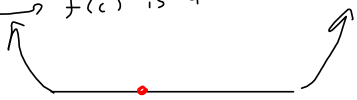
Section 3.1 Maximum and Minimum Values

1 Definition Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geqslant f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leqslant f(x)$ for all x in D.

$f(c)$ is a number.



$(c, f(c))$

c is not necessarily unique!

2 Definition The number $f(c)$ is a

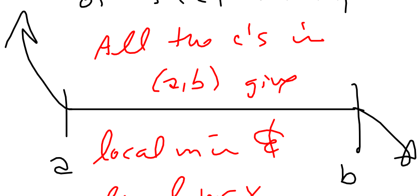
- **local maximum** value of f if $f(c) \geqslant f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leqslant f(x)$ when x is near c.

This is sketchier

Any point in the interior of a horizontal part of the function is both local max and local min.

$f(c) > f(x)\ \forall x$ near c. Better defin of local max

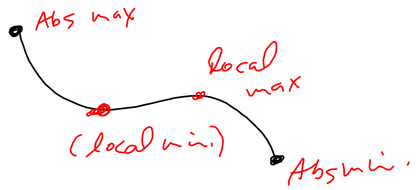
or $f(c) < f(x)$ " " " " " " " local min



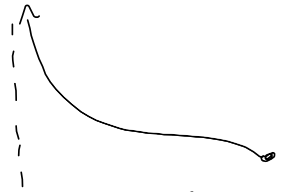
All the c's in (a,b) give local min & local max

3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

By our defin, c & d aren't necessarily unique.

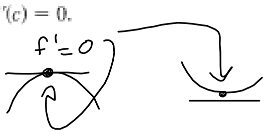


Nonexample



cnt's on $(2, b]$
No abs. max.

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.



Firmah

FirMAH

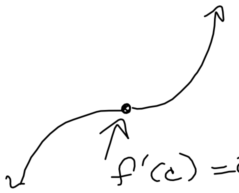
Famous Algebraist,
Fermat's Last Theorem is 100s of years old, but was proven about 10yrs ago.

Converse of Fermat's Theorem Doesn't necessarily hold.

You can have $f'(c) = 0$, but $f(c)$ not be an extreme.

Nonexample $f(x) = x^3$

$f'(x) = 2x^2 = 0$, when $x=0$,
but not a max/min!

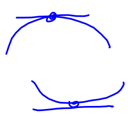


$f'(c) = 0$
NOT an extreme

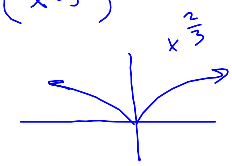
6 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

why $f'(c) = 0$ or $f'(c) \nexists$ as candidates for extrema.

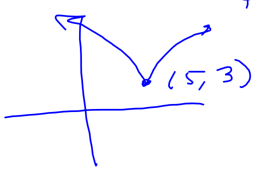
$f'(c)=0$



$(x-5)^{\frac{2}{3}}$



$f(x)$ $(x-5)^{\frac{2}{3}} + 3 \Rightarrow f'(x) = \frac{2}{3}(x-5)^{-\frac{1}{3}}$



$\nexists$ at $x=5$,
and $(5,3)$ is a local (and absolute) min.

7 If f has a local maximum or minimum at c, then c is a critical number of f.

Need f is cnt's here.

The Closed Interval Method To find the *absolute* $\boxed{\text{maximum}}$ and minimum values of a continuous function f on a closed interval $\boxed{[a, b]}$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

1 $\left.\begin{array}{l} f'(x) = 0 \\ f'(x) \nexists \end{array}\right\}$

2 $f(a), f(b)$

1. Question Details SCalc8 3.1.001. [3

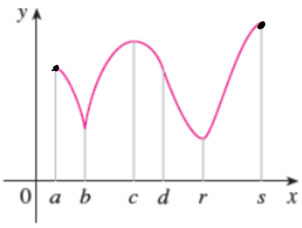
Explain the difference between an absolute minimum and a local minimum.

○ A function f has an **absolute minimum** at $x = c$ if $f(c)$ is the smallest function value on the entire domain of f, whereas f has a **local minimum** at c if $f(c)$ is the smallest function value when x is near c.

Local, not absolute min.

2. Question Details SCalc8 3.1.003. [3354503

For each of the numbers a, b, c, d, r, and s, state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum, or neither a maximum nor a minimum. ~~(Enter your answers as a comma-separated list.)~~

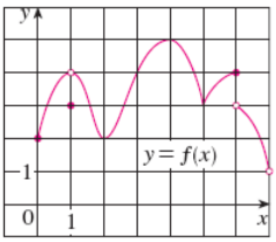


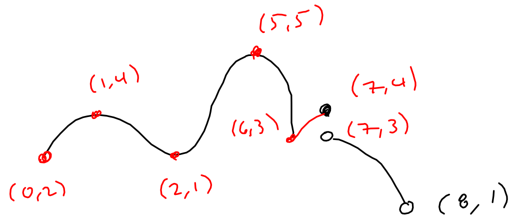
a is sort of a local max, but we don't count it as such, b/c nothing's taking place to its left

a Nada
b Local Min ($f'(b)$ ∄)
c Local Max ($f'(c) = 0$)
d Neither
r local min
s Absolute Max.

