Fermat's Theorem

If $f(c)$ is local max & $f'(c)$ exists, then $f'(c) = 0$

Pf

$\$$ $f(c)$ is a local max & h is "small"

Then $f(c+h) \leq f(c)$

$\Rightarrow f(c+h) - f(c) \leq 0$ Assume $h > 0 \Rightarrow$

$\Rightarrow \dfrac{f(c+h) - f(c)}{h} \leq 0$

$f'(c) = \lim\limits_{h \to 0^+} \dfrac{f(c+h) - f(c)}{h} \leq 0$

Assume $h < 0$.

Then $f(c+h) - f(c) \leq 0$, again, but now $h < 0$.

$\dfrac{f(c+h) - f(c)}{h} \geq 0$

$\Rightarrow f'(c) = \lim\limits_{h \to 0^-} \dfrac{f(c+h) - f(c)}{h} \geq 0$

$0 \leq f'(c) \leq 0 \Rightarrow f'(c) = 0$