1. SCalc8 2.7.002. (3395220) *modified*

A particle moves according to a law of motion $s = f(t)$, $t \geq 0$, where t is measured in seconds and s in feet.

$f(t) = 0.01t^4 - 0.02t^3$

(a) Find the velocity at time t (in ft/s). $\boxed{0.04t^3 - 0.06t^2}$

$V(t) = f'(t) = .04t^3 - .06t^2$

(b) What is the velocity after 1 second(s)? -0.02 ft/s

$V(1) = .04 - .06 = -.02$ ft/s

(c) When is the particle at rest? 0 s (smaller value) 3/2 s (larger value)

Set $f'(t) = 0 \Rightarrow t^2(.04t - .06) \rightarrow \boxed{t = 0 s}$

$.04t^3 - .06t^2$

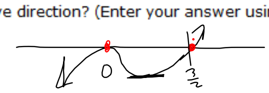
$.04t = .06$

$t = \frac{6}{4} = \frac{3}{2}$

$\boxed{t = \frac{3}{2} s}$

(d) When is the particle moving in the positive direction? (Enter your answer using interval notation.)

$t \in (\frac{3}{2}, \infty)$ $\boxed{\left(\frac{3}{2}, \infty\right)}$



(e) Find the total distance traveled during the first 10 seconds. (Round your answer to two decimal places.)

80.03 ft

$f(t) = .01t^4 - .02t^3 \Rightarrow f(10) = $ Net distance

$.01(10)^4 - .02(10)^3$

$= .0100 - .0200$

$100 - 20 = 80$ ft.

(f) Find the acceleration at time t (in ft/s²). $\boxed{0.12t^2 - 0.12t}$

$a(t) = v'(t) = f''(t) = .12t^2 - .12t$

Find the acceleration after 1 second(s). 0 ft/s²

$.12(1)^2 - .12(1) = 0$

(g) Graph the position, velocity, and acceleration functions for the first 10 seconds.

(h) When, for $0 \leq t < \infty$, is the particle speeding up? (Enter your answer using interval notation.)

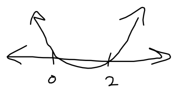
$\boxed{(0, 1) \cup \left(\frac{3}{2}, \infty\right)}$

Wanted $v(t)$ & $a(t)$. Rule:
when $v(t) < 0$ & $a(t) < 0$, it's speeding up.

When, for $0 \leq t < \infty$, is it slowing down? (Enter your answer using interval notation.)

$\boxed{\left(1, \frac{3}{2}\right)}$

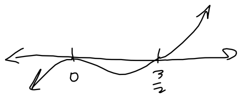
$s(t) = .01t^4 - .02t^3 = .01t^3(t - 2)$



$v(t) = f'(t) = .04t^3 - .06t^2 = s'(t)$

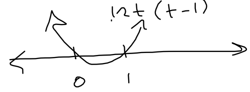
$t^2(.04t - .06) \overset{SET}{=} 0 \Rightarrow t = 0 \quad m = 2$

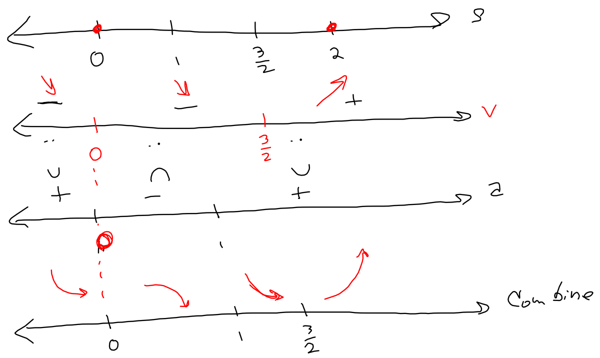
$t = \frac{3}{2} \quad m = 1$



$a(t) = f''(t) = .12t^2 - .12t = s''(t)$

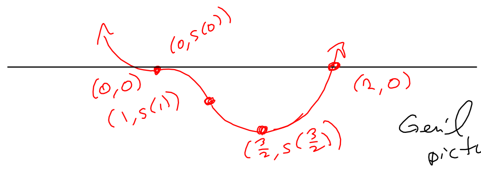
$.12t(t - 1)$





smile! $a(t) > 0$

$a(t) < 0$ frown.



