MAT 201 Test 2 – Fall, 2013 Name__________________
100 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

Test 3

1. (10 pts) Let $f(x) = x^3 - 3x^2 + 2x$. Find all absolute and local extremes of f on the interval [0, 3]. Final answers accurate to the 3rd decimal place are acceptable.

on f(x)-values is not worth much.

$\Rightarrow f'(x) = 3x^2 - 6x + 2 \overset{\text{SET}}{=} 0$

$\Rightarrow a = 3, b = -6, c = 2$

$\Rightarrow b^2 - 4ac = (-6)^2 - 4(3)(2)$

$= 36 - 24$

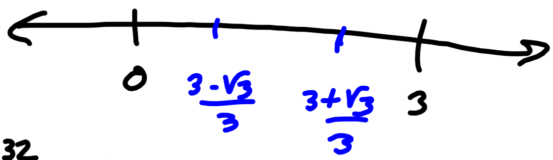
$= 12$

On our test, expect to graph one like this, only w/ nicer answers, hopefully.

& $\sqrt{12} = 2\sqrt{3}$

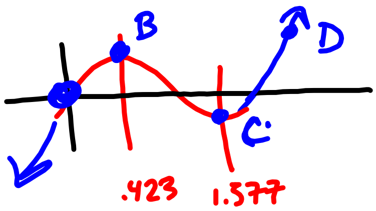
So, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{6 \pm 2\sqrt{3}}{2(3)} = \dfrac{\cancel{2}(3 \pm \sqrt{3})}{\cancel{2}(3)}$



$= \dfrac{3 \pm \sqrt{3}}{3}$ $\quad 1 + \dfrac{1.732}{3} = \dfrac{3 + 1.732}{3} \approx$ 1.577350269

$\approx \dfrac{3 \pm 1.732}{3}$ $\quad 1 - \dfrac{1.732}{3} = \dfrac{3 - 1.732}{3} \approx$ 0.4226497307



$f(0) = 0$ $\quad (0,0)$ Nada (Local Min) A

$f\left(\frac{3-\sqrt{3}}{3}\right) \approx$ 0.38490 $\quad \left(\frac{3-\sqrt{3}}{3}, .385\right)$ local max B

$f\left(\frac{3+\sqrt{3}}{3}\right) \approx$ – 0.38490 $\quad \left(\frac{3+\sqrt{3}}{3}, -.385\right)$ Abs. Min C

$f(3) = 6$ $\quad (3,6)$ Abs max D

2. (10 pts) Confirm that $f(x) = x^3 - 3x^2 + 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval [0, 3]. Then find all values c in (0, 3) that satisfy the conclusion of the theorem.

f is a polynomial; hence, f is cnt^{s} & difb^{l} on its $\mathcal{D} = \mathbb{R}$. Therefore, f is cnt^{s} on [0,3] & Difb^{l} on (0,3)

$\mathcal{R}$ = Range

$\mathbb{R}$ = Reals = (

$$\frac{f(b)-f(a)}{b-a} = \frac{f(3)-f(0)}{3-0} = \frac{6-0}{3} = 2 = m_{AVG}$$

$3^3 - 3(3)^2 + 2(3) = 6$

$f'(x) = 3x^2 - 6x + 2 \overset{SET}{=} m_{AVG} = 2$

$\Rightarrow 3x^2 - 6x = 0$

$\Rightarrow 3x(x-2) = 0$

$x = 0$ OR $x = \boxed{2 = c}$

$\notin (0,3)$

3. (10 pts) Let $f(x) = -2\sin(x)\cos(x) - x$. Find all local extrema in the interval $[0, 2\pi]$.

$$f'(x) = -2\cos x \cos x - 2\sin x(-\sin x) - 1$$
$$= -2\cos^2 x + 2\sin^2 x - 1$$
$$= -2(1-\sin^2 x) + 2\sin^2 x - 1$$
$$= -2 + 2\sin^2 x + 2\sin^2 x - 1$$
$$= 4\sin^2 x - 3 \overset{\text{SET}}{=} 0$$

