

Chain Rule

$$H(x) = f(g(x))$$

$$H'(x) = \frac{df}{dg} \cdot \frac{dg}{dx} = \frac{dH}{dg} \cdot \frac{dg}{dx}$$

Chain rule says

$$\frac{dP}{dt} = \frac{dP}{dS} \cdot \frac{dS}{dt}$$

$$\frac{dP}{dS} = \frac{d}{dS} [20S] = 20$$

$$\frac{dS}{dt} = \frac{d}{dt} [100t] = 100$$

$$\frac{dP}{dS} \cdot \frac{dS}{dt} = 20 \cdot 100 = 2000 \frac{\$}{hr}$$

$$\frac{dP}{dS} = \frac{dP}{d(100t)}$$

Profit: I make \$20 per shirt.

Shirts: I make 100 shirts per hour.

S = # of shirts made in t hours
is $100t$ (shirts)

P = Amt of \$ made by selling
 S shirts = $P(S) = 20S$

P as a function of time
is $20S = 20(100t)$

$$P(t) = 2000t$$

$$P'(t) = \frac{dP}{dt} = 2000 \frac{\$}{hr}$$

1. Question Details

SCalc8 2.5.001. [3]

Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.]

$$y = \sqrt[3]{1+8x}$$

$$u = g(x) = 8x+1$$

$$f(u) = \sqrt[3]{u} = u^{\frac{1}{3}} = y \quad \begin{array}{l} \text{At speed:} \\ \frac{d}{dx}(8x+1)^{\frac{1}{3}} = \frac{1}{3}(8x+1)^{-\frac{2}{3}}(8) \end{array}$$

Find the derivative dy/dx .

$$\frac{dy}{dx} = \frac{dy}{d(g(x))} \cdot \frac{d(g(x))}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{1}{3} u^{-\frac{2}{3}} \right) (8) = \frac{1}{3} (8x+1)^{-\frac{2}{3}} (8)$$

2. Question Details

SCalc8 2.5.003. [3]

Write the composite function in the form $f(g(x))$. [Identify the inner function $u = g(x)$ and the outer function $y = f(u)$.]

$$y = \tan(\pi x)$$

Find the derivative dy/dx .

$$u = \pi x$$

$$\tan u = y$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (\sec^2 u) (\pi)$$

$$\frac{dy}{dx} = \pi$$

$$\frac{dy}{du} = \sec^2 u$$

$$\pi \sec^2(\pi x)$$

3. Question Details

SCalc8 2.5.007. [3354312]

Find the derivative of the function.

$$F(x) = (5x^6 + 2x^3)^4 \implies F'(x) = 4(5x^6 + 2x^3)^3 (30x^5 + 6x^2)$$

4. [Question Details](#) SCalc8 2.5.009.

Find the derivative of the function.

$$f(x) = \sqrt{5x+6} = (5x+6)^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}(5x+6)^{-\frac{1}{2}}(5)$$

5. [Question Details](#) SCalc8 2.5.013.

Find the derivative of the function.

$$f(\theta) = \cos(\theta^2) \Rightarrow f'(\theta) = (-\sin(\theta^2))(2\theta)$$

6. [Question Details](#) SCalc8 2.5.016.

Find the derivative of the function.

$$f(t) = 9t \sin(\pi t) = 9 \sin(\pi t) + 9t \cos(\pi t) \cdot \pi$$

7. Question Details SCalc8 2.5.017.

Find the derivative of the function.

$$f(x) = (2x - 7)^4(x^2 + x + 1)^5 \Rightarrow f'$$

$$f'(x) = 4(2x-7)^3(2)(x^2+x+1)^5 + (2x-7)^4(5(x^2+x+1)^4)(2x+1)$$

8. Question Details SCalc8 2.5.019.

Find the derivative of the function.

$$h(t) = (t + 4)^{2/3}(3t^2 - 1)^3$$

$$\Rightarrow h'(t) = \frac{2}{3}(t+4)^{-1/3}(1)(3t^2-1)^3 + (t+4)^{2/3}(3(3t^2-1)^2)(6t)$$

