Some cheats before we do the grunt work in §2.1:
(to check)

Power Rule: $f(x) = x^n \implies f'(x) = m_{tan} = \frac{dy}{dx} = n x^{n-1} \quad \forall n \neq 0.$

Quotient Rule: $h(x) = \frac{f(x)}{g(x)} \implies h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} = \frac{f'g - fg'}{g^2}$

By props of limits,

$(f \pm g)'(x) = f'(x) \pm g'(x).$

PRODUCT Rule: $(f(x)g(x))'$

$\underline{f'(x)g(x) + f(x)g'(x)}$

$f'g + fg'$

Chain Rule: $h(x) = f(g(x)) \implies$

$\frac{df}{dg} \cdot \frac{dg}{dx}$

Sep 13-1:14 PM

1. Question Details SCalc8 2.1.001. [3354371]

A curve has equation $y = f(x)$.

(a) Write an expression for the slope of the secant line through the points $P(3, f(3))$ and $Q(x, f(x))$.
(x_1, y_1) (x_2, y_2)

(b) Write an expression for the slope of the tangent line at P.

(a) $m_{sec} = \frac{y_2 - y_1}{x_2 - x_1} = \boxed{\frac{f(x) - f(3)}{x - 3}}$ is equivalent to $\frac{f(3+h) - f(3)}{3+h-3} = \frac{f(3+h) - f(3)}{h}$

(b) $m_{tan} = \boxed{\lim_{x\to 3} \frac{f(x) - f(3)}{x - 3}}$ $\qquad \lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$

2. Question Details SCalc8 2.1.003. [3391662] -

Consider the parabola $y = 5x - x^2$. $= f(x)$

(a) Find the slope of the tangent line to the parabola at the point $(1, 4)$.

(b) Find an equation of the tangent line in part (a).

(c) Graph the parabola and the tangent line.

(a) $\frac{f(x+h) - f(x)}{h} = \frac{5(x+h) - (x+h)^2 - (5x - x^2)}{h}$

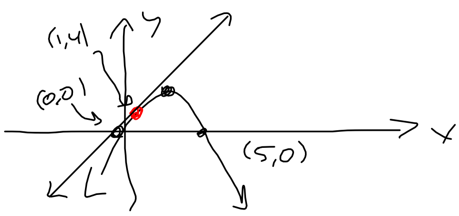
$= \frac{5x + 5h - (x^2 + 2xh + h^2) - 5x + x^2}{h} = \frac{5h - x^2 - 2xh - h^2 + x^2}{h}$

$= \frac{5h - 2xh - h^2}{h} = \frac{h(5 - 2x - h)}{h} = 5 - 2x - h \xrightarrow{h \to 0} 5 - 2x = f'(x)$
$(h \neq 0)$

$\Rightarrow f'(1) = 5 - 2(1) = 5 - 2 = \boxed{3 = m_{tan}}$

(b) $(1, f(1)) = (1, 4) = (x_1, y_1) \Rightarrow y = m(x - x_1) + y_1$
$\boxed{y = 3(x-1) + 4}$ mills. $= \boxed{3x + 1 \quad \text{Book}}$

(c) $5x - x^2 = -x(x-5)$



Cheat:
$f(x) = 5x - x^2 \Rightarrow$ checks
$f'(x) = 5x^0 - 2x^1 = \boxed{5 - 2x}$

3. Question Details SCalc8 2.1.004. [3391765]

Consider the curve $y = x - x^7$.

(a) Find the slope of the tangent line to the curve at the point (1, 0).

(b) Find an equation of the tangent line in part (a).

(c) Graph the curve and the tangent line.

1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

In the previous, I found $f'(x)$ and then plugged in $x = 1$. It's probably quicker, doing these the hard way, as we are, to just do $f'(1)$.