$u = \sin x$

$a = 4, b = 0, c = -3$ $\rightarrow 4u^2 - 3 = 0$

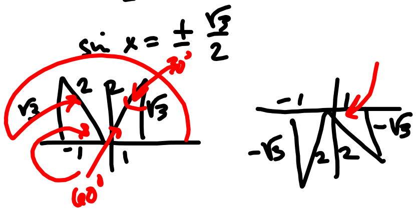
$b^2 - 4ac = 0^2 - 4(4)(-3) = 48$

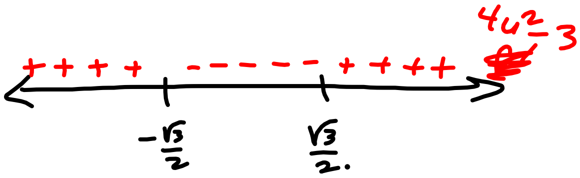
$\cancel{4\sin^2 x - 3} = 4\left(\sin x - \frac{\sqrt{3}}{2}\right)\left(\sin x + \frac{\sqrt{3}}{2}\right)$

$\sqrt{48} = 4\sqrt{3}$

$u = \frac{\pm 4\sqrt{3}}{2(4)} = \pm\frac{\sqrt{3}}{2} = \sin x$

$\sin x = \pm\frac{\sqrt{3}}{2}$



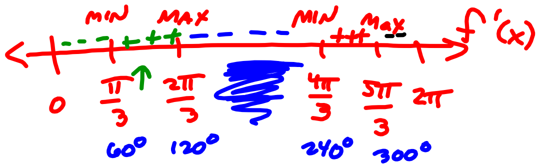


So $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$



$4\sin^2 x - 3 = f'(x)$

$f'(\pi/6) = 4\sin^2\left(\frac{\pi}{6}\right) - 3$
$= 4\left(\frac{1}{2}\right)^2 - 3$
$= 1 - 3 < 0$

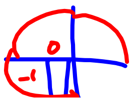
$\frac{\pi}{3} + \frac{\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

$f'\left(\frac{\pi}{2}\right) = 4\sin^2\left(\frac{\pi}{2}\right) - 3$
$= 4(1)^2 - 3$
$= 1$

$\frac{\frac{4\pi}{3} + \frac{5\pi}{3}}{2} = \frac{\frac{9\pi}{3}}{2} = \frac{3\pi}{2}$

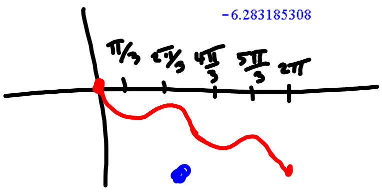
$4\sin^2\pi - 3 = -3$

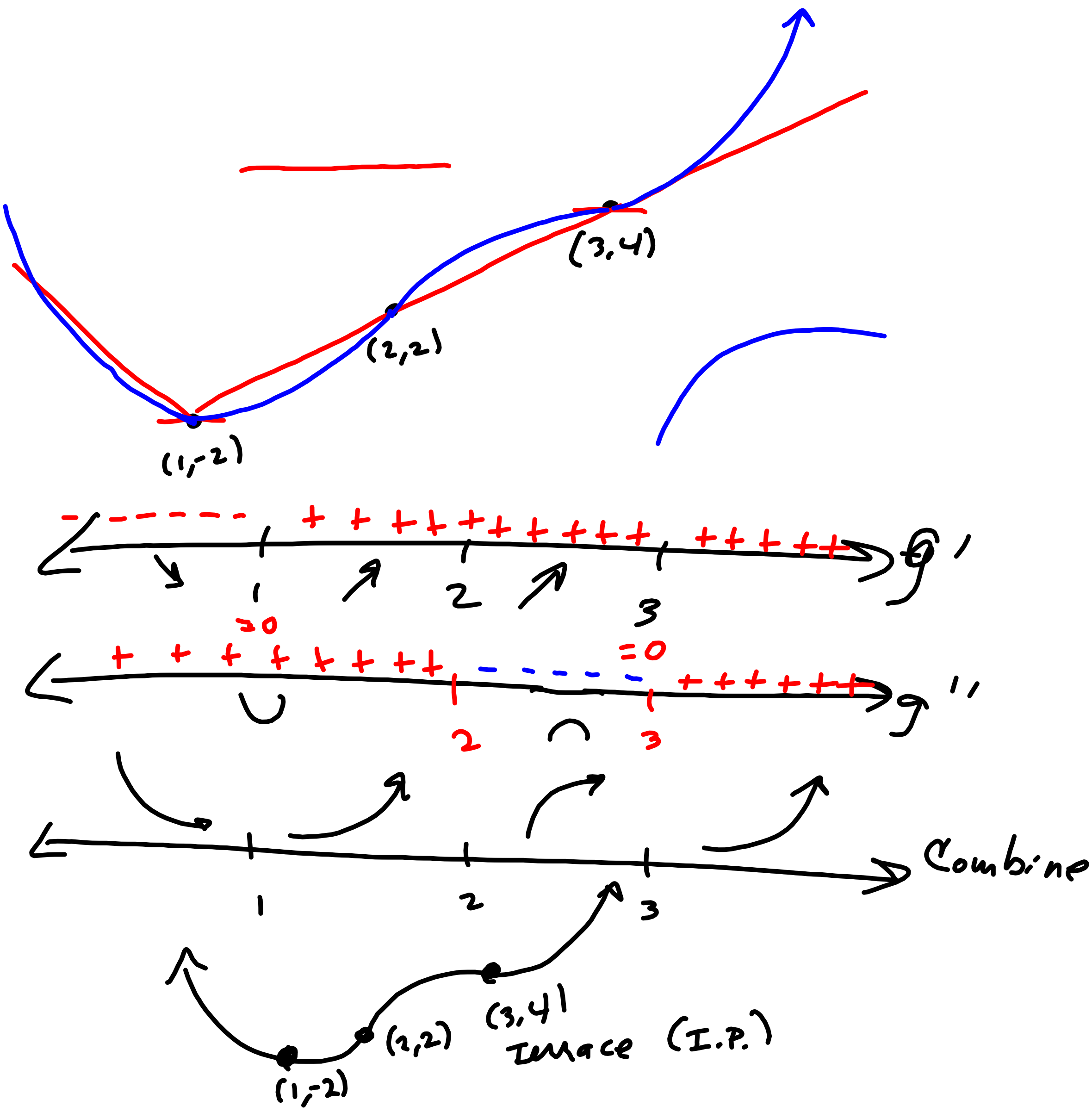
$4\sin^2\left(\frac{3\pi}{2}\right) - 3$
$= 4(-1)^2 - 3 = 1$



