1. Question Details SCalc8 2.8.002. [3354270]

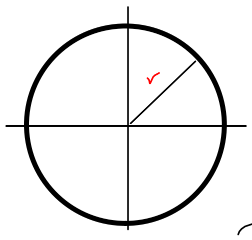
(a) If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

(b) Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 24 m?

when to chain & when not?

a) $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$



b) $\frac{dr}{dt} = 1 \frac{m}{s}$

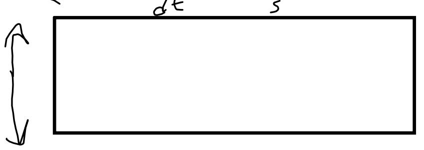
want $\left.\frac{dA}{dt}\right|_{r=24} = \left(2\pi \cdot 24\ m\right)\left(1 \frac{m}{s}\right) = \boxed{48\pi \frac{m^2}{s}}$

2. Question Details SCalc8 2.8.004.MI. [3354090]

The length of a rectangle is increasing at a rate of 6 cm/s and its width is increasing at a rate of 7 cm/s. When the length is 15 cm and the width is 9 cm, how fast is the area of the rectangle increasing?

$\frac{dL}{dt} = 6 \frac{cm}{s}$

$7 \frac{cm}{s} = \frac{dw}{dt}$



w

L

want $\left.\frac{dA}{dt}\right|_{(L,w)=(15,9)}$

$\left.\frac{dA}{dt}\right|_{L=15, w=9}$

$A = LW$

$\frac{dA}{dt} = L'w + Lw'$

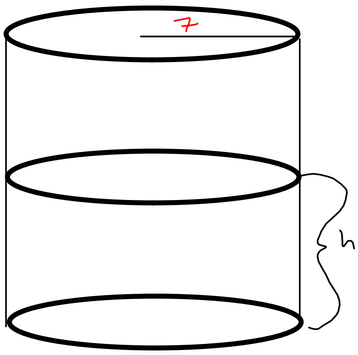
$= \frac{dL}{dt} w + L \frac{dw}{dt}$.

$L=15, w=9 \Rightarrow \frac{dA}{dt} = \underbrace{(6)}_{(cm/s)}\underbrace{(9)}_{cm} + \underbrace{(15)}_{cm}\underbrace{(7)}_{cm/s} = 54 + 105 = \boxed{159 \frac{cm^2}{s} = \left.\frac{dA}{dt}\right|_{L=15, w=9}}$

Sep 28-9:40 AM

3. Question Details SCalc8 2.8.005.MI. [3354134]

A cylindrical tank with radius 7 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?



$$\frac{dV}{dt} = 3\ \frac{m^3}{min}$$

$$V = \pi r^2 h$$

$$V = \pi(7)^2 h = 49\pi h$$

$$\frac{dV}{dt} = 49\pi \frac{dh}{dt} \rightarrow$$

$$3 = 49\pi \frac{dh}{dt} \Rightarrow$$

$$\frac{dh}{dt} = \frac{3}{49\pi} \approx 0.01989436788$$

4. Question Details SCalc8 2.8.006. [3354296]

The radius of a sphere is increasing at a rate of 4 mm/s. How fast is the volume increasing when the diameter is 60 mm?

$$\frac{dr}{dt} = 4\ \frac{mm}{s}$$

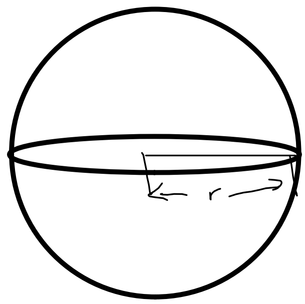
Want $\left.\frac{dV}{dt}\right|_{\substack{diam = 60 \\ r = 30}}$

$$V = \frac{4}{3}\pi r^3$$

5 16

$$\begin{array}{r} 900 \\ \hline 14,400 \end{array}$$

$$\left.\frac{dV}{dt}\right|_{r=30} = \left.4\pi r^2 \frac{dr}{dt}\right|_{r=30} = 4\pi \underset{(mm)^2}{(30)^2}\underset{(\frac{mm}{s})}{(4)} = \boxed{14,400\pi\ \frac{mm^3}{s}}$$



5. Question Details

Suppose $\boxed{y = \sqrt{2x+1}}$, where $\boxed{x \text{ and } y \text{ are functions of } t}$. → So, need chain rule for stuff like $\frac{d}{dt}\left[x^2\right]$

(a) If $dx/dt = 6$, find dy/dt when $x = 4$.