3. Question Details SCalc8 3.1.005. [335456

Use the graph to state the absolute and local maximum and minimum values of the function. (Assume each point lies on the gridlines. Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)



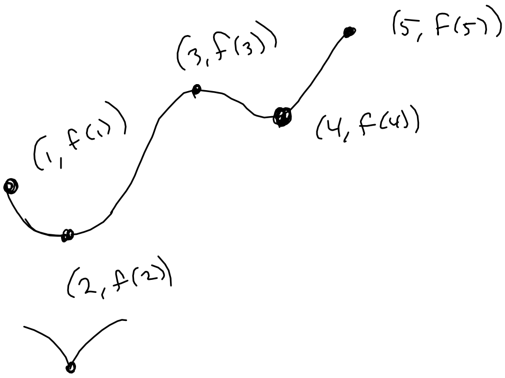


Abs Max: y=5 @ x=5
Abs Min: NONE!

4. Question Details SCalc8 3.1.007.

Sketch the graph of a function f that is continuous on [1, 5] and has the given properties.

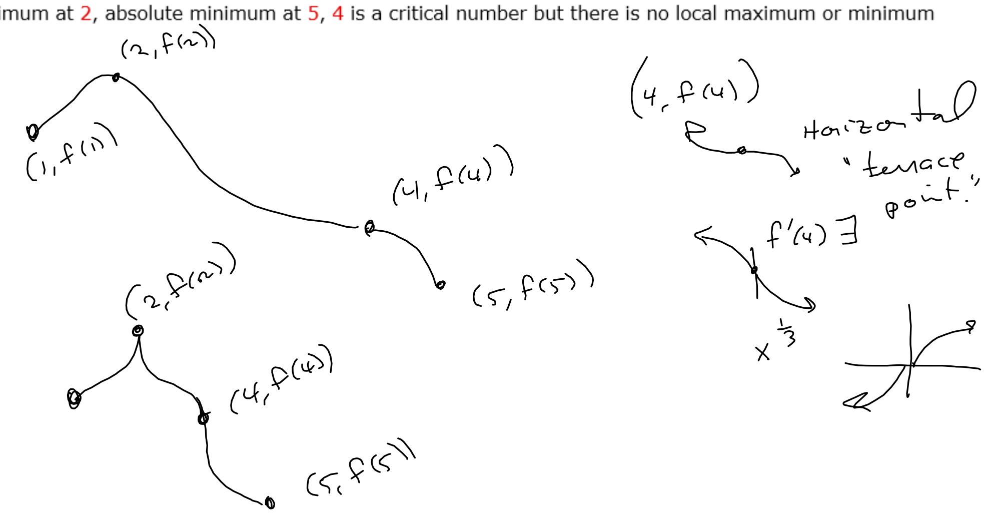
Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4



5. Question Details SCalc8 3.1.010. [33541

Sketch the graph of a function *f* that is continuous on [1, 5] and has the given properties.

Absolute maximum at 2, absolute minimum at 5, 4 is a critical number but there is no local maximum or minimum there.

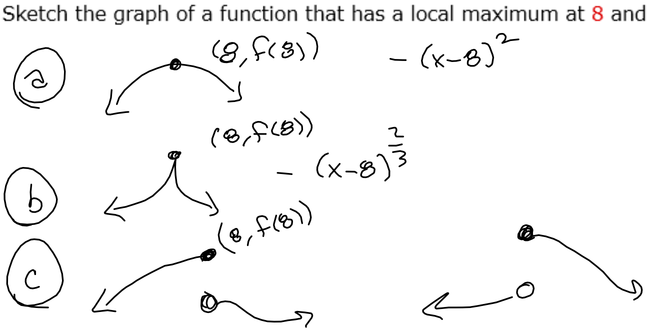


6. Question Details SCalc8 3.1.011.

(a) Sketch the graph of a function that has a local maximum at 8 and is differentiable at 8.

(b) Sketch the graph of a function that has a local maximum at 8 and is continuous but not differentiable at 8.

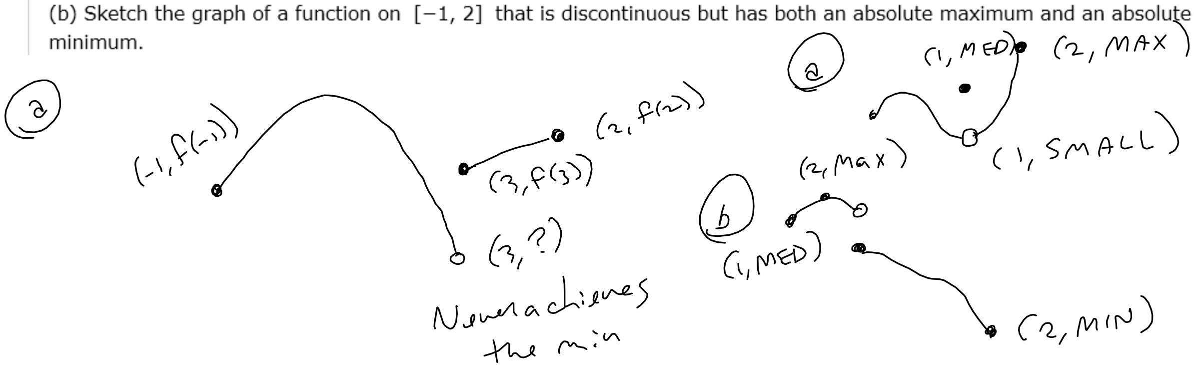
(c) Sketch the graph of a function that has a local maximum at 8 and is not continuous at 8.



