MAT 201 Test 2 – Fall, 2013 Name________________
100 Points Covers Chapter 3 NO GRAPHING CALCULATORS!!!

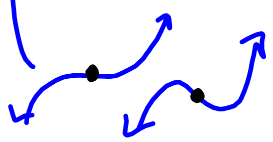
1. (10 pts) Let $f(x) = x^3 - 3x^2 + 2x$. Find all absolute and local extremes of f on the interval [0, 3]. Final answers accurate to the 3rd decimal place are acceptable.

sketch $f(x)$.

$2x^3 - 3x^2 - 72x + 73$

$f(0) = 73$

$\boxed{(0,73) \text{ y-int}}$



Make $f'(x) = 0$ nice

Make either $f(1)$ or $f(2)$ or $f(3) = 0$

to make intercepts nice/doable.

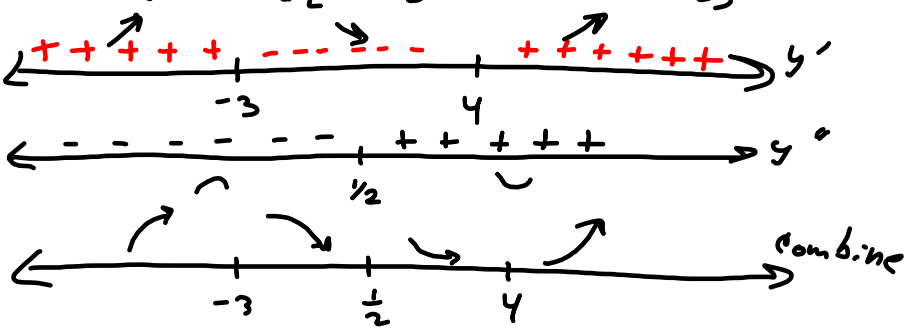
Emphasis not on x-int. (Bonus).

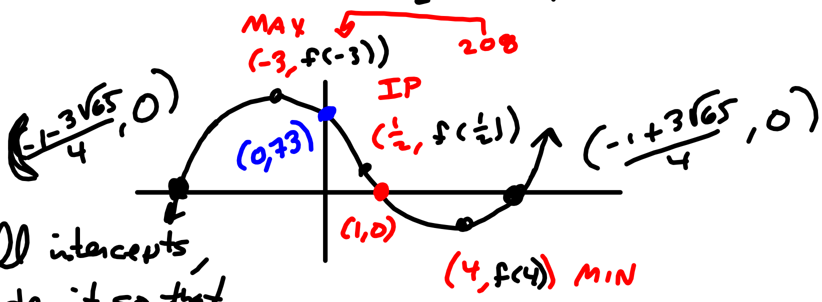
$f'(x) = 6x^2 - 6x - 72 = 6(x^2 - x - 12)$

$= 6(x-4)(x+3) \overset{\text{SET}}{=} 0 \Longrightarrow$

$x \in \{-3, 4\}$

$f''(x) = 6[2x-1] \overset{\text{SET}}{=} 0 \Rightarrow x \in \{\tfrac{1}{2}\}$





Find all intercepts,
Steve made it so that
$f(1)/f(2)/f(3)$ is zero.

$2x^3 - 3x^2 - 72x + 73$

Guess $x = 1$:

1	2	-3	-72	73
		2	-1	-73
	2	-1	-73	0

$273 \cdot 8 = 584$

$(x-1)(2x^2 - x - 73)$

$a = 2, b = 1, c = -3$

$b^2 - 4ac = (-1)^2 - 4(2)(-73)$

$= 1 + 584$

$= 585 = 3^2 \cdot 5 \cdot 13$

$\Longrightarrow \sqrt{585} = 3\sqrt{65}$

$3 \,|\, 585$
$3 \,|\, 195$
$5 \,|\, 65$
13

$f(-3)$:

-3	2	-3	-72	73
		-6	27	135
	2	-9	-45	208

$(-3, 208)$

$$x = \frac{-1 \pm 3\sqrt{65}}{2(2)} = \frac{-1 \pm 3\sqrt{65}}{4}$$

2. (10 pts) Confirm that $f(x) = x^3 - 3x^2 + 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval [0, 3]. Then find all values c in (0, 3) that satisfy the conclusion of the theorem.

Maximize $f(x) = 2x^3 - 3x^2 - 72x + 73$ on $[0,3]$

Find f', set $f' = 0$, etc.