(b) If $dy/dt = 4$, find dx/dt when $x = 24$.

$(2x+1)^{\frac{1}{2}}$

$$\frac{dy}{dt} = \frac{1}{2}(2x+1)^{-\frac{1}{2}}\left(2\frac{dx}{dt}\right)$$

$$= \frac{1}{\sqrt{2x+1}}\frac{dx}{dt}$$

(a) $\frac{dx}{dt} = 6$ & $x = 4$ $\Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{2(4)+1}} \cdot 6 = \frac{6}{\sqrt{9}} = \frac{6}{3} = 2$

(b) $\frac{dy}{dt} = 4$, $x = 24$ $\Rightarrow 4 = \frac{1}{\sqrt{2(24)+1}} \cdot \frac{dx}{dt}$

$\Rightarrow \frac{1}{\sqrt{49}}\frac{dx}{dt} = 4$

$\Rightarrow \boxed{\frac{dx}{dt} = 28}$

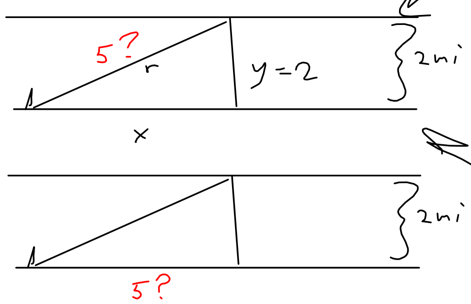
6. Question Details SCalc8 2.8.013. [3354280]

A plane flying horizontally at an altitude of 2 mi and a speed of 590 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 5 mi away from the station. (Round your answer to the nearest whole number.)

510 & 2 ⇒ 442 $\frac{mi}{hr}$
$\stackrel{\wedge}{r}$
& $y=1$

$\frac{dx}{dt} = 590\ \frac{mi}{hr}$

want $\left.\frac{dr}{dt}\right|_{r=5}$



THIS IS THE PIC.

$x^2 + y^2 = r^2$

$2xx' + 2yy' = 2rr'$

$2x\left(590\ \frac{mi}{hr}\right) + 2(2)(0) = 2(5)r' \Rightarrow \frac{2(\sqrt{21})(590)}{10} = \frac{dr}{dt}$

x from pythag. $x^2+y^2=5^2$

$x^2+2^2=5^2$

$x^2 = 21 \Rightarrow x = \sqrt{21}$

$\frac{dr}{dt}$ $118\sqrt{21} \approx$ 540.7439320 ?

OTHER ONE WITH ANS:
510 & $r=2$ & $y=1$

$\frac{2(\sqrt{3})(510)}{2(2)} \approx 442$ ✓
$\uparrow_{r=2}$

1 mile

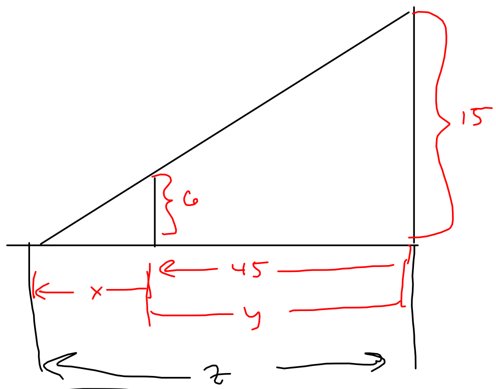
$x^2+y^2=2^2$
$x^2+1^2=2^2$
$x^2=3 \Rightarrow x=\sqrt{3}$

Sep 28-9:43 AM

8

7. Question Details SCalc8 2.8.015. [3354126]

A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 4 ft/s along a straight path. How fast is the tip of his shadow moving when he is 45 ft from the pole?



