1. (15 pts) Find $f'(x)$ by the definition of the derivative (The long way!) for $f(x) = 2x^2 - 5x - 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2 - 5(x+h) - 1 - [2x^2 - 5x - 1]}{h}$$

$$= \frac{2(x^2+2xh+h^2) - 5x - 5h - 1 - 2x^2 + 5x + 1}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - 5h - 2x^2}{h}$$

$$= \frac{4hx + 2h^2 - 5h}{h} = \frac{h[4x + 2h - 5]}{h}$$

$$= 4x + 2h - 5 \quad \boxed{(h \neq 0)} \xrightarrow{h \to 0} 4x - 5$$

$$\frac{x^2+2x+1}{x+1} = \frac{(x+1)(x+1)}{x+1} = x+1$$

$x \neq -1$

2. Find the first derivatives (5 pts each). Do not simplify!

a. $f(x) = x^2 - x^{-3} + 2\sqrt[3]{x} + \frac{2}{\sqrt{x}} + 111.234$

$= x^2 - x^{-3} + 2x^{\frac{1}{3}} + 2x^{-\frac{1}{2}} + \dots$

$\Rightarrow \boxed{f'(x) = 2x + 3x^{-4} + \frac{2}{3}x^{-\frac{2}{3}} - x^{-\frac{3}{2}}}$

b. $g(x) = \dfrac{6x^5 + 2x^3 - 5x}{x^3 - 1}$ $\Longrightarrow$ $\dfrac{f'g - fg'}{g^2} = \left(\dfrac{f}{g}\right)'$

$$g'(x) = \frac{(30x^4 + 6x^2 - 5)(x^3 - 1) - (6x^5 + 2x^3 - 5x)(3x^2)}{(x^3 - 1)^2}$$

c. $h(x) = \sin^2\left(x^2 + \cos(x)\right) = \left[\sin\left(x^2 + \cos(x)\right)\right]^2$

$h(x) = f(g(k(x)))$

$\Longrightarrow h'(x) = \left(2\sin(x^2+\cos(x))\right)\left(\cos(x^2+\cos(x))\right)\left(2x - \sin(x)\right)$

$\frac{dh}{dx} = \left[f(g(k(x)))\right]' = \frac{df}{dg} \cdot \frac{dg}{dK} \cdot \frac{dK}{dx}$

d. $(x^2-7x)\sin(2x)$ $= y \implies$

$y' = (2x-7)(\sin(2x)) + (x^2-7x)(\cos(2x))(2)$

3. (10 pts) Find an equation of the tangent line to $f(x) = \sqrt[3]{x^2}$ at the point $P = (1,1)$

$= x^{\frac{2}{3}}$

(x_1, y_1)

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$

$\Rightarrow f'(1) = (2/3)(1)^{-\frac{1}{3}} = \frac{2}{3} = m_{tan}$

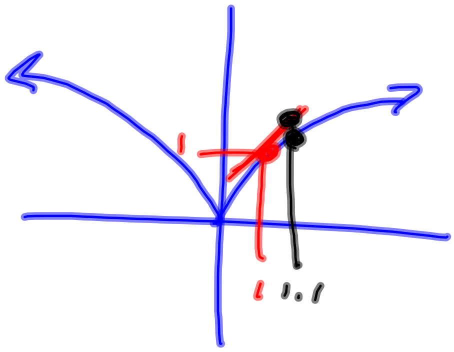
$y = m_{tan}(x - x_1) + y_1$

$\boxed{y = \frac{2}{3}(x-1) + 1}$

4. (5 pts) Estimate $\sqrt[3]{(1.1)^2}$ using the Linearization of a particular function f at a handy value of x.

