What's the deal on all this "continuous on closed interval" and "differentiable on the open interval" stuff? How do we *know* if a function's continuous or not?

BY GETTING A HANDLE ON ALL THE DOMAIN STUFF FROM PREVIOUS CLASSES!!! BASICALLY, IF IT'S IN THE DOMAIN, THEN *F* IS CONTINUOUS THERE, FOR JUST ABOUT ANY FUNCTION WE CAN WRITE, DAGNABBIT!!!

Continuity & Differentiability

$\mathcal{D}(f)$ & $\mathcal{D}(f')$

$\mathbb{Z}$

$\mathbb{Q}$ $\frac{r}{s}$

$\mathbb{R}$

$\mathcal{D}$ = Domain = $\{x \mid f(x) \text{ is defined}\}$

Pretty much everything IS defined, except...

$\sqrt[2n]{\text{Negative}}$ $\frac{\text{Stuff}}{0}$ $\forall n \in \mathbb{Z}$

"for x such that x is an integer."

$\forall x \in \mathbb{Z}$

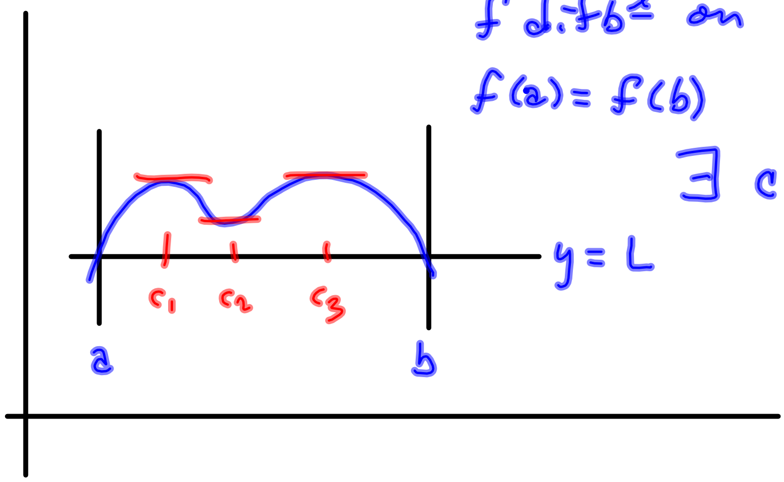
is an element/member of

S'3.1 # 14
I dropped
a sign.

§3.1 & 3.2
Understanding the hypotheses of the theorems.

<u>FINDING</u> that pesky 'c.'

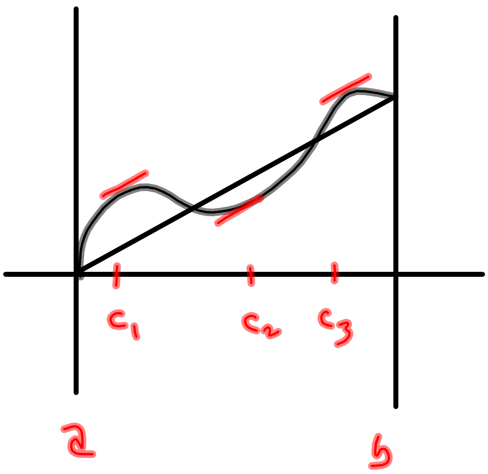
Rolle's Theorem – f cnt's on $[a,b]$,
f dif'ble on (a,b), and
$f(a) = f(b) \implies$
$\exists\, c \in (a,b) \ni f'(c) = 0.$



MVT – f cnt$\underline{s}$ on $[a,b]$,
f difb$\underline{l}$ on (a,b) $\Longrightarrow$

$\exists\ c \in (a,b) \ni f'(c) = m_{avg}$ on $[a,b]$.

$$m_{avg} = \frac{f(b)-f(a)}{b-a}$$



Continuity - Domain for the most part

'Most any function we will see is continuous on its domain.

$\lim_{x \to a} f(x) = f(a) \iff$ cnt$\underline{s}$ @ a.

$\mathcal{D}$: Division by zero, $\sqrt[2n]{\text{negative}}$ } only trouble areas.

Differentiability - Domain of f'

Watch out! $(g(x))^m$, where $0 < m < 1$

f is cnt$\underline{s}$, but f' might not exist.