$z = x + y$

want $\left.\dfrac{dz}{dt}\right|_{y=45}$

$\dfrac{x}{6} = \dfrac{x+y}{15}$

$\dfrac{15x}{6} = \dfrac{5}{2}x = x + y$

$\Rightarrow \dfrac{3}{2}x = y$

$\Rightarrow y' = \dfrac{3}{2}x'$

$4 = \dfrac{3}{2}x'$

$\dfrac{8}{3} = x'$

So $\dfrac{dz}{dt} = \dfrac{d}{dt}(x+y)$

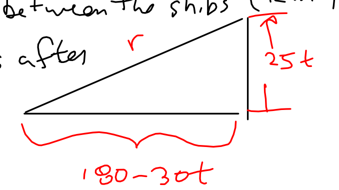
$= x' + y' = \dfrac{8}{3} + 4 = \boxed{\dfrac{20}{3}\ \dfrac{ft}{s}}$

8. Question Details SCalc8 2.8.016. [3354345]

At noon, ship A is 180 km west of ship B. Ship A is sailing east at 30 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

r = distance between the ships (km)

t = # of hours after 12pm



Want $\left.\dfrac{dr}{dt}\right|_{t=4}$

$\boxed{r^2 = (25t)^2 + (180-30t)^2}$ use to find r(4)

$= 625t^2 + (180-30t)^2$

$2r\dfrac{dr}{dt} = 1250t + 2(180-30t)'(-30)$

$2r\dfrac{dr}{dt} = 1250(4) + 2(180-30(4))(-30)$

$= 5000 - 2(30)(60)$

$= 5000 - 3600$

$2rr' = 1400$

$r^2 = (25(4))^2 + (60)^2$

$= 10000 + 3600$

$= 13{,}600 = r^2 \Rightarrow r = 20\sqrt{34} \approx$ 116.6190379

$\Rightarrow 2(20\sqrt{34})r' = 1400$

$r' = \dfrac{1400}{40\sqrt{34}} \approx \boxed{6\ \dfrac{Km}{hr}} \approx 6.00245$

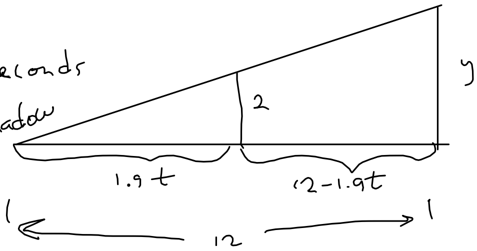
9. Question Details SCalc8 2.8.018.MI. [3354112]

A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.9 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building? (Round your answer to one decimal place.)

LEX:

t = time in seconds

y = height of shadow (meters)



$$\frac{2}{1.9t} = \frac{y}{12}$$

$$\frac{24}{1.9t} = y = \frac{24}{1.9t} = \frac{24}{1.9}t^{-1}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-24}{1.9}t^{-2}$$

Want $\left.\frac{dy}{dt}\right|_{12-1.9t=4}$

$\therefore$ $12 - 1.9t = 4$

$-1.9t = -8$

$t = \frac{-8}{-1.9} = \frac{80}{19} \approx$

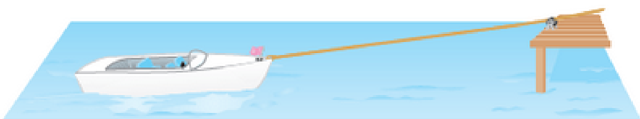
$\approx$ 4.210526316

$\therefore \left.\frac{dy}{dt}\right|_{12-1.9t=4} \approx \frac{-24}{1.9}(4.210526316)^{-2}$

$\boxed{\approx 0.7125000000 \ \frac{m}{s}}$

10. Question Details SCalc8 2.8.022.MI. [3354175]

A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 7 m from the dock? (Round your answer to two decimal places.)

