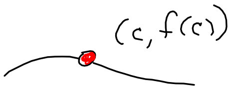
1. Question Details SCalc8 1.8.002. [3354480]

If f is continuous on $(-\infty, \infty)$, what can you say about its graph? (Select all that apply.)

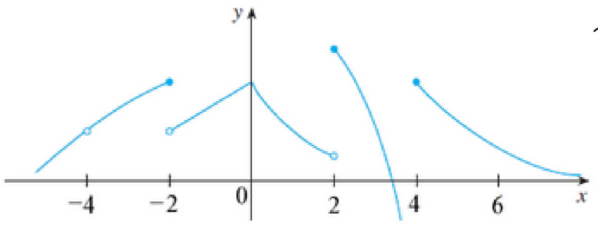
- [ ] The graph of f has a hole.
- [ ] The graph of f has a jump.
- [ ] The graph of f has a vertical asymptote.
- [x] none of these

In general, f is cnt^s at c means $\lim_{x \to c} f(x) = f(c)$



2. Question Details SCalc8 1.8.003.MI. [3354245]

From the graph of f, state each x-value at which f is discontinuous. For each x-value, determine whether f is continuous from the right, or from the left, or neither. (Enter your answers from smallest to largest.)



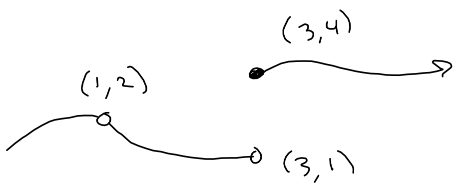
x	cnt^s
−4	NOT
−2	Left
2	RIGHT
4	RIGHT

3. Question Details SCalc8 1.8.007.

Sketch the graph of a function f that is continuous except for the stated discontinuity.

Removable discontinuity at 1, jump discontinuity at 3

HOLE



4. Question Details SCalc8 1.8.010. [

Which of the following are continuous functions? (Select all that apply.)

- [x] The temperature at a specific location as a function of time.
- [ ] The temperature at a specific time as a function of the distance due west from New York City.
- [ ] The altitude above sea level as a function of the distance due west from New York City.
- [ ] The cost of a taxi ride as a function of the distance traveled.
- [ ] The current in the circuit for the lights in a room as a function of time.
- [ ] None of these.

} Too weird

Penny increments jumps!

Switch-on is a jump in current.

5. Question Details SCalc8 1.8.011. [3354334]

Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$f(x) = \left(x + 3x^4\right)^5, \quad a = -1$

$$\lim_{x\to c}\left(\left(x+3x^4\right)^5\right) = \left(\lim_{x\to c}\left(x+3x^4\right)\right)^5$$

$$= \left(\lim_{x\to c} x + \lim_{x\to c}(3x^4)\right)^5 = \ldots \left(\lim_{x\to c} x + 3\left(\lim_{x\to c} x\right)^4\right)^5 = \left(c+3c^4\right)^5$$

$$= f(c)$$

6. Question Details SCalc8 1.8.015. [3354215]

Use the definition of continuity and the properties of limits to show that the function is continuous on the given interval.

$f(x) = x + \sqrt{x-1}, \quad [1, \infty)$

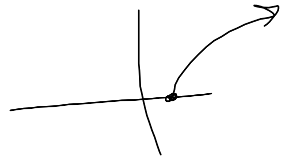
Sums & roots are continuous on their domains.