7. **Question Details** SCalc8 3.1.013. [33

(a) Sketch the graph of a function on [−1, 2] that has an absolute maximum but no absolute minimum.

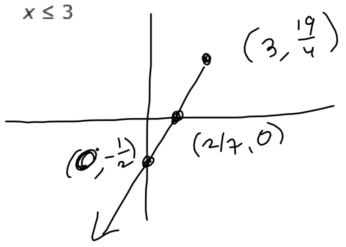
(b) Sketch the graph of a function on [−1, 2] that is discontinuous but has both an absolute maximum and an absolute minimum.



8. **Question Details** SCalc8 3.1.015. [3354432]

Sketch the graph of *f* by hand and use your sketch to find the absolute and local maximum and minimum values of *f*. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$f(x) = \frac{1}{4}(7x - 2), \quad x \le 3$



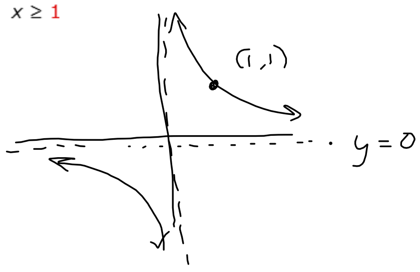
$(3, \frac{19}{4})$ = Abs. Max. Pt.

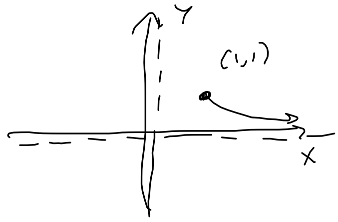
No Min.

9. **Question Details** SCalc8 3.1.017. [3354194]

Sketch the graph of *f* by hand and use your sketch to find the absolute and local maximum and minimum values of *f*. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$f(x) = \frac{1}{x}, \quad x \geq 1$





Abs max: y = 1 @ x = 1

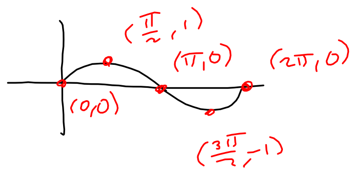
No Abs. MIN.

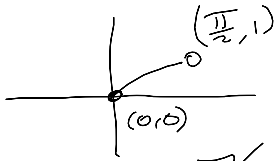
10. **Question Details** SCalc8 3.1.019. [3354154]

Sketch the graph of *f* by hand and use your sketch to find the absolute and local maximum and minimum values of *f*. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$f(x) = \sin(x), \quad 0 \leq x < \frac{\pi}{2}$

Strict!





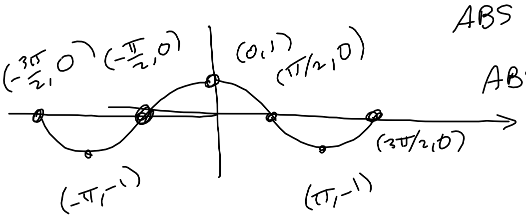
ABS MAX ∄

∴ MIN y = 0 @ x = 0

11. Question Details SCalc8 3.1.022. [3354297]

Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$f(t) = 8\cos(t), \quad -3\pi/2 \le t \le 3\pi/2$



ABS max: $y = 1$ @ $x = 0$

ABS MIN: $y = -1$ @ $x = -\pi, \pi$

local MAX: $y = 1$ @ $x = 0$

" MIN: $y = -1$ @ $x = -\pi, \pi$

12. Question Details SCalc8 3.1.023. [3354

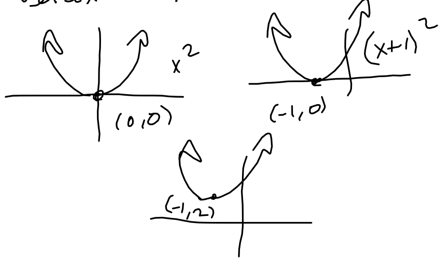
Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (If an answer does not exist, enter DNE.)

