Chain Rule

$H(x) = f(g(x))$

$H'(x) = \frac{df}{dg} \cdot \frac{dg}{dx} = \frac{dH}{dg} \cdot \frac{dg}{dx}$

Chain Rule says

$\frac{dP}{dt} = \frac{dP}{dS} \cdot \frac{dS}{dt}$

$\frac{dP}{dS} = \frac{d}{dS}\left[20S\right] = 20$

$\frac{dS}{dt} = \frac{d}{dt}\left[100t\right] = 100$

$\frac{dP}{dS} \cdot \frac{dS}{dt} = 20 \cdot 100 = 2000 \frac{\$}{hr}$

$\frac{dP}{dS} = \frac{dP}{d(100t)}$

Profit: I make \$20 per shirt.

Shirts: I make 100 shirts per hour.

S = # of shirts made in t hours is $100t$ (shirts)

P = Amt of \$ made by selling S shirts $= P(S) = 20S$

P as a function of time is $20S = 20(100t)$

$P(t) = 2000t$

$P'(t) = \frac{dP}{dt} = 2000 \frac{\$}{hr}$

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1. Question Details SCalc8 2.5.001. [3

Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.]

$y = \sqrt[3]{1 + 8x}$

$u = g(x) = 8x + 1$

$f(u) = \sqrt[3]{u} = u^{\frac{1}{3}} = y$

At speed:

$\frac{d}{dx}(8x+1)^{\frac{1}{3}} = \frac{1}{3}(8x+1)^{-\frac{2}{3}}(8)$

Find the derivative dy/dx.

$$\frac{dy}{dx} = \frac{dy}{d(g(x))} \cdot \frac{d(g(x))}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{1}{3}u^{-\frac{2}{3}}\right)(8) = \frac{1}{3}(8x+1)^{-\frac{2}{3}}(8)$$

2. Question Details SCalc8 2.5.003. [3

Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.]

$y = \tan(\pi x)$ Find the derivative dy/dx.

$u = \pi x$ $\tan u = y$

$\frac{du}{dx} = \pi$ $\frac{dy}{du} = \sec^2 u$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (\sec^2 u)(\pi)$$

$$\pi \sec^2(\pi x)$$

3. Question Details SCalc8 2.5.007. [3354312]

Find the derivative of the function.

$F(x) = (5x^6 + 2x^3)^4 \implies F'(x) = 4(5x^6 + 2x^3)^3(30x^5 + 6x^2)$

4. Question Details SCalc8 2.5.009.

Find the derivative of the function.

$f(x) = \sqrt{5x + 6}$ $= (5x+6)^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}(5x+6)^{-\frac{1}{2}}(5)$

5. Question Details SCalc8 2.5.013.

Find the derivative of the function.

$f(\theta) = \cos(\theta^2)$ $\Rightarrow f'(\theta) = (-\sin(\theta^2))(2\theta)$

6. Question Details SCalc8 2.5.016.

Find the derivative of the function.

$f(t) = 9t \sin(\pi t)$ $= 9 \sin(\pi t) + 9t \cos(\pi t) \cdot \pi$

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7. Question Details SCalc8 2.5.017.

Find the derivative of the function.

$f(x) = (2x - 7)^4(x^2 + x + 1)^5 \Longrightarrow f'$

$f'(x) = 4(2x-7)^3(2)(x^2+x+1)^5 + (2x-7)^4\left(5(x^2+x+1)^4\right)(2x+1)$

8. Question Details SCalc8 2.5.019.

Find the derivative of the function.

$h(t) = (t + 4)^{2/3}(3t^2 - 1)^3$

$\Longrightarrow h'(t) = \frac{2}{3}(t+4)^{-1/3}(1)(3t^2-1)^3 + (t+4)^{\frac{2}{3}}\left(3(3t^2-1)^2\right)(6t)$

9. Question Details SCalc8 2.5.023.

Find the derivative of the function.

$y = \sqrt{\frac{x}{x+3}} = \left(\frac{x}{x+3}\right)^{\frac{1}{2}} \Longrightarrow$

$y' = \frac{1}{2}\left(\frac{x}{x+3}\right)^{-\frac{1}{2}}\left(\frac{1(x+3) - (x)(1)}{(x+3)^2}\right)$

10. Question Details SCalc8 2.5.036.

Find the derivative of the function.

$y = x \sin \frac{6}{x} = x \sin(6x^{-1}) \Longrightarrow y' = \sin\left(\frac{6}{x}\right) + \left(x \cos(6x^{-1})\right)(-6x^{-2})$

11. Question Details SCalc8 2.5.037.

Find the derivative of the function.

$y = \cot^2(\sin(\theta)) \Longrightarrow y' = 2(\cot(\sin\theta))(-\csc(\sin\theta)\cot(\sin\theta))(\cos\theta)$

12. Question Details SCalc8 2.5.042.

Find the derivative of the function.