$f(0) = 73 \rightarrow (0, 73)$ MAX

$f(3) = -116$

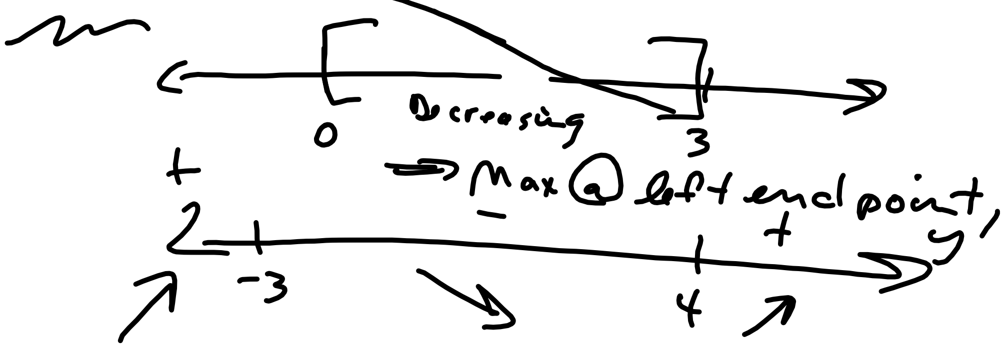
$f'(x) = 6x^2 - 6x - 72$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -72 & 73 \\ & & 6 & 9 & -189 \\ \hline & 2 & 3 & -63 & -116 \end{array}$$

$63 \cdot 3$

$-189 + 73 = -116$

$f'(x) = 0$ @ $x = -3$, $x = 4$



$\Rightarrow$ Max @ left endpoint,

$x^3 - 3x^2 + 2x$

MVT hypotheses on $[0,3]$?

cnt^s on $[0,3]$
dif b^{l} on $(0,3]$ } f is polynomial $\Longrightarrow$ difb^{l} & cnt^s $\forall\, x \in \mathbb{R}$,

$f(0) = 0$

$f(3) = 6$

$$m_{AVG} = \frac{f(3) - f(0)}{3 - 0} = \frac{6 - 0}{3} = 2 = m_{AVG}$$

MVT conclusion is

$$f'(x) = 3x^2 - 6x + 2 \stackrel{SET}{=} 2 = m_{avg}$$

$\exists\, c \in (0,3) \ni$
$f'(c) = m_{avg}$ on $[0,3]$

$$\Longrightarrow 3x^2 - 6x = 0$$

$$\Longrightarrow 3x(x-2) = 0$$

$$\Longrightarrow x \in \{0, 2\}$$

$$\boxed{c = 2 \in (0,3)}$$

3. (10 pts) Let $f(x) = -2\sin(x)\cos(x) - x$. Find all local extrema in the interval $[0, 2\pi]$.

Not endpoints!

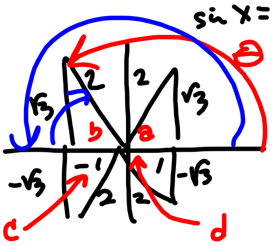
$$f'(x) = -2\left[\cos^2 x - \sin^2 x\right] - 1$$
$$= -2\left[1 - \sin^2 x - \sin^2 x\right] - 1$$
$$= -2\left[1 - 2\sin^2 x\right] - 1$$
$$= -2 + 4\sin^2 x - 1$$
$$= 4\sin^2 x - 3 \overset{\text{SET}}{=} 0$$

$4u^2 - 3 = 0$, etc.

$\rightarrow \sin^2 x = \frac{3}{4}$

$(2\sin x - \sqrt{3})(2\sin x + \sqrt{3})$

$\sin x = \pm \frac{\sqrt{3}}{2}$



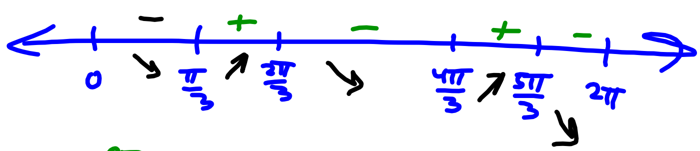
$a = 60° = \frac{\pi}{3}$

$b = 60°$ is reference

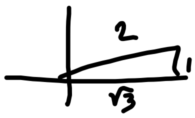
so $\theta = 180° - 60°$
$= 120°$
$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ critical values in $(0, 2\pi)$

Sign pattern



$f'\left(\frac{\pi}{6}\right)$
$= 4\sin^2\left(\frac{\pi}{6}\right) - 3$
$= 4\left(\frac{1}{2}\right)^2 - 3$
$= 1 - 3 < 0$