$x^{2/3}$ $(x+7)^{3/5}$

$f(\theta) = 2\cos\theta + \cos^2\theta, \quad 0 \leqslant \theta \leqslant 2\pi$ $\quad S(x) = x - \sin x, \quad 0 \leqslant x \leqslant 4\pi$

f cnt^s: trigs cnt^s on their domains

f difb^l: $-2\sin\theta + (2\cos\theta)(-\sin\theta)$

trigs cnt^s on $\mathcal{D}$ $\longrightarrow$

$f' \exists.$

Same.

x is poly.

Polys cnt^s on $\mathcal{D}$

$f(\theta) = 2\cos\theta + \cos^2\theta, \quad 0 \leqslant \theta \leqslant 2\pi$ $\quad \boxed{S(x) = x - \sin x,}\ 0 \leqslant x \leqslant 4\pi$

$f'(\theta) = -2\sin\theta - 2\sin\theta\cos\theta$

$f(0) = 2\cos(0) + \cos^2(0)$

$= 2 + 1^2 = 3$

$f(2\pi) = 3$

$\dfrac{f(b) - f(a)}{b - a} = \dfrac{3-3}{2\pi} = \dfrac{0}{2\pi} = 0$

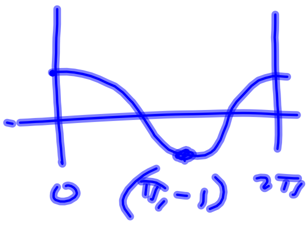
(Rolle's)

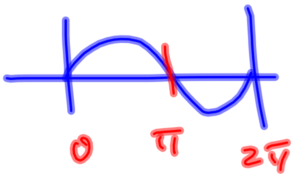
$f'(\theta) = -2\sin\theta - 2\sin\theta\cos\theta \overset{SET}{=} 0$

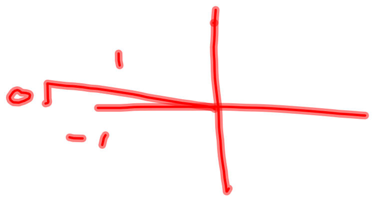
$-2\sin\theta(1 + \cos\theta) = 0$

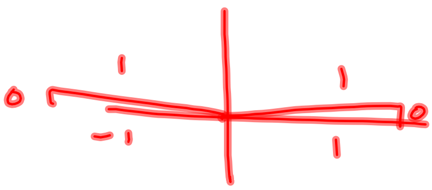
$-2\sin\theta = 0 \quad$ OR $\quad \cos\theta = -1$

$\theta = 0, \pi, 2\pi \qquad\qquad \theta = \pi$









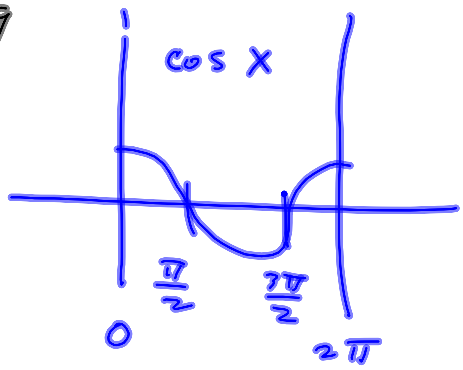
$c \in \{0, \pi, 2\pi\}$

$S(x) = x - \sin x, \quad 0 \leqslant x \leqslant 4\pi$

$S(0) = 0 - \sin(0) = 0$

$S(4\pi) = 4\pi - \sin(4\pi) = 4\pi$

$\frac{f(4\pi) - f(0)}{4\pi - 0} = \frac{4\pi}{4\pi} = 1 = m_{avg}$



$S'(x) = 1 - \cos x \overset{SET}{=} 1$

$-\cos x = 0$

$\cos x = 0$

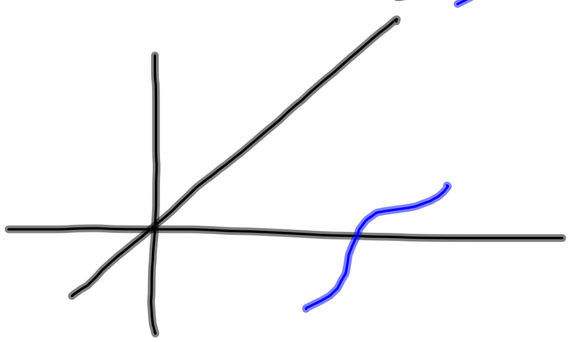
$\rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2},$

