§2.4 Theory Talk

$$\frac{d}{dx}\left[\sin(x)\right] = \cos(x)$$

$\sin(a+b)$
$= \sin a \cos b + \sin b \cos a$
$a = x,\ b = h$

Proof $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sin(x+h) - \sin(x)}{h}$

$$= \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

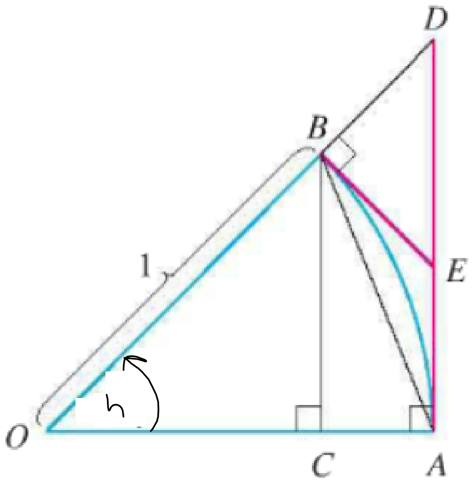
$$= \frac{\sin(x)\cos(h) - \sin(x)}{h} + \frac{\sin(h)\cos(x)}{h}$$

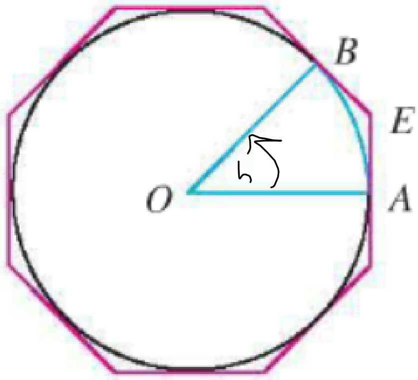
$$= \frac{\sin(x)\left(\cos(h) - 1\right)}{h} + \frac{\sin(h)}{h}\cos(x)$$

$$= \frac{\cos(h)-1}{h}\sin(x) + \frac{\sin(h)}{h}\cos(x) \xrightarrow{h \to 0} \cos(x)\ ?$$

We now show $\dfrac{\cos h - 1}{h} \xrightarrow{h \to 0} 0$ & $\dfrac{\sin h}{h} \xrightarrow{h \to 0} 1$

We prove $\lim_{h \to 0} \frac{\sin h}{h} = 1$, i.e., $\frac{\sin h}{h} \xrightarrow{h \to 0} 1$





① Show $\frac{\sin h}{h} < 1$

② $h =$ arc AB $< \tan h$

③ $\cosh < \frac{\sin h}{h} < 1$ & squeeze.

Recall arc length

$s = r\theta = \theta = h$

$\sin h < h$ by picture

$\frac{\sin h}{h} < 1$

want to show

$\cos h < \frac{\sin h}{h}$ so I can squeeze $\frac{\sin h}{h}$

$\cosh < \frac{\sin h}{h} < 1$

$h \to 0$:

$1 \leq \lim_{h \to 0} \frac{\sin h}{h} \leq 1$

$h < |AD| = \frac{|AD|}{1} = \tan h$

$h < \tan h = \frac{\sin h}{\cos h}$

$\Rightarrow \cos h < \frac{\sin h}{h}$

So, by Squeeze theorem:

$$\boxed{\frac{\sin h}{h} \xrightarrow{h \to 0} 1}$$

Next $\frac{\cos h - 1}{h} \xrightarrow{h \to 0} 0$

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$$\frac{\cos h - 1}{h} \xrightarrow{h \to 0} 0$$

$$\left(\frac{\cos h - 1}{h}\right)\left(\frac{\cos h + 1}{\cos h + 1}\right) = \frac{\cos^2 h - 1}{h(\cos h + 1)} = \frac{1 - \sin^2 h - 1}{h(\cos h + 1)}$$

$$= \frac{-\sin^2 h}{h(\cos h + 1)} = \left(\frac{\sin h}{h}\right)\left(\frac{-\sin h}{\cos(h) + 1}\right) \xrightarrow{h \to 0} (1)\left(\frac{0}{2}\right) = 0$$

So $\frac{d}{dx}[\sin(x)] = \cos x$