$\frac{\frac{4\pi}{3} + \frac{5\pi}{3}}{2} = \frac{\frac{9\pi}{3}}{2} = \frac{3\pi}{2}$

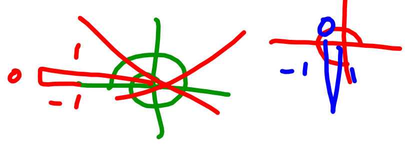
$4\sin^2\left(\frac{3\pi}{2}\right) - 3$
$= 4(-1)^2 - 3 = 1$

$\left(\frac{\pi}{3}, \_\_\right)$ min

$\left(\frac{2\pi}{3}, \_\_\right)$ Max

$\left(\frac{4\pi}{3}, \_\_\right)$ Min

$\left(\frac{5\pi}{3}, \_\_\right)$ Max



evalf(f(0))	0.
$evalf\left(f\left(\frac{Pi}{3}\right)\right)$	−1.913222955
$evalf\left(f\left(\frac{2 \cdot Pi}{3}\right)\right)$	−1.228369699
$evalf\left(f\left(\frac{4 \cdot Pi}{3}\right)\right)$	−5.054815608
$evalf\left(f\left(\frac{5 \cdot Pi}{3}\right)\right)$	−4.369962354
evalf(f(2·Pi))	−6.283185308

4. (10 pts) Suppose a function g satisfies all of the following properties. Sketch a graph of g that incorporates all of the following properties into it:

$g(1) = -2 \quad g(2) = 2 \quad g(3) = 4$
$g'(1) = 0 \quad g'(3) = 0$
$g'(x) > 0$ on $(1,3) \cup (3,\infty)$, $\quad g'(x) < 0$ on $(-\infty,1)$
$g''(x) > 0$ on $(-\infty,2) \cup (3,\infty)$, $\quad g''(x) < 0$ on $(2,3)$

5. (5 pts each) Evaluate the limits:

a. $\lim_{x\to\infty}\left(\sqrt{9x^2+3x+7}-3x\right)$

" Rationalize "

$$\sqrt{9x^2+3x+7}-3x = \left(\frac{\overbrace{\sqrt{9x^2+3x+7}}^{a} \overbrace{-3x}^{-b}}{1}\right)\left(\frac{\overbrace{\sqrt{9x^2+3x+7}}^{a} \overbrace{+3x}^{+b}}{\sqrt{9x^2+3x+7}+3x}\right)$$

$\infty - \infty$

$$= \frac{\left(\sqrt{9x^2+3x+7}\right)^2-(3x)^2}{\sqrt{9x^2+3x+7}+3x} = \frac{9x^2+3x+7-9x^2}{\sqrt{9x^2+3x+7}+3x}$$

$$= \frac{3x\left(1+\frac{7}{3x}\right)}{\sqrt{9x^2}\sqrt{1+\frac{1}{3x}+\frac{7}{9x^2}}+3x} = \frac{3x\left(1+\frac{7}{3x}\right)}{3x\left(\sqrt{1+\frac{1}{3x}+\frac{7}{9x^2}}+1\right)}$$

$$\sqrt{9x^2} = \sqrt{9}\sqrt{x^2} = 3\sqrt{x^2} = 3|x| = 3x$$

$$\xrightarrow{x\to\infty} \frac{1+0}{\sqrt{1+0+0}+1} = \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}}$$

b. $\lim_{x\to\infty}\left(\frac{3x^3-6x+7}{2-5x-7x^3}\right) = \frac{3}{-7}$

6. (10 pts) Find the equation of the oblique asymptote for $R(x)=\dfrac{3x^3-6x+7}{x^2-5}$ This is *sort* of a limit at infinity.

$$\boxed{y=3x}$$

$$\frac{3x^3}{x^2}=3x$$

$$\begin{array}{r} 3x \\ x^2-5 \,\big)\, 3x^3+0x^2-6x+7 \\ -(3x^3 -15x) \\ \hline 9x+7 \end{array}$$

$$R(x)=3x+\frac{9x+7}{x^2-5}$$

$$\frac{28}{3}=9+\frac{1}{3}$$

7. (10 pts) Find the minimum vertical distance between $h(x) = 2x^2 - 5x + 12$ and $k(x) = 1 - 3x - x^2$.

$f(x) = |2x^2 - 5x + 12 - (1 - 3x - x^2)|$ = vertical Dist.