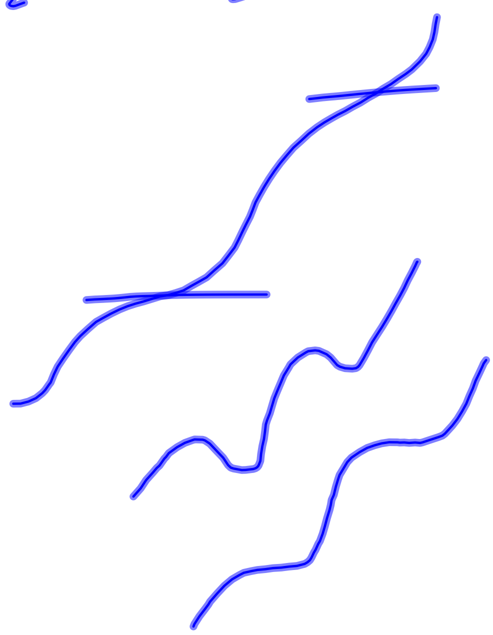
$\left\{ \frac{\pi}{2} + 2n\pi \text{ OR } \frac{3\pi}{2} + 2n\pi,\ n \in \mathbb{Z} \right\}$

$\frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$ $\qquad$ $\frac{3\pi}{2} + 2\pi = \frac{7\pi}{2}$

$\frac{\pi}{2} + 4\pi = \frac{9\pi}{2} \notin (0, 4\pi)$

$c \in \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \right\}$





$f(\theta) = 2\cos\theta + \cos^2\theta, \quad 0 \leqslant \theta \leqslant 2\pi$

f is cnt$^{\underline{s}}$ on $[0, 2\pi]$, because cosine is continuous on its domain, and all positive powers. Also, the sum of two cnt^{s} functions is cnt$^{\underline{s}}$

$f'(\theta) = -2\sin\theta + (2\cos\theta)(-\sin\theta) = -2\sin\theta - 2\sin\theta\cos\theta.$

Defined, because sine & cosine are defined and so is their product.

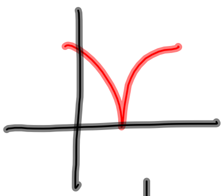
SUBTLE POINT: The Theorem does NOT say that f' has to be continuous (although it usually is.) It JUST has to be DEFINED!

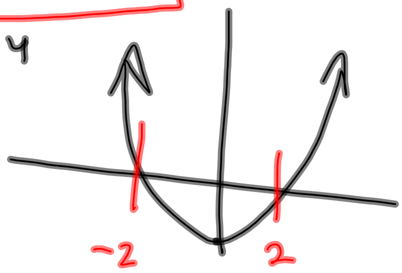
Nonexamples: Funcs that do NOT satisfy the hypotheses of MVT.

$f(x)=\sqrt{x^2-4}$ on $[-1, 3]$

$\boxed{(x-2)^{2/3} \text{ on } [1,3]}$

x^2-4

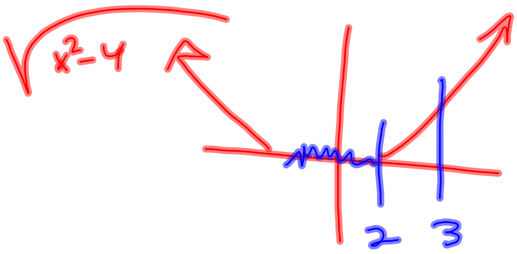




Does not satisfy hypo of MVT.
Not Cnt's on $[-1,3]$!

$\frac{2}{3}(x-2)^{-\frac{1}{3}}$

$= \frac{2}{3\sqrt[3]{x-2}}$



$(x-2)^{2/3}$ on $[1,3]$

$f(1) = (-1)^{2/3} = ((-1)^2)^{\frac{1}{3}} = (1)^{\frac{1}{3}} = 1$

$f(3) = 1^{2/3} = 1$

why no c in $(1,3) \ni f'(c) = 0$

B/c $f'(2) \nexists$.

$2 \in (1,3)$