§3.4 Limits at Infinity

- Some College Algebra Review,
  - Polynomials
  - Sign Patterns
  - Proper/Improper Rational Functions & their asymptotes.

Videos on College Algebra

- Th'ow some Calculus on top of that.

Some quick graph-sketching concepts (Review of Algebra), and sign-pattern demo.

$x^2 - 6x + 7$

$f(x) = x^5 - 18x^4 + 124x^3 - 404x^2 + 615x - 350$

$= (x-2)(x-5)^2(x-3+\sqrt{2})(x-3-\sqrt{2})$

Rational Zeros

$a_n x^n + \cdots + a_0$ Any $\frac{p}{q} \ni f(\frac{p}{q}) = 0$

will satisfy p is factor of a_0

q " " " a_n

$$\begin{array}{r|l} 2 & 350 \\ 5 & 175 \\ 5 & 35 \\ & 7 \end{array}$$

$a_n = a_5 = 1$

$a_0 = -350$

$\frac{p}{q}: \pm 2, \pm 5, \pm 7, \pm 10, \ldots$

Break it down by synthetic division

Divide by $x-2$

$(x-2)(x^4 - 16x^3 + 92x^2 - 220x + 175)$

2	1	-18	124	-404	615	-350
		2	-32	184	-440	350
5	1	-16	92	-220	175	$0 = f(2)$
		5	-55	185	-175	
5	1	-11	37	-35	0	
		5	-30	35		
	1	-6	7	0		

Now we have $(x-2)(x-5)^2(x^2-6x+7)$

$x^2 - 6x + 7$

$b^2 - 4ac = (-6)^2 - 4(1)(7) = 36 - 28 = 8 \rightsquigarrow \sqrt{8} = 2\sqrt{2}$

$x = \frac{6 \pm 2\sqrt{2}}{2(1)} = \frac{2(3 \pm \sqrt{2})}{2} = 3 \pm \sqrt{2}$

$\Rightarrow x^2 - 6x + 7 = (x-(3+\sqrt{2}))(x-(3-\sqrt{2}))$

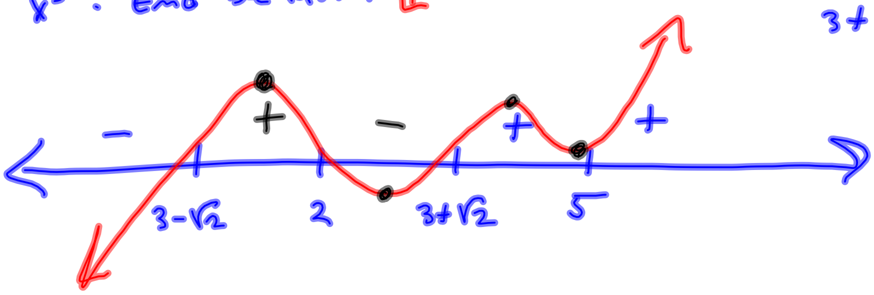
$(x-2)(x-5)^2(x-3-\sqrt{2})(x-3+\sqrt{2})$

$\sqrt{2} \approx 1.4$

$3 - \sqrt{2} \approx 1.6$

$3 + \sqrt{2} \approx 4.4$

x^5 : End-Behavior



End Behavior $\lim_{x \to \pm\infty} f(x)$ $\lim_{|x| \to \infty} f(x)$

Proper/Improper Rational Function.

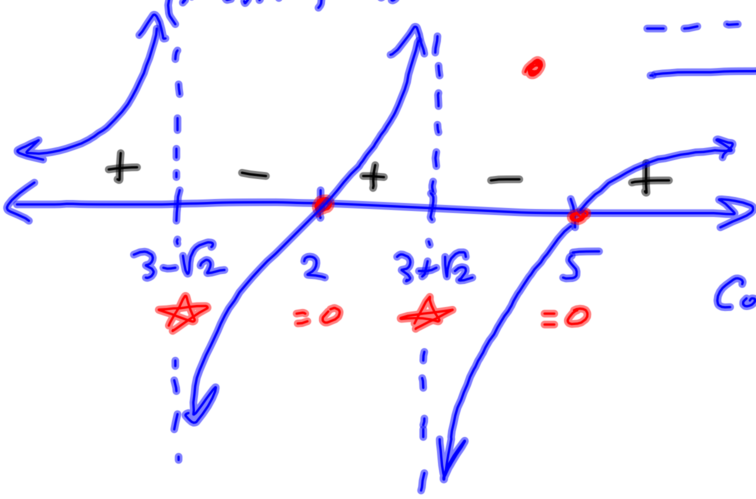
$$\lim_{x\to\infty} \frac{x^2-7x+10}{x^2+6x+7}$$