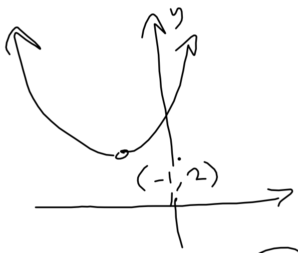
$f(x) = 2 + (x + 1)^2, \quad -2 \le x < 4$

$= (x+1)^2 + 2$

$= a(x-h)^2 + k$

Vertex $= (h,k) = (-1,2)$





I want the (x,y) — pairs, generally "points"

$\left.\begin{array}{l}\text{ABS MAX}\\ \text{LOCAL MAX}\end{array}\right\} \nexists$

$\left.\begin{array}{l}\text{ABS MIN}\\ \text{LOCAL MIN}\end{array}\right\} y = 2$ @ $x = -1$

or just $\rightarrow (-1,2)$

13. Question Details SCalc8 3.1.025. [3354557]

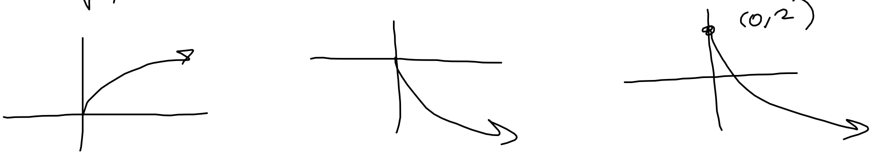
Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. ~~(Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)~~

$f(x) = 2 - \sqrt{x}$

http://harryzaims.com/121-all/videos/03-Writing-Projects/Writing-Project-2/

Link (click)

$\sqrt{x} \longrightarrow -\sqrt{x} \longrightarrow -\sqrt{x} + 2$



ABS Max: (0,2)

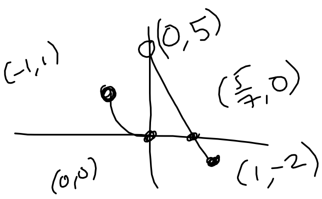
No Mins

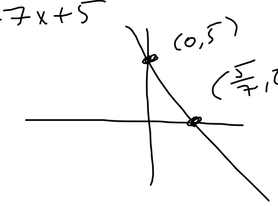
14. Question Details SCalc8 3.1.027. [3435934]

Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum values of f. (~~Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)~~

$$f(x) = \begin{cases} x^2 & \text{if } -1 \le x \le 0 \\ 5 - 7x & \text{if } 0 < x \le 1 \end{cases}$$

$-7x+5$





ABS MAX: ∄

" MIN: $y=-2$ @ $x=1$

(1,-2) Pt.

Local Max ∄

None

Local Min: (0,0)

15. Question Details SCalc8 3.1.029. [335414

Find the critical numbers of the function. (~~Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)~~

$f(x) = 5 + \frac{1}{3}x - \frac{1}{2}x^2 = -\frac{1}{2}x^2 + \frac{1}{3}x + 5 \Rightarrow f'(x) = -x + \frac{1}{3} \overset{\text{SET}}{=} 0$

Crappy Style

$-x = -\frac{1}{3}$

$\boxed{x = \frac{1}{3}}$

16. Question Details SCalc8 3.1.030. [335449

Find the critical numbers of the function. (~~Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)~~

$f(x) = x^3 + 3x^2 - 144x \longrightarrow f'(x) = 3x^2 + 6x - 144 \overset{\text{SET}}{=} 0$

$a=3, b=6, c=-144$

$6^2 - 4(3)(-144)$

$36 + 1728 = 1764$

$x = \frac{-6 \pm \sqrt{1764}}{2(3)}$

$= \frac{-6 \pm 42}{6} = -1 \pm 7$

6, -8

critical #s

$\boxed{x = -8, 6}$

2 | 1764
2 | 882
3 | 441
3 | 147
7 | 49
7

17. **Question Details** SCalc8 3.1.032. [33541

Find the critical numbers of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$f(x) = 4x^3 + x^2 + 4x \Rightarrow f'(x) = 12x^2 + 2x + 4 \overset{\text{SET}}{=} 0$

$\Rightarrow 6x^2 + x + 2 = 0$

$a = 6, b = 1, c = 2$ $\quad b^2 - 4ac = 1^2 - 4(6)(2) < 0$

$\boxed{\nexists \text{ critical \#s}}$

18. **Question Details** SCalc8 3.1.034. [33541

Find the critical numbers of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$g(t) = |5t - 9|$ $\quad 5t - 9 = 0 \Rightarrow t = \frac{9}{5} \Rightarrow$ No derivative.

$$g(t) = \begin{cases} 5t - 9 & \text{if } t \geq \frac{9}{5} \\ -(5t-9) & \text{if } t < \frac{9}{5} \end{cases} \Rightarrow$$

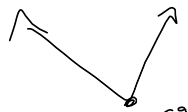
$$g'(t) = \begin{cases} 5 & \text{if } t > \frac{9}{5} \\ -5 & \text{if } t < \frac{9}{5} \end{cases}$$

$g'_-(\frac{9}{5}) = 5$

$g'_+(\frac{9}{5}) = -5$

$g'(\frac{9}{5}) \nexists$, but $g(\frac{9}{5}) = 0$

$t = \frac{9}{5}$ is critical



$(\frac{9}{5}, 0)$

Local & Abs. MIN.

19. Question Details SCalc8 3.1.035.MI. [335421

Find the critical numbers of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$g(y) = \frac{y-3}{y^2-3y+9} \Longrightarrow$

Challenge your algebra skills

$\frac{A}{B} = 0 \Longrightarrow A = 0$

$g'(y) = \frac{(1)(y^2-3y+9) - (y-3)(2y-3)}{(y^2-3y+9)^2}$ SET $= 0$

SET $= 0$ to find where $g'(y)$ ∄

If they're in the domain, then they're critical.

But any solns, here, won't be in $D(g)$.

