

13–16 Use Newton's method to approximate the indicated root of the equation correct to six decimal places.

13. The root of $x^4 - 2x^3 + 5x^2 - 6 = 0$ in the interval $[1, 2]$

$$f'(x) = 4x^3 - 6x^2 + 10x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x^4 - 2x^3 + 5x^2 - 6}{4x^3 - 6x^2 + 10x}$$

15. The positive root of $\sin x = x^2$

$$f(x) = \sin x - x^2$$

$$f'(x) = \cos x - 2x$$

$$x_{n+1} = x_n - \frac{\sin x - x^2}{\cos x - 2x}$$

17–22 Use Newton's method to find all roots of the equation correct to six decimal places.

17. $3 \cos x = x + 1$

$$\Rightarrow f(x) = 3 \cos x - x - 1 = 0$$

$$\Rightarrow f'(x) = -3 \sin x - 1$$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{3 \cos x_n - x_n - 1}{-3 \sin x_n - 1}$$

$$19. \sqrt[3]{x} = x^2 - 1$$

$$x^{\frac{1}{3}} = x^2 - 1$$

$$x^2 - x^{\frac{1}{3}} - 1 = f(x) = 0$$

$$f'(x) = 2x - \frac{1}{3}x^{-\frac{2}{3}}$$

$$x_{n+1} = x_n - \frac{x_n^2 - x_n^{\frac{1}{3}} - 1}{2x_n - \frac{1}{3}x_n^{-\frac{2}{3}}}$$

21. $\cos x = \sqrt{x}$

$$f(x) = \cos x - (x)^{\frac{1}{2}}$$

$$f'(x) = -\sin x - \frac{1}{2}x^{-\frac{1}{2}} = -\sin x - \frac{1}{2x^{1/2}}$$

$$x_{n+1} = x_n - \frac{\cos x - x^{\frac{1}{2}}}{-\sin x - \frac{1}{2}x^{-\frac{1}{2}}}$$