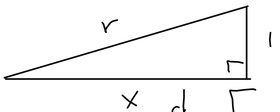


x = distance to dock (m)

r = " " pulley (m)

Want $\left.\frac{dx}{dt}\right|_{x=7m}$

Know $\frac{dr}{dt} = 1\frac{m}{s}$



$$\frac{d}{dt}\left[r^2 = x^2 + 1^2\right]'$$

$$2rr' = 2xx'$$

$x = 7m \Rightarrow r^2 = 7^2 + 1^2 = 50$

$r = \sqrt{50} = 5\sqrt{2}$

$r' = 1$ $\qquad 2(5\sqrt{2})(1) = 2(7)\left.x'\right|_{x=7}$

$$\Rightarrow \left.x'\right|_{x=7} = \frac{10\sqrt{2}}{14} = \frac{5\sqrt{2}}{7}$$

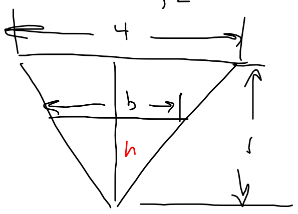
$\approx$ 1.010152544

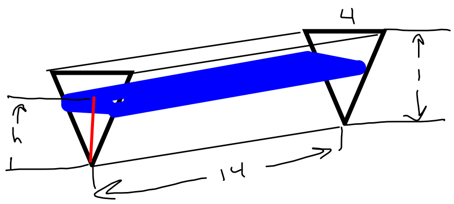
$\boxed{\approx 1.01 \ \frac{m}{s} \approx \left.\frac{dx}{dt}\right|_{x=7}}$

11. Question Details SCalc8 2.8.026.MI. [3354506]

A trough is 14 ft long and its ends have the shape of isosceles triangles that are 4 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 15 ft³/min, how fast is the water level rising when the water is 7 inches deep?

$7'' = \frac{7}{12}\text{ ft}$





similar triangles

$\frac{4}{1} = \frac{b}{h} \Rightarrow 4h = b$

$V = \frac{1}{2} b h \cdot 14$

$= \frac{1}{2}(4h)(h)(14)$

$V = 28h^2$

want $\left.\frac{dh}{dt}\right|_{h=\frac{7}{12}\text{ ft}}$

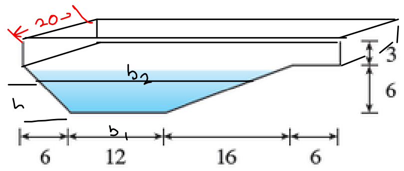
$\frac{56}{12} =$

$\frac{dV}{dt} = 56h\frac{dh}{dt} \Rightarrow 15 = 56\left(\frac{7}{12}\right)\left.\frac{dh}{dt}\right|_{h=\frac{7}{12}}$

$\frac{(15)(12)}{(56)(7)} = \left.\frac{dh}{dt}\right|_{h=\frac{7}{12}} \approx$ 0.4591836735 $\frac{\text{ft}}{\text{min}}$

12. Question Details SCalc8 2.8.028. [3354330]

A swimming pool is 20 ft wide, 40 ft long, 3 ft deep at the shallow end, and 9 ft deep at is deepest point. A cross-section is shown in the figure. If the pool is being filled at a rate of 0.8 ft³/min, how fast is the water level rising whe the depth at the deepest point is 5 ft? (Round your answer to five decimal places.)



Volume = (cross-sec. Area)(width)

= (Area of trap)(20)

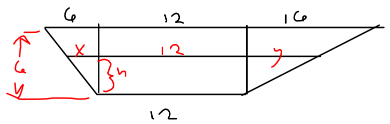
= 20(area of trapezoid)

$= 20\left(\frac{1}{2}(b_1 + b_2)h\right)$

$= 10(12 + b_2)h$

Need b_2 as $f(h)$, if possible

WANT $\left.\frac{dh}{dt}\right|_{h=5}$



similar triangles

$\frac{6}{6} = \frac{x}{h} \Rightarrow x = h$

& $\frac{16}{6} = \frac{y}{h} \Rightarrow 16h = 6y$

$y = \frac{16}{6}h = \frac{8}{3}h$

$\therefore\ b_2 = x + 12 + y = h + 12 + \frac{8}{3}h$

$= \frac{11}{3}h + 12$

$\Rightarrow 10(12 + b_2)h = 10\left(12 + \frac{8}{3}h + 12\right)h = 10\left(24 + \frac{8}{3}h\right)h$