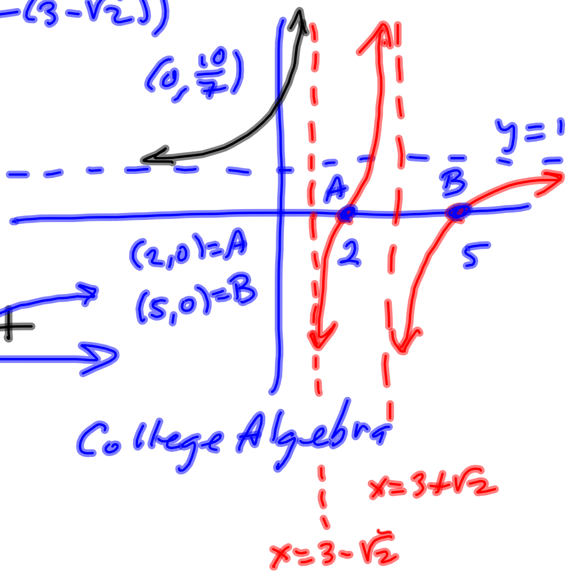
$\frac{x^2}{x^2} = 1 \xrightarrow{x\to\infty} \boxed{1 = y}$ is H.A.

$x^2-7x-10 = (x-2)(x-5)$

$x^2+6x+7 = (x-(3+\sqrt{2}))(x-(3-\sqrt{2}))$

$$y = \frac{(x-2)(x-5)}{(x-(3+\sqrt{2}))(x-(3-\sqrt{2}))}$$





$$f := x \rightarrow \frac{x}{x^2+1}$$

#47 from text

$$fp := \mathrm{D}(f)$$

$$x \rightarrow \frac{1}{x^2+1} - \frac{2x^2}{\left(x^2+1\right)^2}$$

$$normal\left(\frac{1}{x^2+1} - \frac{2x^2}{\left(x^2+1\right)^2}\right)$$

$$-\frac{x^2-1}{\left(x^2+1\right)^2}$$

$$fpp := \mathrm{D}(fp)$$

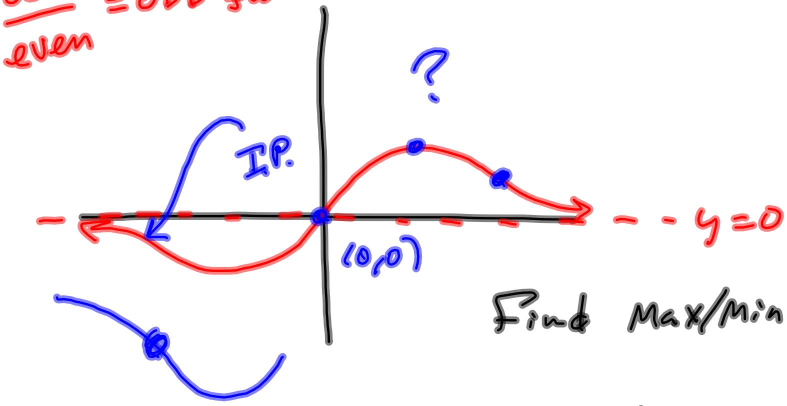
$$x \rightarrow -\frac{6x}{\left(x^2+1\right)^2} + \frac{8x^3}{\left(x^2+1\right)^3}$$

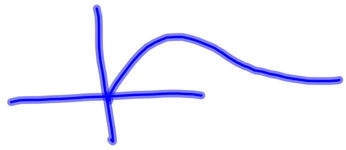
$$normal\left(-\frac{6x}{\left(x^2+1\right)^2} + \frac{8x^3}{\left(x^2+1\right)^3}\right)$$

$$\frac{2x\left(x^2-3\right)}{\left(x^2+1\right)^3}$$

$f(x) = \dfrac{x}{x^2+1}$ Proper $y=0$ is H.A.

$\dfrac{\text{odd}}{\text{even}}$ = ODD func.