9. Question Details SCalc8 2.5.023.

Find the derivative of the function.

$$y = \sqrt{\frac{x}{x+3}} = \left(\frac{x}{x+3}\right)^{1/2} \Rightarrow$$

$$y' = \frac{1}{2} \left(\frac{x}{x+3}\right)^{-1/2} \left(\frac{1(x+3) - (x)(1)}{(x+3)^2}\right)$$

10. Question Details SCalc8 2.5.036.

Find the derivative of the function.

$$y = x \sin \frac{6}{x} = x \sin(6x^{-1}) \Rightarrow y' = \sin\left(\frac{6}{x}\right) + \left(x \cos(6x^{-1})\right)(-6x^{-2})$$

11. Question Details SCalc8 2.5.037.

Find the derivative of the function.

$$y = \cot^2(\sin(\theta)) \Rightarrow y' = 2(\cot(\sin(\theta)))(-\csc(\sin(\theta))\cot(\sin(\theta)))(\cos(\theta))$$

12. Question Details SCalc8 2.5.042.

Find the derivative of the function.

$$y = \sqrt{5x + \sqrt{5x + \sqrt{5x}}} = \left(5x + \left(5x + (5x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$\Rightarrow y' = \frac{1}{2} \left(5x + \left(5x + (5x)^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \left[5 + \frac{1}{2} \left(5x + (5x)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \left(5 + \frac{1}{2} (5x)^{-\frac{1}{2}} (5)\right)\right]$$

13. Question Details SCalc8 2.5.045.

Find the derivative of the function.

$$y = \cos\left(\sqrt{\sin(\tan(9x))}\right) = \cos\left(\left(\sin(\tan(9x))\right)^{\frac{1}{2}}\right)$$

$$\Rightarrow y' = -\sin\left(\left(\sin(\tan(9x))\right)^{\frac{1}{2}}\right) \left(\frac{1}{2} \left(\sin(\tan(9x))\right)^{-\frac{1}{2}} \left(\cos(\tan(9x))\right) \left(\sec^2(9x)\right) (9)\right)$$

14. Question Details

SCalc8 2.5.056. [3943253]

(a) The curve $y = |x|/\sqrt{2-x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point $(1, 1)$. *close to $x=1$, $|x|=x$. $D = (-\sqrt{2}, \sqrt{2})$*

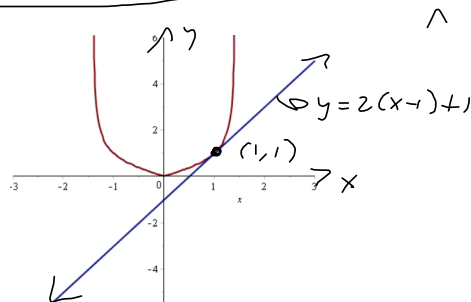
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

$$y = \frac{x}{\sqrt{2-x^2}} = \frac{x}{(2-x^2)^{\frac{1}{2}}} = x(2-x^2)^{-\frac{1}{2}} \implies$$

$$y' = 1(2-x^2)^{-\frac{1}{2}} + x(-\frac{1}{2}(2-x^2)^{-\frac{3}{2}}(-2x))$$

$$y'(1) = 1 + 1(-\frac{1}{2}(1))(-2(1)) = 1 + 1 = 2 = m_{\text{tan}}$$

$$y = m(x-x_1) + y_1 \implies \boxed{y = 2(x-1) + 1}$$



15. Question Details

SCalc8 2.5.057

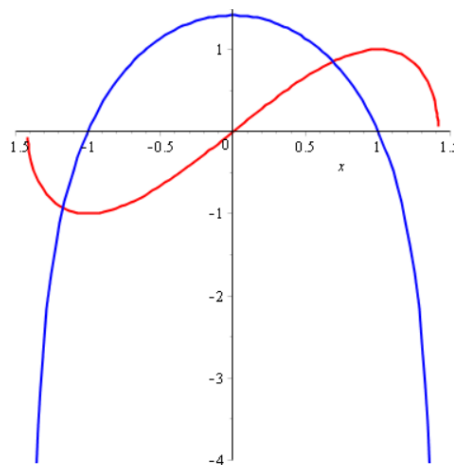
(a) If $f(x) = x\sqrt{2-x^2}$, find $f'(x)$.