(a) $$\frac{f(1+h)-f(1)}{h} = \frac{(1+h)-(1+h)^7-(1-1^7)}{h}$$

$$= \frac{1+h-(1+7h+21h^2+35h^3+35h^4+21h^5+7h^6+h^7)}{h}$$

$$= \frac{1+h-1-7h-21h^2+35h^3+35h^4+21h^5+7h^6+h^7}{h}$$

$$= \frac{h-7h-21h^2-35h^3 \cdots - h^7}{h} = \frac{\cancel{h}(1-7-21h-35h^2-\cdots-h^6)}{\cancel{h}}$$

$$= \boxed{-6}$$

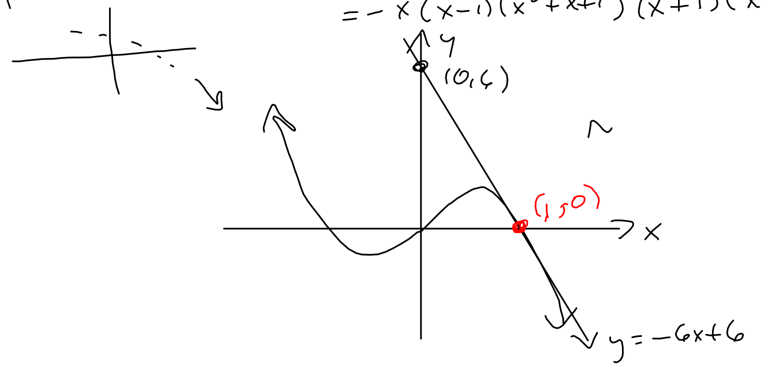
Check $f(x) = x - x^7 \Longrightarrow f'(x) = 1 - 7x^6$

$f'(1) = 1 - 7(1)^6 = -6$ ✓

(b) $y = -6(x-1) + 0 = \boxed{-6x + 6 = y}$

(c) $x - x^7 = -x(x^6 - 1) = -x(x^3-1)(x^3+1)$

$= -x(x-1)(x^2+x+1)(x+1)(x^2+x+1)$



4. Question Details SCalc8 2.1.007.MI. [3354113]

Find an equation of the tangent line to the curve at the given point.

$f(x) = y = \sqrt{x}$, (25, 5)

$$\frac{f(x+h)-f(x)}{h} = \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right) = \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}} \xrightarrow{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Cheat/Check

$$\sqrt{x} = x^{\frac{1}{2}} = f(x) \Rightarrow f'(x) = \tfrac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

$$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{2 \cdot 5} = \frac{1}{10} = m$$

$$\boxed{y = \tfrac{1}{10}(x-25)+5}$$

5. Question Details SCalc8 2.1.010. [3354156]

(a) Find the slope m of the tangent to the curve $y = 7/\sqrt{x}$ at the point where $x = a > 0$.

(b) Find equations of the tangent lines at the points (1, 7) and $\left(4, \frac{7}{2}\right)$.

(c) Graph the curve and both tangents on a common screen.

Here, they're trying to *ease* you into the idea of the derived function, f', as a function in its own right, which I've been kind of doing to you, right along, by taking the derivative in general, then pulugging in a specific value.

cheat: $f(x) = 7x^{-\frac{1}{2}} \Rightarrow f'(x) = -\frac{7}{2}x^{-\frac{3}{2}} = -\frac{7}{2} \cdot \frac{1}{\sqrt{x^3}}$

$$\frac{f(a+h)-f(a)}{h} = \frac{\frac{7}{\sqrt{a+h}} - \frac{7}{\sqrt{a}}}{h} = \frac{1}{h}\left[\frac{7\sqrt{a} - 7\sqrt{a+h}}{\sqrt{a}\sqrt{a+h}}\right]$$

$$= \frac{1}{h}\left[\frac{7\sqrt{a}-7\sqrt{a+h}}{\sqrt{a}\sqrt{a+h}}\right]\left[\frac{7\sqrt{a}+7\sqrt{a+h}}{7\sqrt{a}+7\sqrt{a+h}}\right] = \left[\frac{49a - 49(a+h)}{7\sqrt{a}\sqrt{a+h}(\sqrt{a}+\sqrt{a+h})}\right] \cdot \frac{1}{h}$$