$\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}$

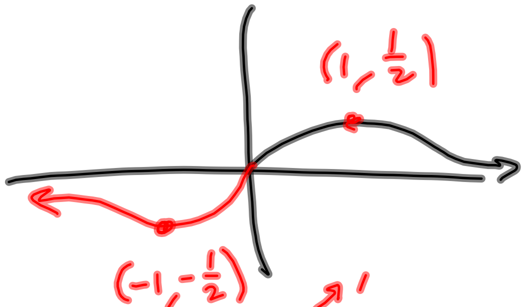
Find Max/Min, IP's.

$$f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$$

$$-x^2+1 \overset{\text{SET}}{=} 0 \Rightarrow x^2-1=0 \Rightarrow (x-1)(x+1)=0$$

$$x = \pm 1$$

$f(1) = \dfrac{1}{1^2+1} = \dfrac{1}{2}$



$$f''(x) = \frac{-2x(x^2+1)^{\cancel{2}\to 1} - (-x^2+1)(2\cancel{(x^2+1)}(2x))}{(x^2+1)^{\cancel{4}\to 3}}$$

SWIDT?

$(-4x^3+4x)$

$$= \frac{-2x(x^2+1) - (-x^2+1)(4x)}{(\ \ )^3}$$

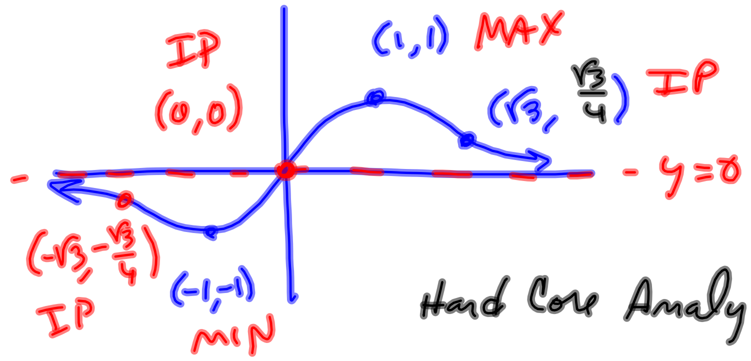
$$= \frac{-2x^3-2x+4x^3-4x}{(\ \ )^3} = \frac{2x^3-6x}{(x^2+1)^3}$$

$$\overset{\text{SET}}{=} 0 \Rightarrow 2x^3-6x=0$$

$$2x(x^2-3)=0$$

$f(\sqrt{3}) = \dfrac{\sqrt{3}}{3+1} =$

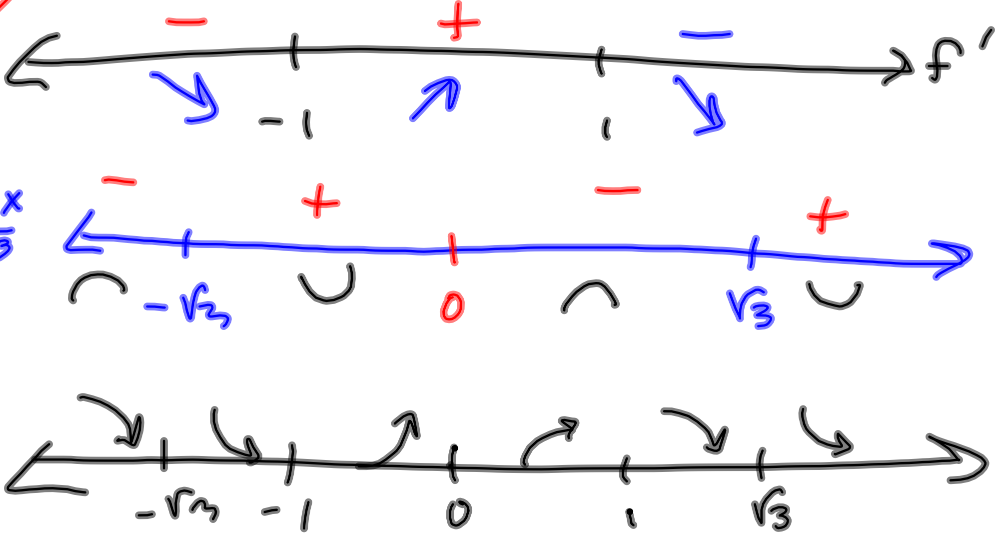
$x=0$ OR $x = \pm\sqrt{3}$



Hard Core Analysis.

$$\frac{-x^2+1}{(x^2+1)^2}$$

$$\frac{2x^3-6x}{(x^2+1)^3}$$



$$2x(x^2-3) = 2x(x-\sqrt{3})(x+\sqrt{3})$$

$$\frac{-x^2+1}{(x^2+1)^2} = -\frac{x^2-1}{(x^2+1)^2} = -\frac{(x-1)(x+1)}{(x^2+1)^2}$$