Let $f(x)$ and $g(x)$ be differentiable functions.

Define $h(x) = f(x)g(x)$

Then $h'(x) = f'(x)g(x) + f(x)g'(x)$

$h' = f'g + fg'$

The Proof:

The difference quotient for $h(x)$ is

$$\frac{h(x+h) - h(x)}{h} = \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{(f(x+h) - f(x))g(x+h)}{h} + \frac{f(x)[g(x+h) - g(x)]}{h}$$

$$= \underbrace{\left(\frac{f(x+h) - f(x)}{h}\right)}_{\downarrow} \underbrace{g(x+h)}_{\downarrow} + \underset{\downarrow}{f(x)} \underbrace{\left(\frac{g(x+h) - g(x)}{h}\right)}_{\downarrow} \xrightarrow{h \to 0}$$

$$= f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$= f'g + fg'$$

Section 2.3 - Product Rule

Mnemonic: $(fg)' = f'g + fg'$

Best way for Calc III students. The book way hurts you in Calc III, imho.

Example $f(x) = x-1$, $g(x) = x-2$

$h(x) = f(x)g(x)$

OLD WAY:

$h'(x) = \frac{d}{dx}\left[(x-1)(x-2)\right] =$

$= \frac{d}{dx}\left[x^2-3x+2\right] = 2x-3$

NEW WAY:

$\frac{d}{dx}\left[(x-1)(x-2)\right] = \frac{d}{dx}\left[f(x)g(x)\right] = (fg)' = f'g + fg' = 1\cdot(x-2) + (x-1)\cdot 1$

$= x-2+x-1 = 2x-3.$

Advantage: You can take the derivatives, separately and simplify. This often beats trying to multiply everything out and take the derivative, afterwards

$h(x) = \underbrace{(x^3+2x^2-7x+5)}_{f}\underbrace{(5x^3-20x^2+15x-8)}_{g} = f(x)g(x)$

$\Rightarrow h'(x) = h' = f'g + fg'$

$= \boxed{(3x^2+4x-7)(5x^3-20x^2+15x-8) + (x^3+2x^2-7x+5)(15x^2-40x+15) = h'(x)}$

(STOP!)