evalf(f(0))	0.
evalf(f(Pi/3))	−1.913222955
evalf(f(2·Pi/3))	−1.228369699
evalf(f(4·Pi/3))	−5.054815608
evalf(f(5·Pi/3))	−4.369962354
evalf(f(2·Pi))	−6.283185308

$(0,0)$ Neither
$\left(\frac{\pi}{3}, -1.91\right)$ local min
$\left(\frac{2\pi}{3}, -1.23\right)$ local max
$\left(\frac{4\pi}{3}, -5.05\right)$ local min
$\left(\frac{5\pi}{3}, -4.37\right)$ local max
$(2\pi, -2\pi) \approx (2\pi, -6.28)$ Abs. Min.





4. (10 pts) Suppose a function g satisfies all of the following properties. Sketch a graph of g that incorporates all of the following properties into it:

$g(1)=-2 \quad g(2)=2 \quad g(3)=4$

$g'(1)=0 \quad g'(3)=0$

$g'(x)>0$ on $(1,3)\cup(3,\infty)$, $\quad g'(x)<0$ on $(-\infty,1)$

$g''(x)>0$ on $(-\infty,2)\cup(3,\infty)$, $\quad g''(x)<0$ on $(2,3)$

$(a-b)(a+b) = a^2 - b^2$

$(3x)^2 = 9x^2$

5. (5 pts each) Evaluate the limits:

a. $\lim_{x\to\infty}\left(\sqrt{9x^2+3x+7}-3x\right)$

$$\left(\frac{\sqrt{9x^2+3x+7}-3x}{1}\right)\left(\frac{\sqrt{9x^2+3x+7}+3x}{\sqrt{9x^2+3x+7}+3x}\right)$$

$$= \frac{9x^2+3x+7-9x^2}{\sqrt{9x^2+3x+7}+3x} = \frac{3x+7}{3x\sqrt{1+\frac{1}{3x}+\frac{7}{9x^2}}+3x}$$

$$= \frac{3x\left(1+\frac{7}{3x}\right)}{3x\left(\sqrt{1+\frac{1}{3x}+\frac{7}{9x^2}}+1\right)} = \frac{1+\frac{7}{3x}}{\sqrt{1+\underset{0}{\frac{1}{3x}}+\underset{0}{\frac{7}{9x^2}}}+1} \xrightarrow{x\to\infty} \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

