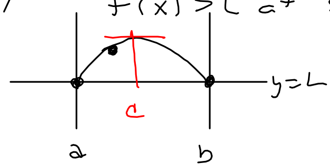


3 cases (i) $f(x)=L$ $\forall\ x\in[a,b]$



$f'(x)=0\ \forall\ x\in(a,b)$ ∎

(ii) / $f(x)>L$ at some pt $t\in(a,b)$.



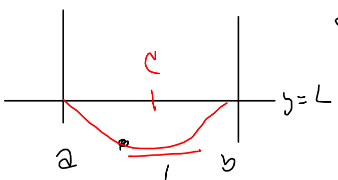
Then f has a max on $[a,b]$

$f(a), f(b)$ aren't, so $f(c)=\max$ at some $c\in(a,b)$.

Extreme Value Theorem

Fermat said, if f difb^{le} & $f(c)$ is max (or min), then $f'(c)=0$ ∎

(iii)



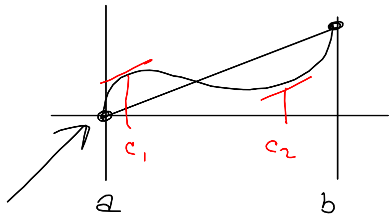
etc.

MVT: f cnt^s on $[a,b]$, difb^l on (a,b)

$\Longrightarrow \exists\, c \in (a,b) \ni f'(c) = m_{AVG} = \dfrac{f(b)-f(a)}{b-a}$

Proof

$$y = \frac{f(b)-f(a)}{b-a}(x-a) + f(a)$$



$$h(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$$

$$h(a) = f(a) - \cancel{\frac{f(b)-f(a)}{b-a}(a-a)} - f(a) = 0$$

$$h(b) = f(b) - \frac{f(b)-f(a)}{(\cancel{b-a})}(\cancel{b-a}) - f(a)$$
$$= f(b) - (f(b)-f(a)) - f(a)$$
$$= f(b) - f(b) + f(a) - f(a) = 0$$

$h(x)$ satisfies Rolle's.

f, $x-a$ are cnt^s on $[a,b]$ & difb^l on (a,b)

& $h(a) = h(b)$ $(=0)$

$\Longrightarrow$ Rolle's Applies!

$$\frac{d}{dx}\left[\frac{f(b)-f(a)}{b-a}(x-a)\right]$$
$$= \frac{f(b)-f(a)}{b-a}\left[\frac{d}{dx}[x-a]\right]$$
$$= \frac{f(b)-f(a)}{b-a}(1) = \frac{f(b)-f(a)}{b-a}$$

$\Longrightarrow \exists\, c \in (a,b) \ni$

$h'(c) = 0$

$$h'(c) = f'(c) - \frac{f(b)-f(a)}{b-a} = 0$$

$$\Longrightarrow f'(c) = \frac{f(b)-f(a)}{b-a} \quad ▨$$