$\lim_{x\to c} f(x) = c + \sqrt{c-1}$

on $(1, \infty)$

& $\sqrt{x-1}$ is cont's from the right @ its left endpoint

$\sqrt{x-1}$



7. Question Details

Explain why the function is discontinuous at the given number a. (Select all that apply.)

$$f(x) = \frac{1}{x+4} \qquad a = -4$$

- [x] $f(-4)$ is undefined.
- [ ] $\lim_{x \to -4^+} f(x)$ and $\lim_{x \to -4^-} f(x)$ are finite, but are not equal.
- [x] $\lim_{x \to -4} f(x)$ does not exist.
- [ ] $f(-4)$ is defined and $\lim_{x \to -4} f(x)$ is finite, but they are not equal.
- [ ] none of the above

8. Question Details

Explain why the function is discontinuous at the given number a. (Select all that apply.)

$$f(x) = \begin{cases} \dfrac{x^2 - 4x}{x^2 - 16} & \text{if } x \neq 4 \\ 1 & \text{if } x = 4 \end{cases} \qquad a = 4$$

$$\frac{x^2-4x}{x^2-16} = \frac{x\cancel{(x-4)}}{(x+4)\cancel{(x-4)}} = \frac{x}{x+4} \; ; \text{if } x \neq 4$$

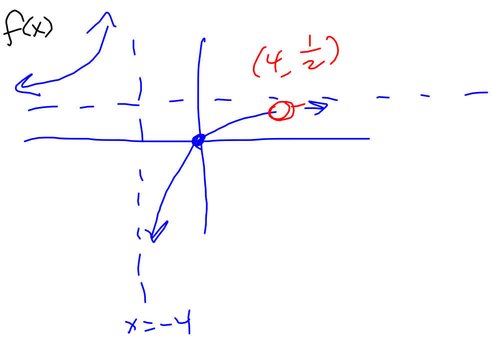
$f^*(x)$

Has a hole @ $x=4$

$$\lim_{x \to 4} f(x) = \frac{4}{4+4} = \frac{1}{2}$$

But $f(4) = 1$?!

So $\lim_{x \to 4} f(x) \neq f(4)$



9. Question Details SCalc8 1.8.025.

Explain, using the theorems, why the function is continuous at every number in its domain.

$$F(x) = \frac{2x^2 - x - 7}{x^2 + 4}$$

F is a quotient of polynomials, i.e., a Rational Function, and they're continuous wherever they're defined, i.e., continuous on their domains.

10. Question Details

Explain, using the theorems, why the function is continuous at every number in its domain.

$$Q(x) = \frac{\sqrt[3]{x-6}}{x^3-6}$$

Q is a composition of continuous functions, and therefore continuous on its domain.

$\sqrt[3]{x}$, $\frac{a}{b}$, x^3-6 are continuous , $x-6$,

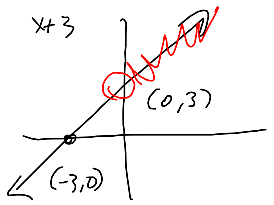
POINT x^3-6 downstairs is the only issue.

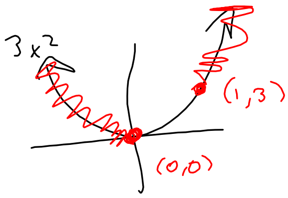
11. Question Details SCalc8 1.8.043. [3354367]

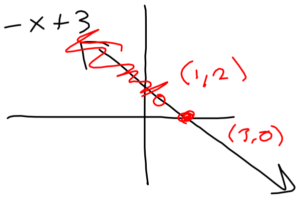
Find each x-value at which f is discontinuous and for each x-value, determine whether f is continuous from the right, or from the left, or neither.

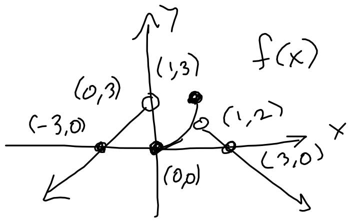
$$f(x) = \begin{cases} x+3 & \text{if } x < 0 \\ 3x^2 & \text{if } 0 \le x \le 1 \\ 3-x & \text{if } x > 1 \end{cases}$$

Sketch the graph.