$\frac{7}{3x} \to 0$

$$\sqrt{9x^2+3x+7} = \sqrt{9x^2\left(1+\frac{3x}{9x^2}+\frac{7}{9x^2}\right)} = \sqrt{9x^2}\sqrt{1+\frac{1}{3x}+\frac{7}{9x^2}}$$

$$= 3|x|\sqrt{\quad} = 3x\sqrt{\quad}$$

b. $\lim_{x\to\infty}\left(\frac{3x^3-6x+7}{2-5x^2-7x^3}\right) = \frac{3x^3}{-7x^3} = \boxed{-\frac{3}{7}}$

$\to 3x^3$

6. (10 pts) Find the equation of the oblique asymptote for $R(x)=\frac{3x^3-6x+7}{x^2-5}$ This is *sort* of a limit at infinity.

$$\frac{3x^3}{x^2}$$

$$\begin{array}{r|l} & 3x \quad r\ 9x+7 \\ x^2-5 & 3x^3+0x^2-6x+7 \\ & -(3x^3 \qquad -15x) \\ \hline & \qquad 9x+7 \end{array}$$

$$\begin{array}{r} 15 \\ -6 \\ \hline 9 \end{array}$$

This says

$$3x^3-6x+7=(x^2-5)(3x)+(9x+7)$$

$$\frac{3x^3-6x+7}{x^2-5}=3x+\frac{9x+7}{x^2-5} \to 0 \text{ as } x\to\pm\infty$$

So it "acts like"

$y=3x$ for big $|x|$.

$\boxed{y=3x \text{ is O.A.}}$

7. (10 pts) Find the minimum vertical distance between $h(x) = 2x^2 - 5x + 12$ and $k(x) = 1 - 3x - x^2$.

want to minimize

$$\left| 2x^2 - 5x + 12 - (1 - 3x - x^2) \right|$$

$$= \left| 2x^2 - 5x + 12 - 1 + 3x + x^2 \right|$$

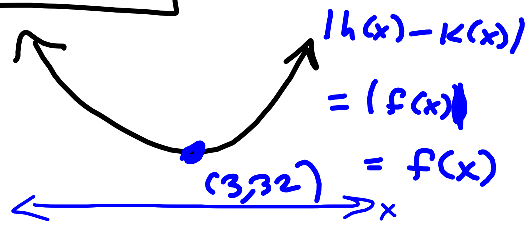
$$= \left| 3x^2 - 2x + 11 \right| = \boxed{3x^2 - 2x + 11 = f(x)} \text{ Min.}$$

$a = 3, b = -2, c = 11$

$$3\left(x^2 - \tfrac{2}{3}x \qquad\right) = -11$$

$$3\left(x^2 - \tfrac{2}{3}x + \left(\tfrac{1}{3}\right)^2\right) = -11 + \tfrac{1}{3} = \text{neg.}$$

$$3\left(x - \tfrac{1}{3}\right)^2 = \text{neg}$$



$$f'(x) = 6x - 2 \overset{\text{SET}}{=} 0 \Rightarrow x = 3$$

$$f(3) = 3(3)^2 - 2(3) + 11$$

$$= 27 - 6 + 11 = \boxed{32}$$ is min distance!

8. (10 pts) Use the graph of the function $f(x)$, on the accompanying sheet, to show how x_2 would be found by Newton's Method, in an attempt to find a root. Derive the formula for x_2, and explain what's going on.

See 3.8 video

See Newton video.

9. (10 pts) Suppose $f''(x) = 40x^3 - 24x^2 + 18x - 2$, and we have the initial conditions $f'(1) = f(1) = 3$. Find $f(x)$.

$f''(x) = 40x^3$ See?

$\Rightarrow f'(x) = 40\frac{x^4}{4} + C$ for any $C \in \mathbb{R}$

$f'(x) = \frac{40}{4}x^4 - \frac{24}{3}x^3 + \frac{18}{2}x^2 - 2x + C$

& $f'(1) = 3$ means

$10(1)^4 - 8(1)^3 + 9(1)^2 - 2(1) + C = 3$

$10 - 8 + 9 - 2 + C = 3$

$9 + C = 3$

$C = -6$

$f'(x) = 10x^4 - 8x^3 + 9x^2 - 2x - 6$

$\Rightarrow f(x) = 5x^5 - 2x^4 + 3x^3 - x^2 - 6x + D$

& $f(1) = 5 - 2 + 3 - 1 - 6 + D = 3$

$-1 + D = 3$

$D = 4$

$\Rightarrow \boxed{f(x) = 5x^5 - 2x^4 + 3x^3 - x^2 - 6x + 4}$