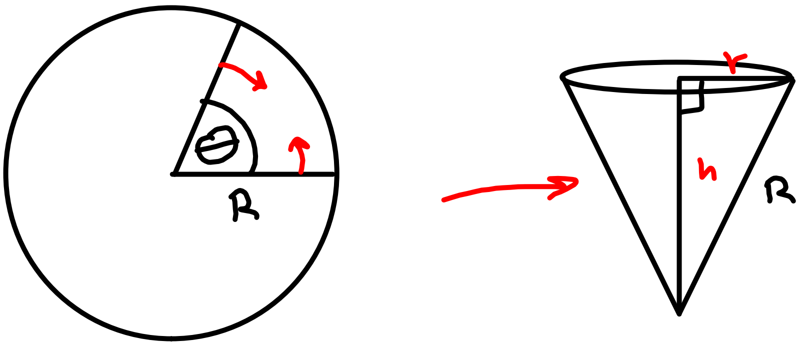


Maximize volume of the cup.

Find the cut

Find θ

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \overset{f}{r^2} \overset{g}{(R^2 - r^2)^{\frac{1}{2}}}$

Auxiliary eq'n to get rid of a variable

$h^2 + r^2 = R^2$

$h^2 = R^2 - r^2$

$h = \pm\sqrt{R^2 - r^2}$

$h = \sqrt{R^2 - r^2}$, since $h > 0$

$\Rightarrow \frac{dV}{dr} = \frac{1}{3}\pi\left[2r(R^2 - r^2)^{\frac{1}{2}} + r^2\left(\frac{1}{2}(R^2 - r^2)^{-\frac{1}{2}}(-2r)\right)\right]$

$= \frac{1}{3}\pi\left[\frac{2r(R^2 - r^2)^{\frac{1}{2}}}{1} \cdot \frac{(R^2 - r^2)^{\frac{1}{2}}}{(R^2 - r^2)^{1/2}} - \frac{r^3}{(R^2 - r^2)^{\frac{1}{2}}}\right]$

$= \frac{1}{3}\pi\left[\frac{2r(R^2 - r^2) - r^3}{(R^2 - r^2)^{1/2}}\right] = \frac{1}{3}\pi\left[\frac{2rR^2 - 2r^3 - r^3}{\sim\sim}\right]$

$= \frac{1}{3}\pi\left[\frac{2rR^2 - 3r^3}{\text{—}}\right] = \frac{1}{3}\pi r\left[\frac{2R^2 - 3r^2}{\sim\sim}\right] \overset{\text{SET}}{=} 0$

$\Rightarrow r = 0$ OR $2R^2 - 3r^2 = 0$

$\cancel{}$

$2R^2 = 3r^2$

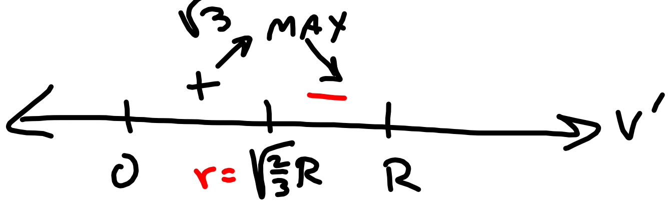
$3r^2 = 2R^2$

$r^2 = \frac{2}{3}R^2$

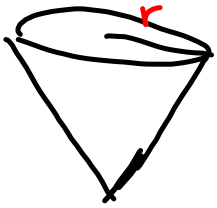
$r = \sqrt{\frac{2}{3}}\,R$

$$\Rightarrow h = \sqrt{R^2 - r^2}$$
$$= \sqrt{R^2 - \left(\sqrt{\tfrac{2}{3}}\,R\right)^2}$$
$$= \sqrt{R^2 - \tfrac{2}{3}R^2}$$
$$= \sqrt{\tfrac{1}{3}R^2} = \sqrt{\tfrac{1}{3}}\,R$$

$$\frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



So $V = \frac{1}{3}\pi r^2 h$
$$= \tfrac{1}{3}\pi\left(\sqrt{\tfrac{2}{3}}\,R\right)^2 \sqrt{\tfrac{1}{3}}\,R$$
$$= \tfrac{1}{3}\pi\left(\tfrac{2}{3}R^2\right)\sqrt{\tfrac{1}{3}}\,R$$
$$= \pi\tfrac{2}{9}\cdot\sqrt{\tfrac{1}{3}}\,R^3$$
$$= \frac{2\pi}{9}\cdot\frac{\sqrt{3}}{3}R^3 = \frac{2\sqrt{3}\,\pi}{27}R^3$$
$$= \text{Vol}$$



C_2 = Circumf of top of cup

= C_1 − Arclength of the cut

$C_2 = 2\pi R - R\theta = 2\pi r$ Solve for θ

$\Longrightarrow -R\theta = 2\pi r - 2\pi R$

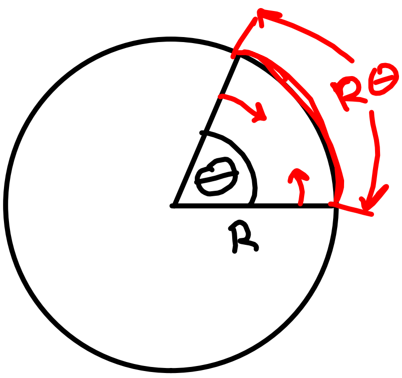
$= 2\pi\sqrt{\frac{2}{3}}R - 2\pi R$

$$\Longrightarrow \theta = \frac{2\pi\sqrt{\frac{2}{3}}R - 2\pi R}{-R}$$

$$= \frac{2\pi R - 2\pi\sqrt{\frac{2}{3}}R}{R}$$

$$= 2\pi - 2\pi\sqrt{\frac{2}{3}}$$

$$= 2\pi\left(1 - \sqrt{\frac{2}{3}}\right) \approx 66.06^{\circ}$$



Arc Length = $R\theta$ (θ in radians)

Find θ

$\exists$ relationship between θ & Circumference. We exploit this:

$C_1 = 2\pi R$ = Circumf. of Big Disc.