$y^2-3y+5-[2y^2-9y+9] = 0$

$y^2-3y+5-2y^2+9y-9 = 0$

$-y^2+6y-4 = 0$

$y^2-6y+4 = 0$

$a=1, b=-6, c=4$

$b^2-4ac = (-6)^2-4(1)(4) = 36-16 = 20$

2, 3, 5, 7, 11, 13, 17, 19

$2 \,|\, 20$
$2 \,|\, 10$
5

$x = \frac{6 \pm 2\sqrt{5}}{2(1)} = \boxed{\frac{6 \pm 2\sqrt{5}}{2}}$

20. Question Details SCalc8 3.1.036. [335442

Find the critical numbers of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$h(p) = \frac{p-5}{p^2+2} \Longrightarrow h'(p) = \frac{(1)(p^2+2) - (p-5)(2p)}{(p^2+2)^2}$ SET $= 0$

$\Longrightarrow p^2+2-(2p^2-10p) = p^2+2-2p^2+10p = -p^2+10p+2 = 0$

$\Longrightarrow p^2-10p-2 = 0$

$\Longrightarrow p^2-10p+5^2-25-2 = (p-5)^2-27 = 0$

$(p-5)^2 = 27$

$p-5 = \pm\sqrt{27} = \pm 3\sqrt{3} \Longrightarrow \boxed{p = 5 \pm 3\sqrt{3}}$

21. **Question Details** SCalc8 3.1.037.MI. [33541

Find the critical numbers of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

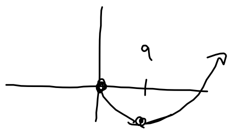
$h(t) = t^{3/4} - 9t^{1/4}$ $\qquad \mathcal{D} = [0, \infty)$ $\qquad$ Algebra Skills!

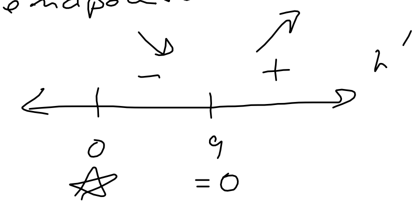
$$h'(t) = \frac{3}{4}t^{-\frac{1}{4}} - \frac{9}{4}t^{-\frac{3}{4}} = \frac{3}{4t^{\frac{1}{4}}} - \frac{9}{4t^{3/4}} = \frac{3}{4t^{\frac{1}{4}}} \cdot \frac{t^{\frac{2}{4}}}{t^{\frac{2}{4}}} - \frac{9}{4t^{3/4}}$$

$$= \frac{3t^{\frac{1}{2}} - 9}{4t^{3/4}} \overset{\text{SET}}{=} 0 \implies 3t^{\frac{1}{2}} - 9 = 0 \implies 3t^{\frac{1}{2}} = 9 \implies t^{\frac{1}{2}} = 3$$

$$\implies \boxed{t = 9}$$

Also, $h'(t) \nexists$ when $4t^{\frac{3}{4}} = 0$, i.e. $\boxed{t = 0}$ is an endpoint.





22. **Question Details** SCalc8 3.1.039. [335442

Find the critical numbers of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$\mathcal{D} = \mathbb{R}$ $\qquad F(x) = x^{4/5}(x - 8)^2 \implies F'(x) = \frac{4}{5}x^{-\frac{1}{5}}(x-8)^2 + x^{\frac{4}{5}}(2(x-8))$

$$= \frac{4(x-8)^2}{5x^{\frac{1}{5}}} + \frac{(2(x-8))(x^{\frac{4}{5}})}{1} \cdot \frac{x^{\frac{1}{5}} \cdot 5}{x^{\frac{1}{5}} \cdot 5}$$

$$= \frac{4(x-8)^2}{5x^{\frac{1}{5}}} + \frac{10x(x-8)}{5x^{\frac{1}{5}}} = \frac{2(x-8)\left[2(x-8) + 5x\right]}{5x^{\frac{1}{5}}}$$

$$= \frac{2(x-8)\left[2x - 16 + 5x\right]}{5x^{\frac{1}{5}}} = \frac{2(x-8)(7x-16)}{5x^{\frac{1}{5}}} \overset{\text{SET}}{=} 0 \implies x \in \left\{\frac{16}{7}, 8\right\}$$

$\implies \nexists$ @ $x = 0$

So, critical #s are $\left\{0, \frac{16}{7}, 8\right\}$

23. **Question Details** SCalc8 3.1.040. [3354398]

Find the critical numbers of the function. (Enter your answers as a comma-separated list. Use n to denote any arbitrary integer values. If an answer does not exist, enter DNE.)

$$g(\theta) = 24\theta - 6\tan(\theta)$$

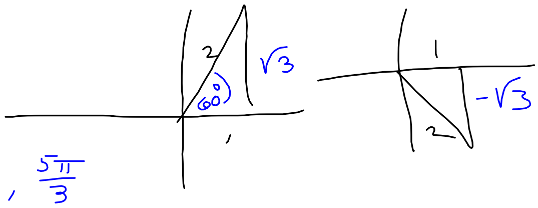
$\Rightarrow g'(\theta) = 24 - 6\sec^2\theta \overset{\text{SET}}{=} 0$