$y = \sqrt{5x + \sqrt{5x + \sqrt{5x}}} = \left(5x + \left(5x + (5x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$\Longrightarrow y' = \frac{1}{2}\left(5x + \left(5x + (5x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \left[5 + \frac{1}{2}\left(5x + (5x)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(5 + \frac{1}{2}(5x)^{-\frac{1}{2}}(5)\right)\right]$

13. Question Details SCalc8 2.5.045.

Find the derivative of the function.

$y = \cos\left(\sqrt{\sin(\tan(9x))}\right) = \cos\left(\left(\sin(\tan(9x))\right)^{\frac{1}{2}}\right)$

$\Longrightarrow y' = -\sin\left(\left(\sin(\tan(9x))\right)^{\frac{1}{2}}\right)\left(\frac{1}{2}\left(\sin(\tan(9x))\right)^{-\frac{1}{2}}(\cos(\tan(9x)))(\sec^2(9x))(9)\right)$

14. Question Details SCalc8 2.5.056. [3943253]

(a) The curve $y = |x|/\sqrt{2-x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point (1, 1).

close to $x=1$, $|x| = x$. $D = (-\sqrt{2}, \sqrt{2})$

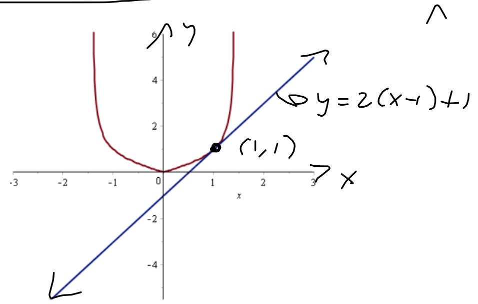
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

$$y = \frac{x}{\sqrt{2-x^2}} = \frac{x}{(2-x^2)^{\frac{1}{2}}} = x(2-x^2)^{-\frac{1}{2}} \Longrightarrow$$

$$y' = 1(2-x^2)^{-\frac{1}{2}} + x\left(-\tfrac{1}{2}(2-x^2)^{-\frac{1}{2}}(-2x)\right)$$

$$y'(1) = 1 + 1\left(-\tfrac{1}{2}(1)\right)(-2(1)) = 1+1 = 2 = m_{tan}$$

$$y = m(x-x_1)+y_1 \Longrightarrow \boxed{y = 2(x-1)+1}$$



15. Question Details SCalc8 2.5.057

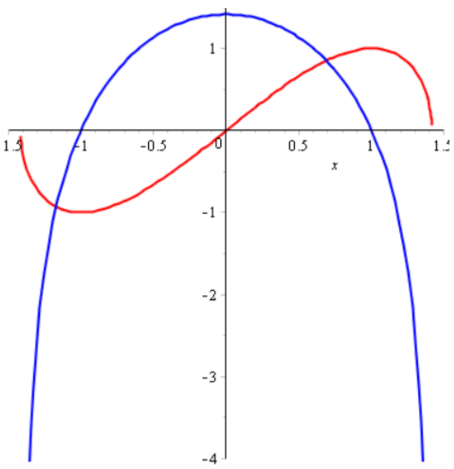
(a) If $f(x) = x\sqrt{2-x^2}$, find $f'(x)$.

(b) Check to see that your answer in part (a) is reasonable by comparing the graphs of f and f'.

$$(a)\quad f(x) = x(2-x^2)^{\frac{1}{2}} \Longrightarrow$$

$$f'(x) = 1(2-x^2)^{\frac{1}{2}} + x\left(\tfrac{1}{2}(2-x^2)^{-\frac{1}{2}}\right)(-2x)$$

$$= (2-x^2)^{\frac{1}{2}}\left(\frac{(2-x^2)^{\frac{1}{2}}}{(2-x^2)^{\frac{1}{2}}}\right) - \frac{x^2}{(2-x^2)^{\frac{1}{2}}} = \frac{2-x^2-x^2}{\sqrt{2-x^2}} = \frac{2-2x^2}{\sqrt{2-x^2}}$$



16. Question Details SCalc8 2.5.059. [3694806]

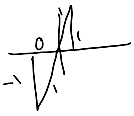
Find all points on the graph of the function $f(x) = 2\sin(x) + \sin^2(x)$ at which the tangent line is horizontal. (Use n as your arbitrary integer.)

$$\Rightarrow f'(x) = 2\cos x + 2\sin x\cos x \overset{SET}{=} 0$$

$$\Rightarrow (2\cos x)[1 + \sin x] = 0$$

$2\cos x = 0$

$\cos x = 0$



$\frac{\pi}{2} + n\pi \ \forall n \in \mathbb{Z}$

$\sin x = -1$

$\frac{\pi}{2} + 2n\pi \ \forall n \in \mathbb{Z} \rightarrow$ So $\left\{ \frac{\pi}{2} + n\pi \mid n \in \mathbb{Z} \right\}$