12. Question Details SCalc8 1.8.045.MI.

For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 4x & \text{if } x < 3 \\ x^3 - cx & \text{if } x \ge 3 \end{cases}$$

worried about "suture point," where the two pieces meet ; $x = 3$.

continuous (polynomial)

$c(3)^2 - 4(3) = 3^3 - c \cdot 3$

$9c - 12 = 27 - 3c$

$12c = 39$

$\boxed{c = \frac{39}{12}}$

13. Question Details SCalc8 1.8.047.

Suppose f and g are continuous functions such that $g(8) = 2$ and $\lim_{x \to 8} [3f(x) + f(x)g(x)] = 30$. Find $f(8)$.

f cnts $\Longrightarrow \lim_{x \to 8} f(x) = f(8)$

$$\lim_{x \to 8} \left[3f(x) + f(x)g(x)\right] = 3f(8) + f(8)g(8) = 3f(8) + f(8)\cdot 2$$

$$= 5f(8) = 30$$

$$f(8) = \frac{30}{5} = \boxed{6 = f(8)}$$

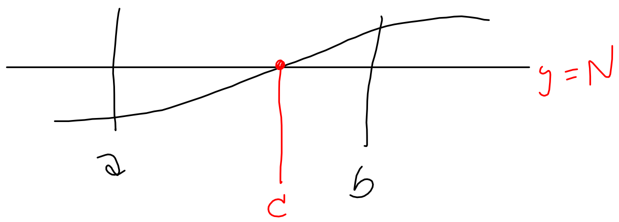
14. Question Details SCalc8 1.8.050. [3354505] -

Suppose that a function f is continuous on $[0, 1]$ except at 0.25 and that $f(0) = 2$ and $f(1) = 4$. Let $N = 3$. Sketch a possible graph of f showing that f might not satisfy the conclusion of the Intermediate Value Theorem.

We illustrate why continuity is such an important piece of the hypotheses for Intermediate Value Theorem. Build a non-example of IVT.

IVT: If the chicken crossed the road, then he crossed the stripe in the middle of the road.

IVT: f cnts on $[a, b]$, with $f(a) \neq f(b)$ & $N =$ some # between $f(a)$ & $f(b)$ $\Longrightarrow \exists\, c \in (a, b) \ni f(c) = N$.

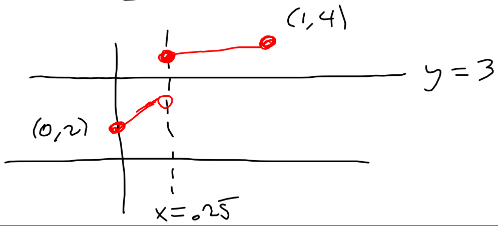


Nothing in IVT about HOW to find c.

It's just a claim that such a c exists!

$f(0) = 2$, $f(1) = 4$. Discontinuous @ $x = .25$.

Doesn't touch $y = 3$.



15. Question Details SCalc8 1.8.053. [33

Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$f(x) = x^4 + x - 6 = 0$, $(1, 2)$

$f(x) = 0$

$$\left.\begin{aligned} f(1) &= 1^4 + 1 - 6 = -4 < 0 \\ f(2) &= 2^4 + 2 - 6 = 12 > 0 \end{aligned}\right\} \Longrightarrow \exists\, c \in (1, 2) \ni f(c) = 0,$$

b/c $f(x)$ is cnts (Polynomial!) $\forall\, x \in \mathbb{R}$.

16. Question Details SCalc8 1.8.057.

Consider the following.

$\cos(x) = x^3$

(a) Prove that the equation has at least one real root. (0, 1).

(b) Use your calculator to find an interval of length 0.01 that contains a root. (Enter your answer using interval notation. Round your answers to two decimal places.)

(a) $f(x) = x^3 - \cos(x)$

Then $f(2) = 2^3 - \cos(2) = 8 - \cos(2) \geq 7 > 0$

$f(-2) = (-2)^3 - \cos(-2) = -8 - \cos(2) \leq -7 < 0$

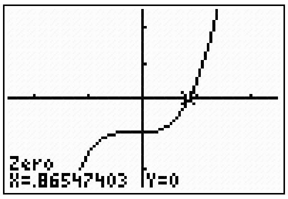
$\Rightarrow \exists\, c \in (-2,2) \ni f(c) = 0$,

i.e., $x^3 = \cos(x)$

(b) $.865 + .005 = .870$

$.865 - .005 = .860$

$I = (.860, .870)$



17. Question Details SCalc8 1.8.061.

Prove, without graphing, that the graph of the function has at least two x-intercepts in the specified interval.