$$= \frac{1}{\cancel{h}}\left[\frac{-\cancel{49h}^{\,7}}{\cancel{7}\sqrt{a}\sqrt{a+h}(\sqrt{a}+\sqrt{a+h})}\right] \xrightarrow{h \to 0} \frac{-7}{\sqrt{a}\sqrt{a}(\sqrt{a}+\sqrt{a})}$$

$$(\sqrt{a})^2 = a$$

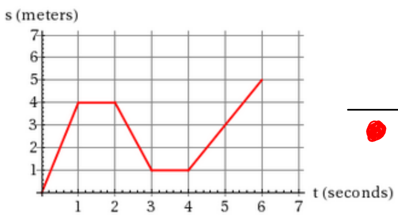
$$\sqrt{a^2} = |a|$$

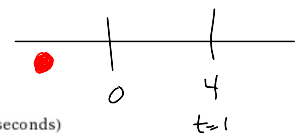
$$= \frac{-7}{a(2\sqrt{a})} = \frac{-7}{2a \cdot a^{1/2}}$$

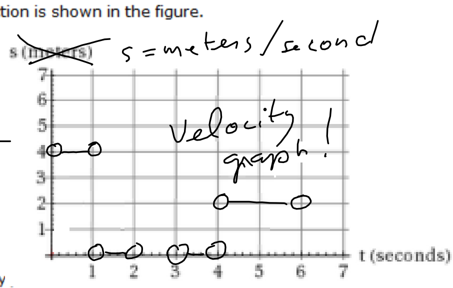
$$= \boxed{-\frac{7}{2}a^{-\frac{3}{2}}}$$ is fine

6. Question Details SCalc8 2.1.011. [3425289]

(a) A particle starts by moving to the right along a horizontal line; the graph of its position function is shown in the figure.







For which of the following time intervals is the particle moving to the right? (Select all that apply

For which of the following time intervals is the particle moving to the left? (Select all that apply.)

For which of the following time intervals is the particle standing still? (Select all that apply.)

(b) Draw a graph of the velocity function.

0–0 $y=-3$

R $(0,1)\cup(4,6)$

L $(2,3)$

$(1,2)\cup(3,4)$

7. Question Details SCalc8 2.1.013.MI. [3354522]

If a ball is thrown into the air with a velocity of 39 ft/s, its height (in feet) after t seconds is given by $y = 39t - 16t^2$. Find the velocity when $t = 1$.

Cheat: $f'(t) = -32t + 39$

$y = -16t^2 + 39t$

$f'(1) = -32 + 39 = \boxed{7\ ft/sec}$

$h = \frac{1}{2}gt^2 + v_0 t + h_0$

By defn: $\dfrac{-16(t+h)^2 + 39(t+h) - (-16t^2 + 39t)}{h}$

$= \dfrac{-16(t^2 + 2th + h^2) + 39t + 39h + 16t^2 - 39t}{h}$

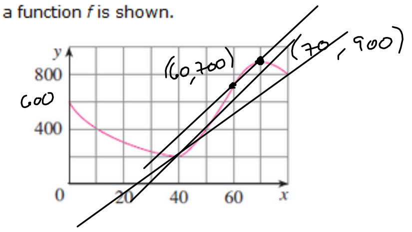
$= \dfrac{-16t^2 - 32th - 16h^2 + 39t + 39h + 16t^2 - 39t}{h}$

$= \dfrac{-32th - 16h^2 + 39h}{h} = -32t - 16h + 39 \xrightarrow{h \to 0} -32t + 39 = v(t)$

$= f'(t)$

8. Question Details SCalc8 2.1.018. [3354520]

The graph of a function *f* is shown.



a) $m_{AVG} = \dfrac{900-700}{70-60} = \dfrac{200}{10} = \boxed{20 = m}$

on [60, 70]

(a) Find the average rate of change of *f* on the interval [60, 70]. (Round your answer to the nearest integer.)

