Build a cubic function with the zeros I want:

$f := x \rightarrow (x-1)\cdot(x-2)\cdot(x-3)$

$$f := x \mapsto (x-1)\cdot(x-2)\cdot(x-3) \quad \mathbf{(1)}$$

The polynomial written in expanded form, in descending order (of powers of *x*):

$expand(f(x))$

$$x^3 - 6x^2 + 11x - 6 \quad \mathbf{(2)}$$

Take the derivative of *f* with respect to *x*. This looks like the CAS is using the product rule.

$fp := \mathrm{D}(f)$

$$fp := x \mapsto (x-2)\cdot(x-3) + (x-1)\cdot(x-3) + (x-1)\cdot(x-2) \quad \mathbf{(3)}$$

$expand(fp(x))$

$$3x^2 - 12x + 11 \quad \mathbf{(4)}$$

Another Example (Product Rule Example).

$f := x \rightarrow x^3 + 2\cdot x^2 - 7\cdot x + 5$

$$f := x \mapsto x^3 + 2\cdot x^2 - 7\cdot x + 5 \quad \mathbf{(5)}$$

$g := x \rightarrow 5\cdot x^3 - 20\cdot x^2 + 15\cdot x - 8$

$$g := x \mapsto 5\cdot x^3 - 20\cdot x^2 + 15\cdot x - 8 \quad \mathbf{(6)}$$

$h := x \rightarrow f(x)\cdot g(x)$

$$h := x \mapsto f(x)\cdot g(x) \quad \mathbf{(7)}$$

With the product rule, $h' = f\,'g + f g'$:

$hp := \mathrm{D}(h)$

$$hp := x \mapsto \left(3\cdot x^2 + 4\cdot x - 7\right)\cdot g(x) + f(x)\cdot\left(15\cdot x^2 - 40\cdot x + 15\right) \quad \mathbf{(8)}$$

$hp(x)$

$$\left(3x^2 + 4x - 7\right)\left(5x^3 - 20x^2 + 15x - 8\right) + \left(x^3 + 2x^2 - 7x + 5\right)\left(15x^2 - 40x + 15\right) \quad \mathbf{(9)}$$

$expand(hp(x))$

$$30x^5 - 50x^4 - 240x^3 + 561x^2 - 442x + 131 \quad \mathbf{(10)}$$

Now check this against the "old way:"

$k := x \rightarrow expand(h(x))$

$$k := x \mapsto expand(h(x)) \quad \mathbf{(11)}$$

This function, $k(x)$, is just $h(x)$, expanded:

$k(x)$

$$5x^6 - 10x^5 - 60x^4 + 187x^3 - 221x^2 + 131x - 40 \quad \mathbf{(12)}$$

Taking the derivative of $k(x)$ by the rule for polynomials gives the following:

$kp := \mathrm{D}(k)$

$$kp := x \mapsto 30 \cdot x^5 - 50 \cdot x^4 - 240 \cdot x^3 + 561 \cdot x^2 - 442 \cdot x + 131 \quad \textbf{(13)}$$

You see that computing it either way yields the same derivative.