$y = \sin(x^3)$, (1, 2)

x^3 cont's, $\sin(*)$ cont's

$f(1) \approx .8 > 0$ $c_1 \in (1, 1.5)$

$f(1.5) \approx -.2 < 0$ $c_2 \in (1.5, 1.9)$

$f(1.9) \approx .5 > 0$

$\sin(x^3)$ is composition of cont's fns; $\circ\circ$, cont's on its domain.

18. Question Details SCalc8 1.8.065.

A function f is continuous at a if and only if

$\lim_{h\to 0} f(a + h) = f(a)$.

Hmmm. Must be missing the question, here!

$$\frac{f(x) - f(a)}{x - a} = \frac{f(a+h) - f(a)}{h}$$

I'll just try to convince you that this is equivalent to the definition of continuity, previously given.

if $x = a+h$, it's the same thing

$\lim_{h\to 0} f(a+h)$ is THE SAME as

$\lim_{x\to a} f(x)$

Use this to show $\cos(x) = f(x)$ is cont's

$\cos(a+h) = \cos(a)\cos(h) - \sin(a)\sin(h) \xrightarrow{h\to 0} \cos(a)\cdot\cos(0)$

$- \sin(a)\sin(0) = \cos(a)$.

$$\lim_{h\to 0} \cos(a+h) = \lim_{h\to 0}\left(\cos(a)\cos(h) - \sin(a)\sin(h)\right) \ldots$$

$$= \cos(a)$$

Sep 6-2:48 PM

19. Question Details SCalc8 1.8.067.

For what value(s) of x is f continuous?

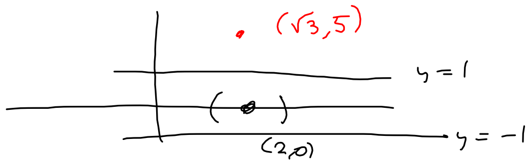
$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 5 & \text{if } x \text{ is irrational} \end{cases}$$

Mainly of theoretical interest, motivating why we define continuity the way we do.

No place where f is cont^s.

& $x = 2$ works.

Any neighborhood (interval) containing $x=2$ will contain irrational #s, and so $\varepsilon = 1$ is enough to defeat you

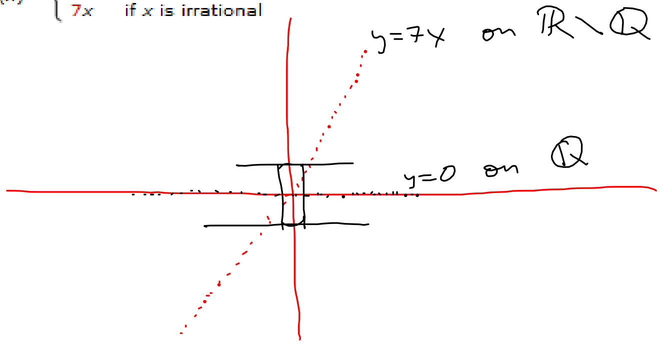


20. Question Details SCalc8 1.8.068.

For what value(s) of x is g continuous?

$$g(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 7x & \text{if } x \text{ is irrational} \end{cases}$$

A function that's continuous at exactly one point!?:



$\delta = \frac{\varepsilon}{7}$ will work for any $\varepsilon > 0$

x is rational $\Rightarrow$

$|g(x) - 0| = 0 < \varepsilon$

x irrational $\Rightarrow$

$|g(x) - 0| < \varepsilon$ whenever

$0 < |x - 0| < \frac{\varepsilon}{7} = \delta$