(b) Identify an interval on which the average rate of change of *f* is 0. b) (15, 40)

(c) Which interval gives a larger average rate of change, [40, 50] or [40, 80]? c) [40, 50]

(d) Compute $\dfrac{f(40) - f(10)}{40 - 10} = \dfrac{200-400}{30} = -\dfrac{200}{30} = -\dfrac{20}{3}$ $\boxed{= -6.\overline{6}}$.

What does this value represent geometrically?

This quotient is the average slope of *f* from x = 10 to x = 40.

9. Question Details SCalc8 2.1.020. [3943310]

Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 4$ if $g(4) = -3$ and $g'(4) = 6$. (Enter your answer as an equation in terms of y and x.)

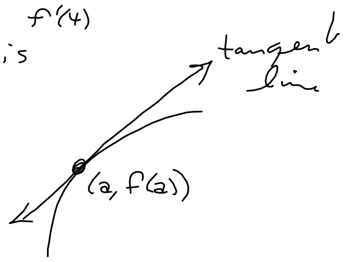
f(x) f(4) f'(4)

Tan. Line thru $(x_1, y_1) = (a, f(a))$ is

$y = m(x - x_1) + y_1$

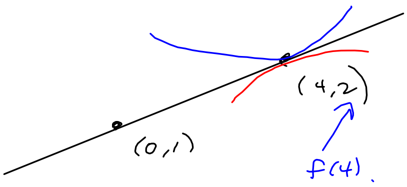
$= f'(a)(x - a) + f(a)$

$\boxed{y = 6(x-4) - 3}$



10. Question Details SCalc8 2.1.022. [3354563]

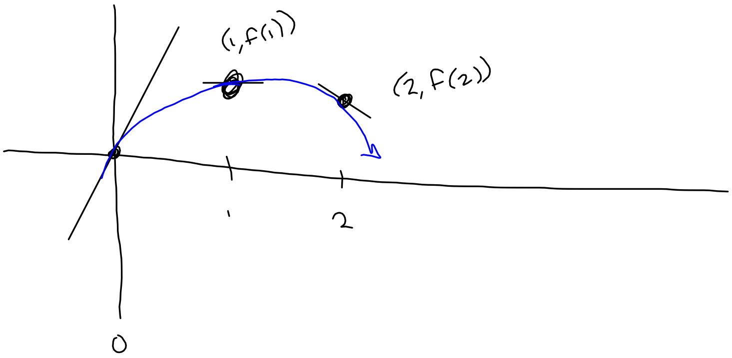
If the tangent line to $y = f(x)$ at $(4, 2)$ passes through the point $(0, 1)$, find $f(4)$ and $f'(4)$.



$f(4)$. $f'(4) = m_{tan} = \frac{2-1}{4-0} = \boxed{\frac{1}{4} = f'(4)}$

11. Question Details

Sketch the graph of a function f for which $f(0) = 0$, $f'(0) = 3$, $f'(1) = 0$, and $f'(2) = -1$.



12. Question Details SCalc8 2.1.033.MI. [3354474]

Find ~~f'(a)~~. $h'(a)$

$h(t) = \cancel{f(t)} = \dfrac{6t+8}{t+7}$

Cheat: $h(t) = \dfrac{f(t)}{g(t)} \Rightarrow h' = \dfrac{f'g - fg'}{g^2}$

$$h'(t) = \frac{6(t+7) - (6t+8)(1)}{(t+7)^2} = \frac{6t + 42 - 6t - 8}{(t+7)^2}$$

$$\boxed{= \frac{34}{(t+7)^2}}$$

By defin:

$$\frac{h(t+h) - h(t)}{h} = \frac{\dfrac{6(t+h)+8}{(t+h)+7} - \dfrac{6t+8}{t+7}}{h} = \frac{1}{h}\left[\frac{(6(t+h)+8)(t+7) - (6t+8)(t+h+7)}{(t+7)(t+h+7)}\right]$$