17. Question Details SCalc8 2.5.061.

If $F(x) = f(g(x))$, where $f(-1) = 5$, $f'(-1) = 6$, $f'(4) = 4$, $g(4) = -1$, and $g'(4) = 8$, find $F'(4)$.

$$F'(x) = \frac{dF}{dx} = \frac{df}{dg}\cdot\frac{dg}{dx} = f'(g(x))\cdot g'(x)$$

$$F'(4) = f'(g(4))\,g'(4) = f'(-1)\cdot 8 = 6\cdot 8 = 48$$

18. Question Details

A table of values for f, g, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = f(g(x))$, find $h'(3)$. $= f'(g(3))\,g'(3) = 2\cdot 9 = 18$

(b) If $H(x) = g(f(x))$, find $H'(1)$.

$= g'(f(1))\,f'(1) = 9\cdot 3 = 27$

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19. Question Details SCalc8 2.5.076. [3354234]

If the equation of motion of a particle is given by $s = A\cos(\omega t + \delta)$, the particle is said to undergo *simple harmonic motion*.

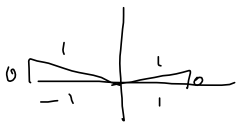
(a) Find the velocity of the particle at time t.

(b) When is the velocity 0? (Use n as the arbitrary integer.)

(a) $s'(t) = \frac{ds}{dt} = -A\sin(\omega t + \delta)\cdot\omega$

(b) SET $= 0 \Rightarrow -A\omega\sin(\omega t+\delta) = 0$

$\Rightarrow \sin(\omega t + \delta) = 0$



$\omega t + \delta = n\pi \quad \forall\, n \in \mathbb{Z}$

$\omega t = n\pi - \delta$

$t = \frac{n\pi - \delta}{\omega} \quad \forall\, n \in \mathbb{Z}$

Sol'n Set

$\left\{ \frac{n\pi - \delta}{\omega} \,\middle|\, n \in \mathbb{Z} \right\}$

20. Question Details SCalc8 2.5.077. [3354362]

A Cepheid variable star is a star whose brightness alternately increases and decreases. For a certain star, the interval between times of maximum brightness is 5.8 days. The average brightness of this star is 3.0 and its brightness changes by ±0.45. In view of these data, the brightness of the star at time t, where t is measured in days, has been modeled by the function

$$B(t) = 3.0 + 0.45\sin\left(\frac{2\pi t}{5.8}\right).$$

(a) Find the rate of change of the brightness after t days.

(b) Find, correct to two decimal places, the rate of increase after one day.

Period $= T = 5.8$

want $bx = 2\pi$ when $x = 5.8$

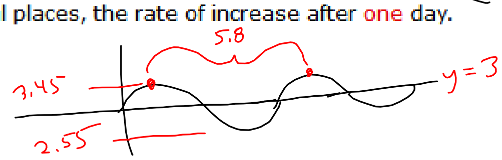
$5.8b = 2\pi$

$b = \frac{2\pi}{5.8}$

$u = \frac{2\pi t}{5.8}$

(a) $B'(t) = (.45)\underbrace{\sin\left(\frac{2\pi t}{5.8}\right)}_{\frac{dy}{du}}\underbrace{\left(\frac{2\pi}{5.8}\right)}_{\frac{du}{dx}}$

$= (.45)\left(\frac{2\pi}{5.8}\right)\sin\left(\frac{2\pi t}{5.8}\right)$



$\sin\left(\frac{2\pi}{5.8}\right)$

(b) $B'(1) = (.45)\left(\frac{2\pi}{5.8}\right)\sin\left(\frac{2\pi}{5.8}\right) \approx 0.4307019749$

$\approx 0.43 \ \frac{\text{BRIGHTNESS UNITS}}{\text{DAY}}$

21. Question Details SCalc8 2.5.078. [3354573]

A model for the length of daylight (in hours) in Philadelphia on the tth day of the year is

$$L(t) = 12 + 2.8 \sin\left[\frac{2\pi}{365}(t - 80)\right].$$

$L'(t)$

Use this model to compare how the number of hours of daylight is increasing in Philadelphia on April 21 and May 21. (Assume there are 365 days in a year. Round your answers to four decimal places.)

$$L'(t) = (2.8)\left(\frac{2\pi}{365}\right)\cos\left(\frac{2\pi}{365}(t-80)\right)$$

$$L'(111) = L'(\text{April } 21) \approx 0.04149812957 \approx 0.0415$$

$\dfrac{\text{hours of daylight}}{\text{day}}$

May 21^{st}: $L'(141) = \ldots \approx 0.02398002990$

June 23^{rd}: $L'(174) = \ldots \approx -0.002280880417$

After the Solstice, Days getting shorter.