$= 10\left(\frac{8}{3}h^2 + 24h\right)$

Volume $= V = V(h) = \frac{80}{3}h^2 + 240h$

$\frac{dV}{dt} = .8\ \frac{\text{ft}^3}{\text{min}} = \frac{160}{3}hh' + 240h'$

$\Rightarrow h'\left(\frac{160}{3}h + 240\right) = .8$

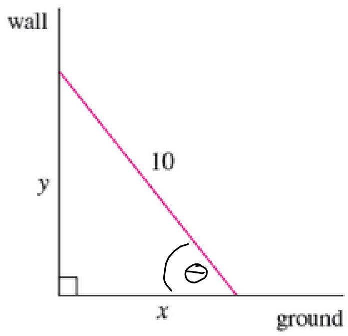
$h' = \frac{.8}{\frac{160}{3}h + 240}$

want $\left.\frac{dh}{dt}\right|_{h=5} = \frac{.8}{\frac{160}{3}(5) + 240} \approx$ 0.001578947368 ft/min

13. Question Details SCalc8 2.8.032. [3426404]

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1.2 ft/s, how fast is the angle between the ladder and the ground changing when the bottom of the ladder is 6 ft from the wall? (That is, find the angle's rate of change when the bottom of the ladder is 6 ft from the wall.)

Want $\left.\frac{d\theta}{dt}\right|_{x=6}$



$\frac{dx}{dt} = \frac{1.2 \text{ ft}}{s}$

$\frac{y}{x} = \tan\theta$

$y = x\tan\theta$

$\boxed{\frac{dy}{dt}} = \frac{dx}{dt}\tan\theta + x\sec^2(\theta)\frac{d\theta}{dt}$

unknown

$x^2 + y^2 = 10^2 = 10$

$2xx' + 2yy' = 0$

$y' = -\frac{x}{y}$

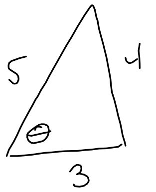
Now $x = 6 \Rightarrow$

$6^2 + y^2 = 100$

$y^2 = 100 - 36 = 64$

$y = \pm 8 \Rightarrow y = 8$

$\therefore\ y' = \left.\frac{dy}{dt}\right|_{x=6} = -\frac{6}{8} = -\frac{3}{4}$



$\sec\theta = \frac{5}{3}$

$\sec^2\theta = \frac{25}{9}$

$\&\ \theta = \arctan\frac{y}{x}$
$= \arctan\left(\frac{8}{6}\right) = \arctan\left(\frac{4}{3}\right) \approx$ 0.9272952179

Put it together:

$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta + x\sec^2(\theta)\frac{d\theta}{dt}$

$-\frac{3}{4} = \frac{6}{5}\tan\left(\arctan\left(\frac{4}{3}\right)\right) + (6)\left(\frac{25}{9}\right)\frac{d\theta}{dt}$

$\frac{\cancel{6}^{2}}{5}\left(\frac{4}{\cancel{3}}\right) + \frac{50}{3}\frac{d\theta}{dt} = -\frac{3}{4}$

$\Rightarrow \frac{8}{5} + \frac{50}{3}\theta' = -\frac{3}{4}$

$\Rightarrow \frac{50}{3}\theta' = -\frac{3}{4} - \frac{8}{5} = \frac{-15-32}{20} = \frac{-47}{20}$

$\Rightarrow \theta' = \left(-\frac{47}{20}\right)\left(\frac{3}{50}\right) = -\frac{141}{1000} = \boxed{-.141\ \frac{\text{radians}}{\text{sec}} = \left.\frac{d\theta}{dt}\right|_{x=6}}$

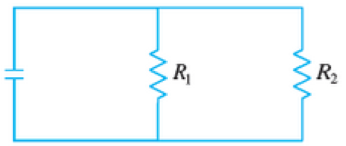
Sep 28-9:48 AM

14. Question Details SCalc8 2.8.039. [3354186]

If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure below, then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If R_1 and R_2 are increasing at rates of 0.3 Ω/s and 0.2 Ω/s, respectively, how fast is R changing when R_1 = 100 Ω and R_2 = 120 Ω? (Round your answer to three decimal places.)