Scratch

$$(t+7)(6t+6h+8) = 6t^2 + 6th + 8t + 42t + 42h + 56$$
$$= \cancel{6t^2} + \cancel{6th} + \cancel{50t} + 42h + 56$$

$$-\ (6t+8)(t+h+7) = (6t^2 + 6th + 42t + 8t + 8h + 56)(-1)$$
$$= -(\cancel{6t^2} + \cancel{6th} + \cancel{50t} + 8h + 56)$$

Main

$$= \left[\frac{34\cancel{h}}{(t+7)(t+h+7)}\right]\frac{1}{\cancel{h}} \xrightarrow{h \to 0} \frac{34}{(t+7)^2}$$

13. Question Details SCalc8 2.1.035. [3354299]

Find $f'(a)$.

$f(x) = \sqrt{2-4x}$

Chain Rule

$$= (2-4x)^{\frac{1}{2}} \Rightarrow f'(x) = \tfrac{1}{2}(2-4x)^{-\frac{1}{2}} \cdot (-4)$$

$$= \frac{-2}{\sqrt{2-4x}}$$

$h(x) = f(u(x))$

$u = 2-4x \Rightarrow \dfrac{du}{dx} = -4$

$$f(2-4x) = f(u) \Rightarrow \frac{df}{du} = \frac{d}{du}\left[u^{\frac{1}{2}}\right] = \tfrac{1}{2}u^{-\frac{1}{2}} = \tfrac{1}{2}(2-4x)^{-\frac{1}{2}}$$

$$f'(a) = \tfrac{1}{2}(2-4a)^{-\frac{1}{2}}(-4)$$

$$\frac{f(x+h) - f(x)}{h} = \left(\frac{\sqrt{2-4(x+h)} - \sqrt{2-4x}}{h}\right)\left(\frac{\sqrt{2-4(x+h)} + \sqrt{2-4x}}{\sqrt{2-4(x+h)} + \sqrt{2-4x}}\right)$$

$$= \frac{2-4(x+h) - (2-4x)}{h(\sim\sim)} = \frac{2 - 4x - 4h - 2 + 4x}{h(\sim\sim)} = \frac{-4h}{h(\sim\sim)}$$

$$= \frac{-4}{\sqrt{2-4(x+h)} + \sqrt{2-4x}} \xrightarrow{h \to 0} \frac{-4}{\sqrt{2-4x} + \sqrt{2-4x}} = \frac{-4}{2\sqrt{2-4x}} = \frac{-2}{\sqrt{2-4x}}$$

$$f'(a) = \frac{-2}{\sqrt{2-4a}}$$

Sep 13-12:47 PM

14. Question Details — SCalc8 2.1.041. [3354567]

The limit represents the derivative of some function f at some number a. State such an f and a.

$$\lim_{h\to 0}\frac{\cos(\pi+h)+1}{h}$$

$\cos(\pi) = -1$

$\lim_{h\to 0}\frac{\cos(\pi+h)-(-1)}{h} = \frac{\cos(\pi+h)-\cos\pi}{h}$

$\boxed{f(x)=\cos x,\ a=\pi}$

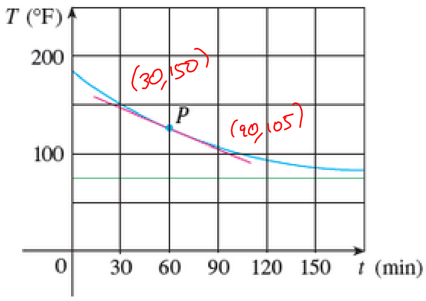
15. Question Details — SCalc8 2.1.046. [3354277]

A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F. The graph shows how the temperature of the turkey decreases and eventually approaches room temperature. By measuring the slope of the tangent, estimate the rate of change of the temperature after an hour.

[ ] °F/min

Newton's Law of Cooling.

Later in book: Applications.