$f(x) = \sqrt[3]{x^2}$

$L_1(x) = \frac{2}{3}(x-1) + 1$



$\sqrt[3]{1^2} = \sqrt[3]{1} = (1, 1)$

$L_1(1.1) = \frac{2}{3}(1.1-1) + 1$

$\boxed{= \frac{2}{3}(.1) + 1}$

$\approx (.66667)(.1) + 1$

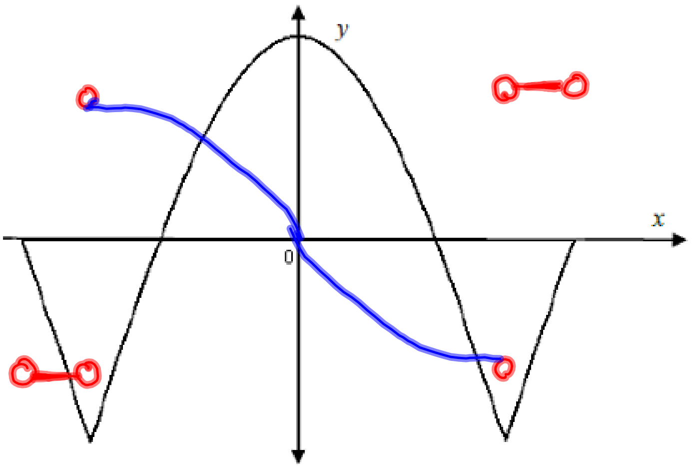
$= .066667 + 1 = 1.066667$

$= .\bar{6}(.1) + 1$

$= .0\bar{6} + 1$

$= 1.0\bar{6}$

5. (10 pts) The graph of a function f is shown. Sketch the graph of $f\,'$ on the same set of axes.



6. (15 pts) Find $\frac{dy}{dx}$, given $x^2y^3 - 5x^2 - 5y^2 = y^3 + 11.3$

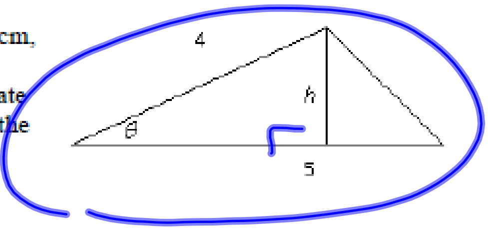
$\frac{dy}{dx} \to y'$

$2xy^3 + x^2(3y^2y') - 10x - 10y\,y' = 3y^2y'$

$y'[3x^2y^2 - 10y - 3y^2] = -2xy^3 + 10x$

$$\boxed{y' = \frac{-2xy^3 + 10x}{3x^2y^2 - 10y - 3y^2}}$$

7. (15 pts) Two sides of a triangle are 4 cm and 5 cm, respectively. The 3rd side keeps changing, as the angle between the other two sides increases at a rate of 0.5 radians per second. Find the rate at which the *area* of the triangle is changing when the angle between the sides of fixed length is $\frac{\pi}{3}$.



Want $\left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}} \to \frac{cm^2}{s}$, where A = Area of triangle (cm^2)

Given $\frac{d\theta}{dt} = \frac{.5 \text{ radians}}{\text{sec.}}$

$A = \frac{1}{2}bh = \frac{1}{2}\cdot 5 \cdot h = \frac{5}{2}h$

$\frac{dA}{dt} = \frac{5}{2}\cdot\frac{dh}{dt}$, but still don't have $\frac{dh}{dt}$

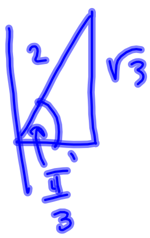
$\frac{h}{4} = \sin\theta \Rightarrow h = 4\sin\theta$

$\Rightarrow A = \frac{5}{2}\cdot 4\sin\theta = 10\sin\theta$

$\frac{dA}{dt} = \frac{dA}{d\theta}\cdot\frac{d\theta}{dt} = (10\cos\theta)\left(\frac{d\theta}{dt}\right)$

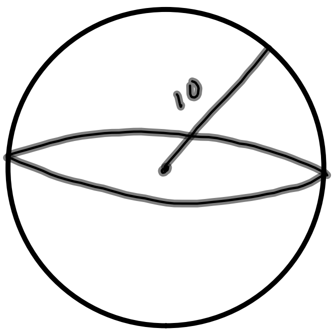
$= (10\cos\theta)(.5) = 5\cos\theta$

$\left.\frac{dA}{dt}\right|_{\theta=\frac{\pi}{3}} = 5\cos\frac{\pi}{3} = 5\left(\frac{1}{2}\right) = \boxed{\frac{5}{2}\ \frac{cm^2}{s}}$



8. (10 pts) The radius of a sphere is measured as 10 cm, with a possible error in measure of 0.15 cm. Use differentials to estimate the maximum possible error in the measurement of the volume of the sphere. Hint: The volume of a sphere is $\frac{4}{3}\pi r^3$.

$10 \pm .15$



want ΔV

$\Delta V \approx dV = \frac{dV}{dr} \cdot dr$

$= V'(r)dr$

$V = \frac{4}{3}\pi r^3 \implies$

$\frac{dV}{dr} = 4\pi r^2$, i.e. $dV = \pi r^2 dr$

$= \pi (10)^2 (\pm .15)$

$= \boxed{\pm 15\pi \text{ cm}^3 \approx \Delta V}$

Actual: $\max \{ |V(10.15) - V(10)|, |V(9.85) - V(10)| \}$