$$R_{TOTAL} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

RESISTANCE IN PARALLEL

$\frac{dR_1}{dt} = .3 \frac{\Omega}{s}$

$\frac{dR_2}{dt} = .2 \frac{\Omega}{s}$

Want $\frac{dR}{dt}$ | $R_1 = 100\,\Omega$, $R_2 = 120\,\Omega$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \sum_{k=1}^{2} \frac{1}{R_k}$$

$$R^{-1} = R_1^{-1} + R_2^{-1}$$

$$-R^{-2}\frac{dR}{dt} = -R_1^{-2}\frac{dR_1}{dt} - R_2^{-2}\frac{dR_2}{dt}$$

$2\lfloor 100$, $2\lfloor 50$, $5\lfloor 25$, 5

$2\lfloor 120$, $2\lfloor 60$, $2\lfloor 30$, $3\lfloor 15$, 5

$LCD = 2^3 \cdot 3 \cdot 5^2$

$$\frac{1}{R} = \frac{1}{100} + \frac{1}{120} = \frac{1}{2^2 5^2}\cdot\frac{2\cdot 3}{2\cdot 3} + \frac{1}{2^3\cdot 3\cdot 5}\cdot\frac{5}{5} = \frac{6+5}{2^3\cdot 3\cdot 5^2} = \frac{11}{600} = \frac{1}{R}$$

24

$$\Rightarrow \boxed{R = \frac{600}{11}}$$

$$-\left(\frac{600}{11}\right)^{-2}\frac{dR}{dt} = -(100)^{-2}(.3) - (120)^{-2}(.2)$$

$$\Rightarrow \left(-(100^{-2})(.3) - (120)^{-2}(.2)\right)\left(-\left(\frac{600}{11}\right)^2\right) = \frac{dR}{dt}$$

$$\Rightarrow \frac{dR}{dt} = \left((100^{-2})(.3) + (120^{-2})(.2)\right)\left(\frac{600}{11}\right)^2$$

$$\boxed{\approx 0.1305785124 \ \frac{\Omega}{s} \approx \frac{dR}{dt}}$$ | $R_1 = 100$, $R_2 = 120$

Sep 29-10:32 AM

15. Question Details — SCalc8 2.8.043. [3354501]

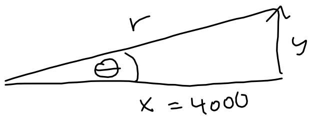
A television camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 600 ft/s when it has risen 3000 ft. (Round your answers to three decimal places.)

(a) How fast is the distance from the television camera to the rocket changing at that moment?

(b) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment?

y = height of rocket in feet.

(a) Want $\left.\frac{dr}{dt}\right|_{\substack{y=3000\text{ ft}\\ y'=600\text{ ft/s}}}$



(b) Want $\left.\frac{d\theta}{dt}\right|_{\substack{y=3000\text{ ft}\\ y'=600\text{ ft/s}}}$

$$y^2 + x^2 = r^2 = (4000)^2 + y^2$$

$$2r\left.\frac{dr}{dt}\right|_{\substack{y=3000\\ y'=600}} = 2y\frac{dy}{dt} = 2(3000)(600)$$

$r = ?$

$$\left.r\right|_{y=3000} = \sqrt{4000^2 + 3000^2} = \sqrt{4^2\,1000^2 + 3^2(1000^2)}$$

$$((4)(1000))^2 = 4^2(1000^2) \qquad = \sqrt{1000^2(4^2+3^2)}$$

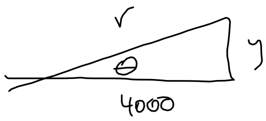
$$= 1000\sqrt{16+9}$$

$$= 1000\sqrt{25}$$

$$= 5000 = r$$

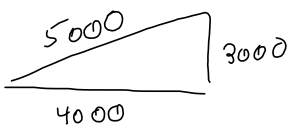
So, $2(5000)\frac{dr}{dt} = 3600000$

$$\frac{dr}{dt} = \frac{36\cancel{00000}}{1\cancel{0000}} = \boxed{360 \text{ ft/s}}$$



$$\frac{y}{4000} = \tan\theta$$

$$y = 4000\tan\theta$$



$$\frac{dy}{dt} = (4000\sec^2\theta)\frac{d\theta}{dt} = 600(4000)$$

$\theta = \arctan\frac{3}{4}$

$$4000\left(\frac{5}{4}\right)^2\frac{d\theta}{dt} = 2400000$$

$$\frac{d\theta}{dt} = \left(\frac{2400000}{4000}\right)\left(\frac{16}{25}\right) = \text{etc.}$$