$m = \frac{105-50}{90-30} \Rightarrow \frac{55}{60} = \frac{11}{10} = m_{\tan}$ @ $t=60$, i.e., $f'(60)$

$\frac{11}{10}\ \frac{°F}{min}$.

16. Question Details — SCalc8 2.1.051. [3391775]

The cost (in dollars) of producing x units of a certain commodity is $C(x) = 5000 + 11x + 0.15x^2$.

(a) Find the average rate of change of C with respect to x when the production level is changed from $x = 100$ to the given value. (Round your answers to the nearest cent.)

(i) $x = 104$

(ii) $x = 101$

(b) Find the instantaneous rate of change of C with respect to x when $x = 100$. (This is called the *marginal cost*.)

(a) m_{AVG} :

(i) $x_2 = 104 \Rightarrow m_{avg} = \frac{C(104)-C(100)}{104-100} = \boxed{41.6}$

(ii) $x_2 = 101 \Rightarrow m_{avg} =$ Economist's DEFINITION of marginal cost @ $x = 100$ $= \frac{C(101)-C(100)}{101-100}$ $\boxed{=41.15}$

```
Plot1 Plot2 Plot3
\Y1■.15X²+11X+50
00
\Y2■Y1(100)
\Y3■(Y1(X)-Y2)/(
X-100)
\Y4=
\Y5=
```

$\leftarrow \frac{C(x)-C(100)}{x-100}$

```
Y3(104)
                41.6
Y3(101)
               41.15
■
```

(b) $C'(x) = .30x + 11 \Rightarrow$

$C'(100) = .30(100) + 11$

$= 30 + 11 = \boxed{41 = C'(100)}$

Very close to

$\frac{C(101)-C(100)}{1}$

17. Question Details SCalc8 2.1.053. [3354491]

The cost of producing x ounces of gold from a new gold mine is $C = f(x)$ dollars.

(a) What is the meaning of the derivative $f'(x)$? What are its units?

(b) What does the statement $f'(700) = 17$ mean?

(c) Do you think the values of $f'(x)$ will increase or decrease in the short term? What about the long term? Explain.

- ○ $f'(x)$ is the total amount of gold produced. Its units are ounces.
- ○ $f'(x)$ is the average production cost with respect to the number of ounces of gold produced. Its units are dollars per ounce.

(a) $f'(x) =$ rate of change in cost in $\frac{\$}{oz}$

(b) $f'(700) = 17$ means $\frac{17\ \$}{oz}$

(c) Increase in short term (from $x = 700$) b/c $17 > 0$

Long term? I dunno. Go up? Inflation?

18. Question Details SCalc8 2.1.056. [3354478]

The quantity (in pounds) of a gourmet ground coffee that is sold by a coffee company at a price of p dollars per pound is $Q = f(p)$.

(a) What is the meaning of the derivative $f'(5)$?

(a) It's the rate of change in the quantity of coffee sold with respect to change in price.

What are the units of $f'(5)$?

$\frac{\text{pounds}}{\text{dollar}}$ are the units

(b) In general, will $f'(5)$ be positive or negative?

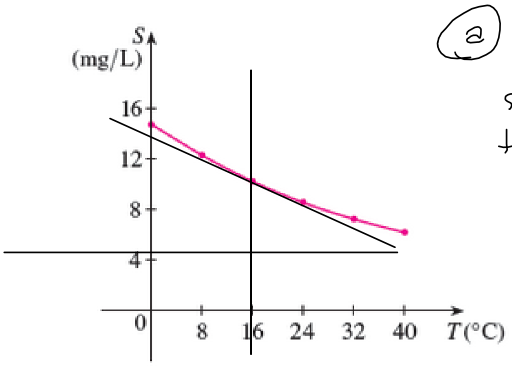
Expect $f'(5) < 0$

Increasing price $\Rightarrow$ Decrease in quantity.

Getting repetitive.

19. Question Details SCalc8 2.1.057. [3354307]

The quantity of oxygen that can dissolve in water depends on the temperature of the water. (So thermal pollution influences the oxygen content of water.) The graph shows how oxygen solubility S varies as a function of the water temperature T.