$\Rightarrow 6\sec^2\theta = 24$

$\Rightarrow \sec^2\theta = 4$

$\sec\theta = \pm 2$

$\cos\theta = \pm \frac{1}{2}$



$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

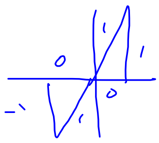
$= 60°, 300°$

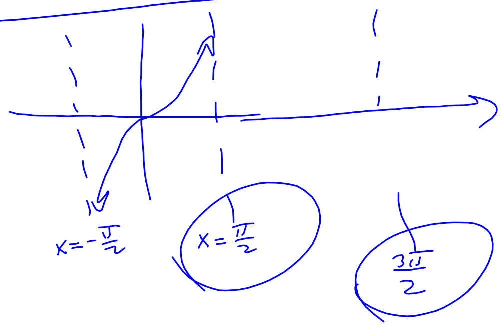
$\boxed{\theta = \frac{\pi}{3} + 2n\pi,\ \frac{5\pi}{3} + 2n\pi,\ n \in \mathbb{Z}}$

Look @ $\sec^2\theta$ $\nexists$:

when $\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \notin D(\tan\theta)$





24. **Question Details** SCalc8 3.1.042. [33545

Find the critical numbers of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$$g(x) = \sqrt{9 - x^2}$$

$= (9-x^2)^{\frac{1}{2}} \Rightarrow$

$g'(x) = \frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{9-x^2}} \overset{\text{SET}}{=} 0 \Rightarrow x = 0$

$\sqrt{9-x^2} = 0 \Rightarrow x = \pm 3$

$9 - x^2 = 0$

$9 = x^2$

$\boxed{\{0, \pm 3\}}$

25. Question Details SCalc8 3.1.047.

Find the absolute maximum and absolute minimum values of *f* on the given interval.

$f(x) = 2x^3 - 3x^2 - 36x + 9, \quad [-3, 4]$

$\Rightarrow f'(x) = 6x^2 - 6x - 36 \overset{SET}{=} 0$

$\Rightarrow x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\Rightarrow x = -2, 3$

Check Criticals:

$$\boxed{\begin{aligned} f(-2) &= 53 \\ f(3) &= -72 \end{aligned}}$$

Check endpoints

$$\boxed{\begin{aligned} f(-3) &= 36 \\ f(4) &= -23 \end{aligned}}$$

$f(-2) = 53$ Abs MAX

$f(3) = -72$ " MIN

$f(4) = -23$ Rel. Min

$f(-3) = 36$ Rel. Max

26. Question Details SCalc8 3.1.049.

Find the absolute maximum and absolute minimum values of *f* on the given interval.

$f(x) = 6x^4 - 8x^3 - 24x^2 + 1, \quad [-2, 3]$

$\Rightarrow f'(x) = 24x^3 - 24x^2 - 48x \overset{SET}{=} 0$

$\Rightarrow 24x(x^2 - x - 2) = 0$

$\Rightarrow 24x(x-2)(x+1) = 0$

$\Rightarrow x \in \{-1, 0, 2\}$

$f(-2) = 65$

$$\begin{array}{r|rrrrr} -2 & 6 & -8 & -24 & 0 & 1 \\ & & -12 & 40 & -32 & 64 \\ \hline & 6 & -20 & 16 & -32 & 65 \end{array}$$

$f(3) = 55$

$$\begin{array}{r|rrrrr} 3 & 6 & -8 & -24 & 0 & 1 \\ & & 18 & 30 & 18 & 54 \\ \hline & 6 & 10 & 6 & 18 & 55 \end{array}$$

$f(-1) = -9$

$$\begin{array}{r|rrrrr} -1 & 6 & -8 & -24 & 0 & 1 \\ & & -6 & 14 & 10 & -10 \\ \hline & 6 & -14 & -10 & 10 & -9 \end{array}$$

$f(0) = 1$

$f(2) = -63$

$$\begin{array}{r|rrrrr} 2 & 6 & -8 & -24 & 0 & 1 \\ & & 12 & 8 & -32 & -64 \\ \hline & 6 & 4 & -16 & -32 & -63 \end{array}$$

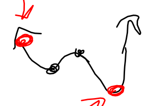
$\boxed{f(-2) = 65}$ Abs Max

$f(-1) = -9$ local MIN

$f(0) = 1$ local MAX

$\boxed{f(2) = -63}$ Abs MIN

$f(3) = 55$ ENDPOINT Neither.



27. Question Details SCalc8 3.1.052.

Find the absolute maximum and absolute minimum values of *f* on the given interval.