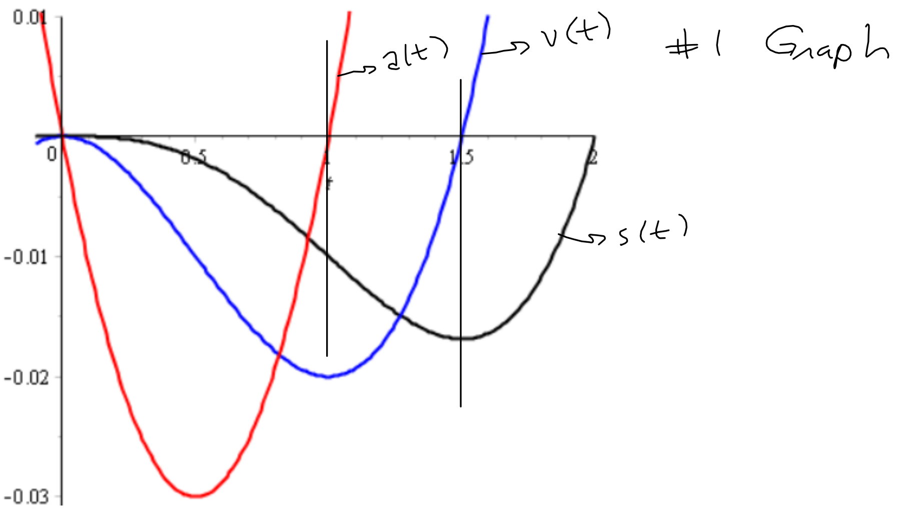
You can sub in $s(1)$ & $s(\frac{3}{2})$

Gen'l picture.

Speeding up:

$x \in (0,1) \cup (\frac{3}{2}, \infty)$

Slowing Down: $x \in (1, \frac{3}{2})$

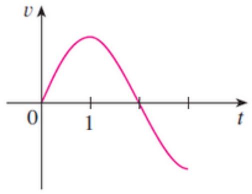

0.01
0
0.5
1
1.5
2
t
-0.01
-0.02
-0.03
a(t)
v(t)
s(t)
#1 Graph

2. SCalc8 2.7.005. (3354441)*omized, slightly modified*

Graphs of the *velocity* functions of two particles are shown, where *t* is measured in seconds.

Velocity, not position.

(a)



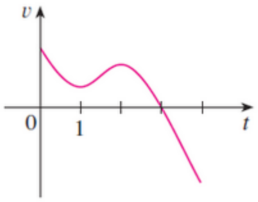
When is the particle in figure (a) speeding up? (Enter your answer using interval notation.)

$\boxed{(0, 1) \cup (2, 3)}$ $(0,1) \cup (2,3)$

When is the particle in figure (a) slowing down? (Enter your answer using interval notation.)

$\boxed{(1, 2)}$ $(1,2)$

(b)



When is the particle in figure (b) speeding up? (Enter your answer using interval notation.)

$\boxed{(1, 2) \cup (3, 4)}$ $(1,2) \cup (3,4)$

When is the particle in figure (b) slowing down? (Enter your answer using interval notation.)

$\boxed{(0, 1) \cup (2, 3)}$ $(0,1) \cup (2,3)$

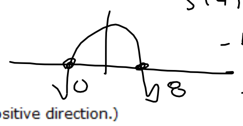
3. SCalc8 2.7.008. (3354531)

If a ball is thrown vertically upward with a velocity of 128 ft/s, then its height after t seconds is $s = 128t - 16t^2$.

(a) What is the maximum height reached by the ball?

256 ft

$t=4$ $\quad -16t^2+128t = -16t(t-8)$



$s(4) = 128(4)$
$-16(16)$
$512-256$
$=256$

(b) What is the velocity of the ball when it is 240 ft above the ground on its way up? (Consider up to be the positive direction.)

32 ft/s

$32\ ft/s \quad s(t)=240 \Longrightarrow$

What is the velocity of the ball when it is 240 ft above the ground on its way down?

-32 ft/s

$s(t) = 240 = -16t^2+128t \Longrightarrow$

$-16t^2+128t-240=0 \Longrightarrow (t-5)(t-3)=0$

$-16(t^2-8t+15)=0$

up $\quad t=3$

$v(t)=s'(t) = -32t+128$

$\Longrightarrow v(3) = -32(3)+128$

$= -96+128 = 32\ \frac{ft}{s}$

on its way down: $t=5$

$v(5) = -32(5)+128$

$= -160+128$

$= -32\ ft/s$

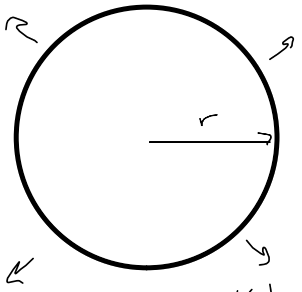
4. SCalc8 2.7.014. (3354421) odified

A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 40 cm/s. Find the rate at which the area within the circle is increasing after each of the following.