(a) $S'(T)$ is the rate of change of solubility of O_2 with respect to change in temperature.

units are $\frac{mg/L}{°C}$

(b) 10

(0, 14)

(40, 5)

$m_{tan} \approx \frac{9}{-40} \frac{mg/L}{°C}$

(a) What is the meaning of the derivative $S'(T)$?

(b) Estimate the value of $S'(16)$. (Round your answer to three decimal places.) xygen solubility.

Off/Topic science fact:

CO2 dissolves better in COLD water. C02 causes Carbonic Acid to form in water. So higher temperatures do NOT cause limestone to erode faster. It erodes more slowly at higher temps.

This is the opposite of what our intuition tells us about how heating up a solution allows you to dissolve MORE salt/sugar/anything. Just not CO2.

Sep 13-12:52 PM

20. Question Details SCalc8 2.1.059. [3354198]

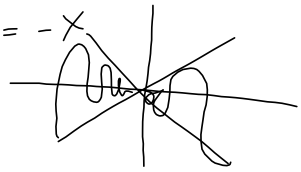
Determine whether $f'(0)$ exists.

$$f(x) = \begin{cases} x\sin\left(\frac{4}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is cnt's @ $x=0$, but not differentiable

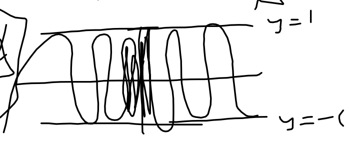
Topologist's sine curve $f(x) = \sin\left(\frac{1}{x}\right)$

$x\sin\left(\frac{4}{x}\right)$ oscillates between $y=x$ & $y=-x$



$x \neq 0$

$$\frac{f(0+h)-f(0)}{h} = \frac{f(h)-f(0)}{h} = \frac{h\sin\left(\frac{4}{h}\right)}{h} = \sin\left(\frac{4}{h}\right) \xrightarrow{h\to 0} \nexists$$

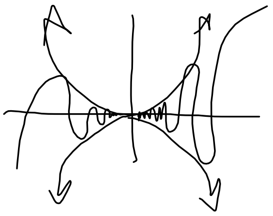


$\boxed{\text{No } f'(0)}$

21. Question Details SCalc8 2.1.060. [3354123]

Determine whether $f'(0)$ exists.

$$f(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$



Yes $f'(0) = 0$!

$$\frac{f(h)-f(0)}{h} = \frac{h^2\sin\left(\frac{1}{h}\right) - 0}{h} = h\sin\left(\frac{1}{h}\right) \xrightarrow{h\to 0} 0$$

$h \neq 0$

22. Question Details SCalc8 2.1.510.XP. [3391693

The limit represents the derivative of some function f at some number a. State such an f and a.

$$\lim_{x\to 2}\frac{4^x-16}{x-2} = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{4^{2+h}-4^2}{h}$$

is $f'(2)$ for $f(x)=4^x$

$$= \lim_{h\to 0}\frac{4^2(4^h-1)}{h} = 16\boxed{\left(\frac{4^h-1}{h}\right)}$$

$x=2+h$

Alternate

Need more tools to handle $h\to 0$.

23. Question Details SCalc8 2.1.AE.001. [339018

From the WebAssign

$$f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

$$= \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

EXAMPLE 1 Find an equation of the tangent line to the parabola $y = 5x^2$ at the point $P(1, 5)$ using this definition.

SOLUTION Here we have $a = 1$, and $f(x) = 5x^2$, so the slope is

$$m = \lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$$

$$\frac{f(x)-f(1)}{x-1} = \frac{5x^2-5(1)^2}{x-1} = \frac{5(x^2-1)}{x-1} = 5\frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

$$= 5(x+1) \xrightarrow{x\to 1} 5(2) = 10$$

Check $f'(x) = 2(5)x^{2-1} = 10x^1$

$f'(1) = 10$ ✓