$f(x) = \dfrac{x}{x^2 - x + 16}, \quad [0, 12]$

$f'(x) = \dfrac{1(x^2 - x + 16) - x(2x-1)}{(x^2 - x + 16)^2} \overset{SET}{=} 0 \Rightarrow$

$x^2 - x + 16 - 2x^2 + x = 0 \Rightarrow -x^2 + 16 = (4-x)(4+x) = 0 \Rightarrow$

$x \in \{-4, 4\}$

Denom $(f') = 0 \Rightarrow$ Denom $(f) = 0$

No worries for $x^2 - x + 16 = 0$

$f(4) = \dfrac{4}{16 - 4 + 16} = -1$

$f(12) = \dfrac{12}{144 - 12 + 16} = \dfrac{12}{148} = \dfrac{6}{74} = \dfrac{3}{37}$

~~$f' = 0$ $f(-4)$~~ NADA $f(0) = 0$

ENDPT $f(0)$

$f' = 0$ $f(4)$

ENDPT $f(12)$

ABS. MIN $f(4) = -1$

ABS. MAX $f(12) = \dfrac{3}{37}$

28. Question Details SCalc8 3.1.056.

Find the absolute maximum and absolute minimum values of f on the given interval.

$f(t) = 9t + 9\cot(t/2)$, $[\pi/4, 7\pi/4]$ is cnts & difb^{le}

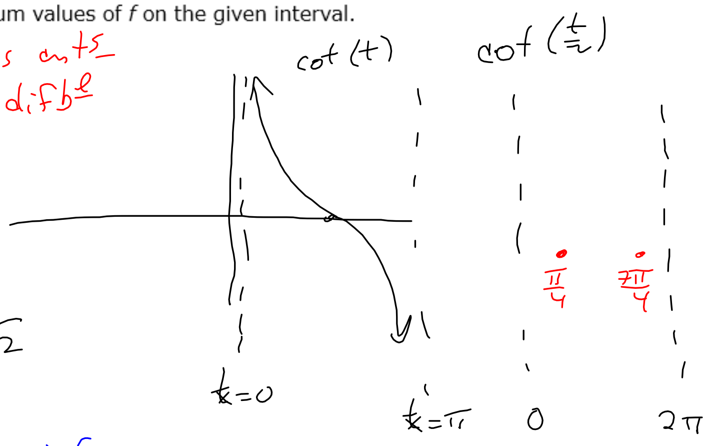
$\Longrightarrow f'(t) = 9 - \frac{1}{2}\csc^2\left(\frac{t}{2}\right)$

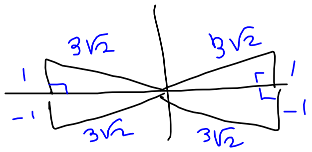
$\underset{=}{\text{SET}}\ 0 \Longrightarrow -\frac{1}{2}\csc^2\left(\frac{t}{2}\right) = -9$

$\csc^2\left(\frac{t}{2}\right) = 18$

$\csc\left(\frac{t}{2}\right) = \pm\sqrt{18} = \pm 3\sqrt{2}$

$\sin\left(\frac{t}{2}\right) = \pm\frac{1}{3\sqrt{2}}$

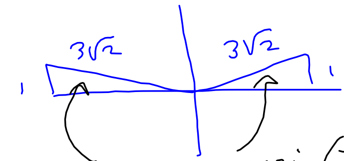




$t \in \left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$

$\frac{t}{2} \in \left[\frac{\pi}{8}, \frac{7\pi}{8}\right]$

So, really, $\frac{t}{2}$ pic is this:



$\pi - \arcsin\left(\frac{1}{3\sqrt{2}}\right) \approx 2.903651529$

$\arcsin\left(\frac{1}{3\sqrt{2}}\right) \approx 0.2379411248$

$f\left(\arcsin\left(\frac{1}{3\sqrt{2}}\right)\right)$ ABS MAX ≈ 77.4332

$f\left(\pi - \arcsin\left(\frac{1}{3\sqrt{2}}\right)\right) \approx 27.2087$ ABS MIN

$f\left(\frac{\pi}{4}\right) \approx 28.7965$

$f\left(\frac{7\pi}{4}\right) \approx 27.7522$

$f\left(\frac{\text{Pi}}{4}\right)$ $\quad \frac{9}{4}\pi + 9\cot\left(\frac{1}{8}\pi\right)$ $\quad 28.79650553$

$\mathit{evalf}\left(f\left(\arcsin\left(\frac{1}{3\cdot\text{sqrt}(2)}\right)\right)\right)$ $\quad 77.43318695$

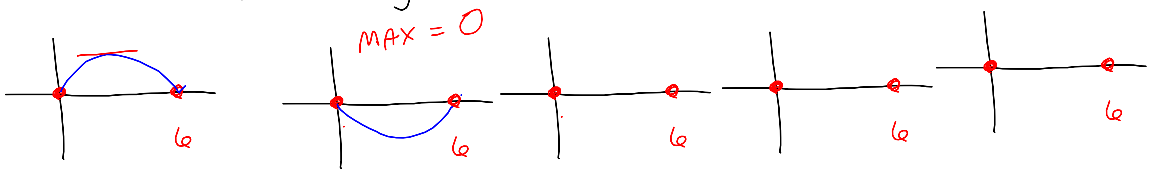
$\mathit{evalf}\left(f\left(\text{Pi} - \arcsin\left(\frac{1}{3\cdot\text{sqrt}(2)}\right)\right)\right)$ $\quad 27.20867932$

$\mathit{evalf}\left(f\left(\frac{7\cdot\text{Pi}}{4}\right)\right)$ $\quad 27.75216224$

29. Question Details SCalc8 3.1.057.

If a and b are positive numbers, find the maximum value of $f(x) = x^a(6 - x)^b$ on the interval $0 \le x \le 6$.