(a) after 1 s $\quad 3200\pi$ cm²/s

(b) after 4 s $\quad 12800\pi$ cm²/s

(c) after 6 s $\quad 19200\pi$ cm²/s



$\frac{dr}{dt} = 40\ \frac{cm}{s}$

Chain Rule.

$A = \pi r^2$

want

$A'(t) = \frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi r r'$

$r(t) = 40t$

$\Longrightarrow A(t) = \pi r^2(t) = \pi(40t)^2$

$\Longrightarrow A'(t) = 2\pi r r' = 2\pi(40t)(40)$

$A'(t) = 3200\pi t$

$\Longrightarrow$ a) $A'(1) = 3200\pi\ \frac{cm^2}{s}$

(b) $A'(4) = 3200\pi(4) = 12,800\pi\ \frac{cm^2}{s}$

(c) $A'(6) = 3200\pi(6) = 19,200\pi\ \frac{cm^2}{s}$

$32 \times 6 = 192$

5. SCalc8 2.7.016. (3354162) ed

(a) The volume V of a growing spherical cell is $V = \frac{4}{3}\pi r^3$, where the radius is measured in micrometers (1 µm = 10^{-6}m).

(i) 2 to 5 µm Find the average rate of change of V with respect to r when r changes from 2 to each of the following. (Round your answers to one decimal place.)

[redacted] µm³/µm

(ii) 2 to 3 µm

[redacted] µm³/µm

(iii) 2 to 2.1 µm

[redacted] µm³/µm

$$(i)\quad \frac{V(5)-V(2)}{5-2} = m_{sec} = m_{avg} = \frac{\frac{4}{3}\pi(5)^3 - \frac{4}{3}\pi(2)^3}{5-2}$$

$$= \frac{\frac{4}{3}\pi[5^3 - 2^3]}{3} = \frac{\frac{4}{3}\pi[125-8]}{3} = \frac{\frac{4}{3}\pi[\overset{39}{\cancel{117}}]}{\cancel{3}_1} = \frac{156\pi}{3}\ \frac{\mu m^3}{\mu m}$$

(b) Find the instantaneous rate of change of V with respect to r when r = 2 µm. (Round your answer to one decimal place.)

[redacted] µm³ / µm

$$(iii)\quad \frac{V(2.1)-V(2)}{2.1-2} = \frac{\frac{4}{3}\pi[2.1^3 - 2^3]}{.1}$$

$$\approx 16.813333\ \frac{\mu m^3}{\mu m}$$

$$\frac{dV}{dr} = V'(r) = 4\pi r^2 \Rightarrow$$

$$V'(2) = 4\pi(2)^2 = 16\pi\ \frac{\mu m^3}{\mu m}$$

6. SCalc8 2.7.018.MI. (3354440)

If a tank holds 3500 gallons of water, which drains from the bottom of the tank in 50 minutes, then Toricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 3500\left(1 - \frac{1}{50}t\right)^2 \quad 0 \le t \le 50.$$

Chain Rule

Find the rate at which water is draining from the tank after the following amounts of time.
(Remember that the rate must be negative because the amount of water in the tank is decreasing.)

(a) 5 min -126 gal/min

(b) 10 min -112 gal/min

(c) 20 min -84 gal/min

(d) 50 min 0 gal/min

$$V'(t) = \frac{dV}{dt} = 700\cancel{0}\left(1 - \frac{1}{50}t\right)\left(-\frac{1}{5\cancel{0}}\right) = -\frac{700}{5}\left(1 - \frac{1}{50}t\right)$$

$$-140\left(1 - \frac{1}{50}t\right) = -140 + \frac{14\cancel{0}}{5\cancel{0}}t$$

$$= \frac{14}{5}t - 140 \le 0 \text{ on } [0, 50]\ ✓$$

$$(a)\quad V'(5) = \frac{14}{5}(5) - 140 = 14 - 140 = -126\ \frac{gal}{min}$$

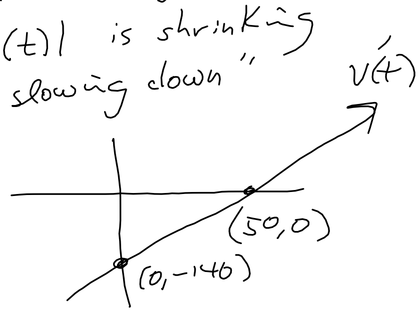
(b) $V'(t)$ is negative.
$|V'(t)|$ is shrinking
"slowing down"

At what time is the water flowing out the fastest? 0 min

From the above : $t = 5$ In general, $t = 0$.

