Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

8 $y \sin(8x) = x \cos(2y)$, $(\pi/2, \pi/4)$

$\frac{d}{dx}[\;\;]$: $y' \sin(8x) + y(\overset{12}{8}\cos(8x)) = 1\cos(2y) + x(-2y'\sin(2y))$

$(\sin(\overset{12}{8}x) + 2x\sin(2y))y' = \cos(2y) - \overset{12}{8}y\cos(8x)$

$$y' = \frac{\cos(2y) - \overset{12}{8}y\cos(8x)}{\sin(8x) + 2x\sin(2y)}$$

$$\Rightarrow y'\Big|_{\substack{(x,y) = (\frac{\pi}{2}, \frac{\pi}{4}) \\ \text{or} \\ x = \frac{\pi}{2} \\ y = \frac{\pi}{4}}} = \frac{\cos(2\cdot\frac{\pi}{4}) - \overset{12}{8}(\frac{\pi}{4})\cos(8\cdot\frac{\pi}{2})}{\sin(\underset{12}{8}\cdot\frac{\pi}{2}) + 2(\frac{\pi}{2})\sin(2\cdot\frac{\pi}{4})}$$

$$= \frac{\cos(\frac{\pi}{2}) - 2\pi(\cos(\overset{6\pi}{4\pi}))}{\sin(\underset{6\pi}{4\pi}) + \pi(\sin(\frac{\pi}{2}))}$$

$$= \frac{0 - 2\pi(1)}{0 + \pi(1)} = -\frac{2\pi}{\pi} = -2 = m_{tan}$$

$y = L(x) = m_{tan}(x - x_1) + y_1$

§ 2.1

$= -2(x - \frac{\pi}{2}) + \frac{\pi}{4}$

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Find the points on the lemniscate where the tangent is horizontal.

$2(x^2 + y^2)^2 = 25(x^2 - y^2)$ $\Rightarrow$ $2(x^4 + 2x^2y^2 + y^4) = 25x^2 - 25y^2$

$(x, y) = ($ $)$ (smaller x-value, smaller y-value)

$(x, y) = ($ $)$ (smaller x-value, larger y-value)

$(x, y) = ($ $)$ (larger x-value, smaller y-value)

$(x, y) = ($ $)$ (larger x-value, larger y-value)

$\Rightarrow$

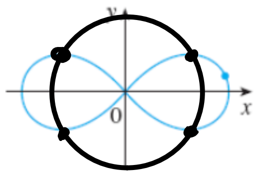
$2x^4 + 4x^2y^2 + 2y^4 = \text{SAME} \Rightarrow$

$8x^3 + 8xy^2 + 8x^2y \cdot y' + 8y^3y'$

$= 50x - 50yy'$

$8x^2yy' + 50yy' = 50x - 8x^3 - 8xy^2$

$y' = \dfrac{50x - 8x^3 - 8xy^2}{8x^2y + 50y}$ SET 0



$\Rightarrow x[50 - 8x^2 - 8y^2] = 0$

$8x^2 + 8y^2 = 50$

$x^2 + y^2 = \frac{50}{8} = \frac{25}{4}$ $(0,0)$ $r = \frac{5}{2}$

$y = \pm\sqrt{\frac{25}{4} - x^2}$

Plug in to lemniscate

$2(x^2+y^2)^2 = 25(x^2 - y^2)$

$2\left(x^2 + \frac{25}{4} - x^2\right)^2 = 25\left(x^2 - \left(\frac{25}{4} - x^2\right)\right)$

$2\left(\frac{25}{4}\right)^2 = 25\left(x^2 - \frac{25}{4} + x^2\right)$

$\left(\frac{2(25^2)}{4^2}\right)\left(\frac{1}{25}\right) = 2x^2 - \frac{25}{4}$

$\frac{25}{8} = 2x^2 - \frac{25}{4}$

$2x^2 = \frac{25}{8} + \frac{25}{4}\cdot\frac{2}{2} = \frac{75}{8}$

$x^2 = \frac{75}{16}$

$x = \pm\sqrt{\frac{75}{16}} = \pm\frac{5\sqrt{3}}{4}$

$y = \pm\sqrt{\frac{25}{4} - x^2}$

$= \pm\sqrt{\frac{25}{4} - \left(\frac{5\sqrt{3}}{4}\right)^2}$

$= \pm\sqrt{\frac{25}{4} - \frac{25\cdot 3}{16}} = \pm\sqrt{\frac{25}{4}\cdot\frac{4}{4} - \frac{75}{16}} = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$

$\left(\pm\frac{5\sqrt{3}}{4}, \pm\frac{5}{4}\right)$

$\left(-\frac{5\sqrt{3}}{4}, -\frac{5}{4}\right), \left(-\frac{5\sqrt{3}}{4}, \frac{5}{4}\right), \left(\frac{5\sqrt{3}}{4}, -\frac{5}{4}\right), \left(\frac{5\sqrt{3}}{4}, \frac{5}{4}\right)$