Holy Moley!



$f'(x) = ax^{a-1}(6-x)^b + x^a\left(b(6-x)^{b-1}(-1)\right)$

$= x^{a-1}(6-x)^{b-1}\left[a(6-x) + bx(-1)\right]$

$= \boxed{x^{a-1}(6-x)^{b-1}}\left[6a - 6x - bx\right] \overset{\text{SET}}{=} 0$

ENDPOINTS

$\Rightarrow -(6+b)x = -6a$

$(b+6)x = 6a \Rightarrow \boxed{x = \frac{6a}{6+b}}$

$f\left(\frac{6}{6+b}\right) = ?$

Computer Algebra Says:

$f := x \to x^a \cdot (6 - x)^b$

$$x \to x^a\,(6 - x)^b$$

$fp := \mathrm{D}(f)$

$$x \to \frac{x^a\,a\,(6 - x)^b}{x} - \frac{x^a\,(6 - x)^b\,b}{6 - x} = ax^{a-1}(6-x)^b - bx^a(6-x)^{b-1}$$

$solve(fp(x) = 0, x)$

$$\frac{6\,a}{a + b}$$ Same as I got

$f\left(\frac{6 \cdot a}{a + b}\right)$

$$\left(\frac{6\,a}{a + b}\right)^a \left(6 - \frac{6\,a}{a + b}\right)^b$$

I thought it'd be something nice, but it just plugged in $\frac{6a}{a+b}$ & went no further.

$simplify(\%)$

$$\boxed{6^{a + b} \left(\frac{a}{a + b}\right)^a \left(\frac{b}{a + b}\right)^b}$$

FACTORED 6 out of both factors.
BIG DEAL.

30. Question Details SCalc8 3.1.059

Consider the following.

$f(x) = x^5 - x^3 + 7, \quad -1 \le x \le 1$

(a) Use a graph to find the absolute maximum and minimum values of the function to two decimal places.

(b) Use calculus to find the exact maximum and minimum values.

(a) ——//——

(b) $f'(x) = 5x^4 - 3x^2 = x^2(5x^2 - 3) \underset{=}{\text{SET}} 0$ OK

$\Rightarrow x = 0$ OR $5x^2 - 3 = 0$

$5x^2 = 3$

$x^2 = \frac{3}{5}$

$\Rightarrow x = \pm\sqrt{\frac{3}{5}} = \pm\frac{\sqrt{15}}{5}$

$f(1) = f(-1) = 7$

$f\left(\frac{\sqrt{15}}{5}\right) = \left(\frac{\sqrt{15}}{5}\right)^5 - \left(\frac{\sqrt{15}}{5}\right)^3 + 7$

$\boxed{\text{MIN } f\left(\frac{\sqrt{15}}{5}\right) = -\frac{6}{125}\sqrt{15} + 7}$

≈ 6.814

$\boxed{\text{MAX } f\left(-\frac{\sqrt{15}}{5}\right) = \frac{6}{125}\sqrt{15} + 7}$

≈ 7.186

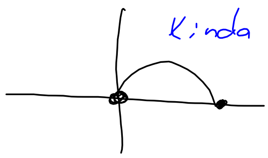
31. Question Details SCalc8 3.1.061.

Consider the following.

$f(x) = 8x\sqrt{x - x^2} = 8x(x - x^2)^{\frac{1}{2}}$

~~(a) Use a graph to find the absolute maximum and minimum values of the function to two decimal places.~~

(b) Use calculus to find the exact maximum and minimum values.



$x - x^2 \ge 0$

$x(1 - x) \ge 0$

$- \quad + \quad -$

$0 \qquad 1$

$D = [0, 1]$!

$f'(x) = 8(x - x^2)^{\frac{1}{2}} + 8x\left(\frac{1}{2}(x - x^2)^{-\frac{1}{2}}(1 - 2x)\right)$

$\underset{=}{\text{SET}} 0 \Rightarrow \frac{(x - x^2)^{\frac{1}{2}}}{1} \cdot \frac{2(x - x^2)^{\frac{1}{2}}}{2(x - x^2)^{\frac{1}{2}}} + \frac{1 - 2x}{2(x - x^2)^{\frac{1}{2}}}$

$\frac{2(x - x^2) + 1 - 2x}{2(x - x^2)^{\frac{1}{2}}} = \frac{2x - 2x^2 + 1 - 2x}{2(x - x^2)^{\frac{1}{2}}} = \frac{-2x^2 + 1}{2(x - x^2)^{\frac{1}{2}}} = 0$

$\Rightarrow 2x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

$f(x) = 8x\sqrt{x - x^2}$

$f(0) = 0$

$f(1) = 8\left(\sqrt{1 - 1^2}\right) = 0$

ABS MIN

$\boxed{f\left(\frac{1}{\sqrt{2}}\right) = \frac{8}{\sqrt{2}}\sqrt{\sqrt{2} - 2}}$ ABS MAX.

~~$f\left(-\frac{1}{\sqrt{2}}\right)$~~ $-\frac{1}{\sqrt{2}} \notin D = [0, 1]$