At what time is the water flowing out the slowest? 50 min

$t = 50$



7. SCalc8 2.7.019. (3394362)

The quantity of charge Q in coulombs (C) that has passed through a point in a wire up to time t (measured in seconds) is given by $Q(t) = t^3 - 2t^2 + 4t + 4$. [See the example. The unit of current is an ampere (1 A = 1 C/s).]

(a) Find the current when $t = 0.3$ s. 3.07 A

(b) Find the current when $t = 1$ s. 3 A

At what time is the current lowest? 2/3 s

$Q'(t) = 3t^2 - 4t + 4$

$\cancel{(a)\ Q'(3) = 3(3)^2 - 4(3) + 4 = 27 - 12 + 4 = \boxed{19 \text{ Amps}}}$ (C/s)

(a) $Q'(.3) = 3(.3)^2 - 4(.3) + 4$
$= 3(.09) - 1.2 + 4$
$= .27 + 2.8$
$= 3.07\ C/s$

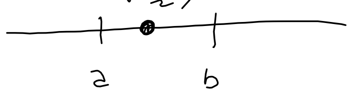
(b) $Q'(1) = 3(1)^2 - 4(1) + 4$
$= 3\ A$

when is $Q'(t)$ lowest?
$3t^2 - t + 4$. Find vertex!

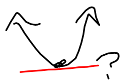
$(3)(4) = 12$
$b^2 - 4ac = (-1)^2 - 4(3)(4) = 1 - 48 = -47$

$t = \dfrac{1 \pm \sqrt{-47}}{\quad}$ No real soln!

$\left(\frac{a+b}{2}\right)$



How find vertex?



Take $Q''(t) \overset{SET}{=} 0$

$Q''(t) = 6t - 4 \overset{SET}{=} 0$
$6t = 4$
$t = \frac{4}{6} = \boxed{\frac{2}{3}\,s}$

8. SCalc8 2.7.020. (3354119)

Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the bodies.

(a) Find dF/dr. $-\dfrac{2GmM}{r^3}$

What is the meaning of dF/dr?

What does the minus sign indicate?

(b) Suppose it is known that the earth attracts an object with a force that decreases at the rate of 2 N/km when r = 20,000 km.

How fast does this force change when r = 10,000 km? -16 N/km

9. SCalc8 2.7.023. (3394370)

Boyle's Law states that when a sample of gas is compressed at a constant temperature, the product of the pressure and the volume remains constant: $PV = C$.

(a) Find the rate of change of volume with respect to pressure.

(b) A sample of gas is in a container at low pressure and is steadily compressed at constant temperature for 11 minutes. Is the volume decreasing more rapidly at the beginning or the end of the 11 minutes? Explain.

From the formula for $\frac{dV}{dP}$, we see that as P increases, the absolute value of $\frac{dV}{dP}$.

Thus, the volume is more rapidly at the beginning.

(c) Write the formula for isothermal compressibility (see this example) in terms of P.

10. SCalc8 2.7.035. (3354360)

In the study of ecosystems, *predator-prey models* are often used to study the interaction between species. Consider populations of tundra wolves, given by $W(t)$, and caribou, given by $C(t)$, in northern Canada. The interaction has been modeled by the equations

$$\frac{dC}{dt} = aC - bCW \qquad \frac{dW}{dt} = -cW + dCW.$$

(a) What values of dC/dt and dW/dt correspond to stable populations?

(b) How would the statement "The caribou go extinct" be represented mathematically?

(c) Suppose that $a = 0.03$, $b = 0.001$, $c = 0.03$, and $d = 0.0001$. Find all population pairs (C, W) that lead to stable populations.

0, 0

300, 30

According to this model, is it possible for the two species to live in balance or will one or both species become extinct?

11. SCalc8 2.7.503.XP. (3389951)

If a stone is thrown vertically upward from the surface of the moon with a velocity of 6 m/s, its height (in meters) after t seconds is $h = 6t - 0.83t^2$.

(a) What is the velocity of the stone after 2 s? 2.68 m/s

(b) What is the velocity of the stone after it has risen 7 m? 3.57 m/s

(Round your answers to two decimal places.)