$= |2x^2 - 5x + 12 + x^2 + 3x - 1|$

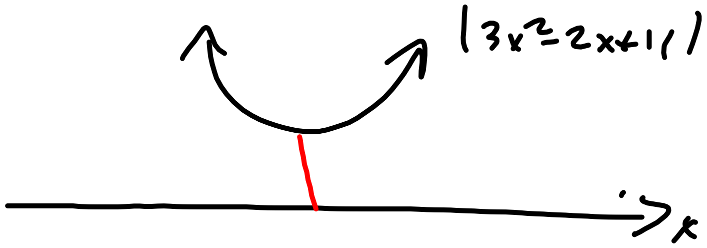
$= |3x^2 - 2x + 11| = \begin{cases} 3x^2 - 2x + 11 & \text{if } 3x^2 - 2x + 11 \geq 0 \\ -(3x^2 - 2x + 11) & \text{if } 3x^2 - 2x + 11 < 0 \end{cases}$

$b^2 - 4ac = (-2)^2 - 4(3)(11)$

$< 0 \Rightarrow$ No ~~roots~~ real roots.

$= 3x^2 - 2x + 11$, always!

$|3x^2 - 2x + 11|$



$f'(x) = 6x - 2 \overset{\text{SET}}{=} 0 \Rightarrow$

$6x = 2$

$x = \frac{1}{3}$ gives min distance.

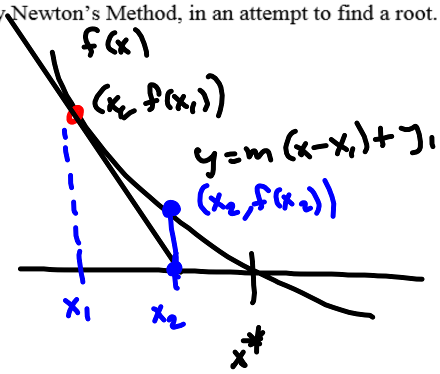
min Distance

$= f(\frac{1}{3}) = 3(\frac{1}{3})^2 - 2(\frac{1}{3}) + 11$

$= \frac{1}{3} - \frac{2}{3} + \frac{33}{3}$

$= \boxed{\frac{32}{3} = \text{min distance @ } x = \frac{1}{3}}$

8. (10 pts) Use the graph of the function $f(x)$, on the accompanying sheet, to show how x_2 would be found by Newton's Method, in an attempt to find a root. Derive the formula for x_2, and explain what's going on.



Make a guess: x_1

Tangent line @ x_1 is

$$y = f'(x_1)(x - x_1) + f(x_1)$$

x_2 is where the tan line crosses the x-axis:

$$f'(x_1)(x-x_1) + f(x_1) = 0$$

Solve for x!

$$f'(x_1)(x-x_1) = -f(x_1)$$

$$x - x_1 = -\frac{f(x_1)}{f'(x_1)}$$

$$x = x_1 - \frac{f(x_1)}{f'(x_1)}$$

We want x^* = where $f(x)=0$

$$f'(x_1)x - f'(x_1)x_1 = -f(x_1)$$

$$f'(x_1)x = f'(x_1)x_1 - f(x_1)$$

$$x = \frac{f'(x_1)x_1}{f'(x_1)} - \frac{f(x_1)}{f'(x_1)}$$

$$x = x_1 - \frac{f(x_1)}{f'(x_1)} \equiv x_2$$

In general

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

9. (10 pts) Suppose $f''(x) = 40x^3 - 24x^2 + 18x - 2$, and we have the initial conditions $f'(1) = f(1) = 3$. Find $f(x)$.

$\Longrightarrow f'(x) = 40\frac{x^4}{4} - 24\frac{x^3}{3} + 18\frac{x^2}{2} - 2x + C$

$= 10x^4 - 8x^3 + 9x^2 - 2x + C$

$f'(1) = 10 - 8 + 9 - 2 + C = 3$

$= C + 9 = 3$

$\Longrightarrow \boxed{C = -6}$

So $f'(x) = 10x^4 - 8x^3 + 9x^2 - 2x - 6$

$\Longrightarrow f(x) = 2x^5 - 2x^4 + 3x^3 - x^2 - 6x + D$

$f(1) = 2 - 2 + 3 - 1 - 6 + D = 3$

$\Longrightarrow D - 4 = 3$

$D = 7$

$\Longrightarrow \boxed{f(x) = 2x^5 - 2x^4 + 3x^3 - x^2 - 6x + 7}$