(b) Check to see that your answer in part (a) is reasonable by comparing the graphs of f and f' .

$$(2) f(x) = x(2-x^2)^{\frac{1}{2}} \implies$$

$$f'(x) = 1(2-x^2)^{\frac{1}{2}} + x(\frac{1}{2}(2-x^2)^{-\frac{1}{2}})(-2x)$$

$$= (2-x^2)^{\frac{1}{2}} - \frac{x^2}{(2-x^2)^{\frac{1}{2}}} = \frac{2-x^2-x^2}{\sqrt{2-x^2}} = \frac{2-2x^2}{\sqrt{2-x^2}}$$



16. Question Details

SCalc8 2.5.059. [3694806]

Find all points on the graph of the function $f(x) = 2 \sin(x) + \sin^2(x)$ at which the tangent line is horizontal. (Use n as your arbitrary integer.)

$$\Rightarrow f'(x) = 2 \cos x + 2 \sin x \cos x \stackrel{\text{SET}}{=} 0$$

$$\Rightarrow (2 \cos x)(1 + \sin x) = 0$$

$$2 \cos x = 0$$

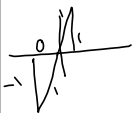
$$\cos x = 0$$

$$\frac{\pi}{2} + n\pi \quad \forall n \in \mathbb{Z}$$

$$\sin x = -1$$

$$\frac{3\pi}{2} + 2n\pi \quad \forall n \in \mathbb{Z}$$

$$\rightarrow \text{So } \left\{ \frac{\pi}{2} + n\pi \mid n \in \mathbb{Z} \right\}$$



17. Question Details

SCalc8 2.5.061.

If $F(x) = f(g(x))$, where $f(-1) = 5$, $f'(-1) = 6$, $f'(4) = 4$, $g(4) = -1$, and $g'(4) = 8$, find $F'(4)$.

$$F'(x) = \frac{dF}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx} = f'(g(x)) \cdot g'(x)$$

$$F'(4) = f'(g(4)) g'(4) = f'(-1) \cdot 8 = 6 \cdot 8 = 48$$

18. Question Details

A table of values for f , g , f' , and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = f(g(x))$, find $h'(3)$. $= f'(g(3)) g'(3) = 2 \cdot 9 = 18$

(b) If $H(x) = g(f(x))$, find $H'(1)$.

$$= g'(f(1)) f'(1) = 9 \cdot 3 = 27$$

19.  Question Details

SCalc8 2.5.076. [3354234]

If the equation of motion of a particle is given by $s = A \cos(\omega t + \delta)$, the particle is said to undergo *simple harmonic motion*.

- (a) Find the velocity of the particle at time t .
- (b) When is the velocity 0? (Use n as the arbitrary integer.)

20.  Question Details

SCalc8 2.5.077. [3354362]

A Cepheid variable star is a star whose brightness alternately increases and decreases. For a certain star, the interval between times of maximum brightness is 5.8 days. The average brightness of this star is 3.0 and its brightness changes by ± 0.45 . In view of these data, the brightness of the star at time t , where t is measured in days, has been modeled by the function

$$B(t) = 3.0 + 0.45 \sin\left(\frac{2\pi t}{5.8}\right).$$

- (a) Find the rate of change of the brightness after t days.
- (b) Find, correct to two decimal places, the rate of increase after one day.

21. [Question Details](#)

SCalc8 2.5.078. [3354573]

A model for the length of daylight (in hours) in Philadelphia on the t th day of the year is

$$L(t) = 12 + 2.8 \sin\left[\frac{2\pi}{365}(t - 80)\right].$$

Use this model to compare how the number of hours of daylight is increasing in Philadelphia on **April 21** and **May 21**. (Assume there are 365 days in a year. Round your answers to four decimal places.)