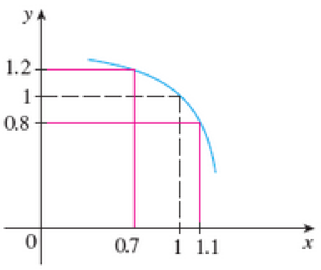
#s 23 - 28 are "just in time" algebra review. You may want to jump to those.

1. Question Details

Use the given graph of f to find a number δ such that

if $|x - 1| < \delta$ then $|f(x) - 1| < 0.2$

δ = 0.1

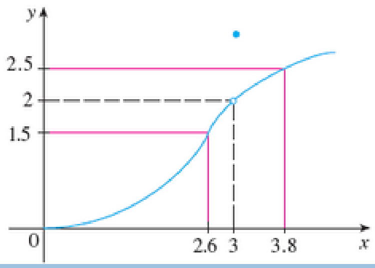


2. Question Details

Use the given graph of f to find a number δ such that

if $0 < |x - 3| < \delta$ then $|f(x) - 2| < 0.5.$

δ = 0.4

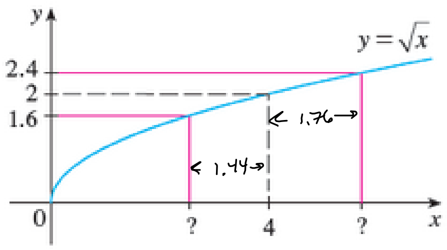


3. Question Details

Use the given graph of $f(x) = \sqrt{x}$ to find a number δ such that

if $|x - 4| < \delta$ then $\left|\sqrt{x} - 2\right| < 0.4.$

δ = 1.44



2.56 5.76

$\sqrt{x} = 2.4$

$x = 2.4^2$

$= 5.76$

$\sqrt{x} = 1.6$

$x = 1.6^2 = 2.56$

$$\begin{array}{r} 24 \\ \times 24 \\ \hline 96 \\ 480 \\ \hline 576 \end{array}$$

4. Question Details

Use the given graph of $f(x) = x^2$ to find a number δ such that

if $|x - 1| < \delta$ then $|x^2 - 1| < \frac{1}{2}$.

(Round your answer down to three decimal places.)

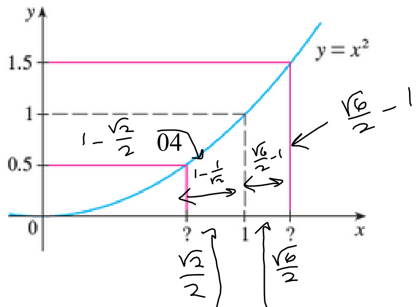
$\delta =$ .225

$x^2 = .5 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$x^2 = 1.5 = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$



0.2928932190 0.224744872 $\approx .225$

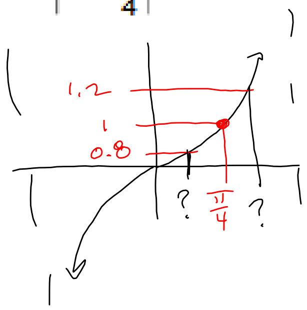
5. Question Details

A graphing calculator is recommended.

Use a graph to find a number δ such that

if $\left|x - \frac{\pi}{4}\right| < \delta$ then $|\tan(x) - 1| < 0.2$.

$\boxed{\delta \approx .0906598}$



The ?'s will give us x's

The x's will gives δ, via $|x - \frac{\pi}{4}|$. Pick smallest.

$\tan x = 0.8$ $\tan x = 1.2$

0.674741 0.876058

$|x - \frac{\pi}{4}| \approx |-0.110657|$ $|x - \frac{\pi}{4}| \approx 0.0906598$

6. Question Details SCalc8 1.7.007. [3394361]

A graphing calculator is recommended.

For the limit

$$\lim_{x \to 3} (x^3 - 3x + 4) = 22$$

what we've BEEN doing finds the biggest possible δ.

illustrate the definition by finding the largest possible values of δ that correspond to $\varepsilon = 0.2$ and $\varepsilon = 0.1$. (Round your answers to four decimal places.)

$\varepsilon = 0.2$ $\delta =$

$\varepsilon = 0.1$ $\delta =$

$\varepsilon = 0.2$, so $21.8 < y < 22.2$ is what we want

$x^3 - 3x + 4 \overset{\text{SET}}{=} 21.8$ $x^3 - 3x + 4 \overset{\text{SET}}{=} 22.2$

$\Rightarrow x \approx$ 2.991640485 3.008307429

$3 - 3.008307429 \approx -0.008307429$

So, $\delta = .0083$

$3 - 2.991640485 \approx 0.008359515$

7. Question Details SCalc8 1.7.011. [3354

A machinist is required to manufacture a circular metal disk with area 1100 cm^2.

(a) What radius produces such a disk? (Round your answer to four decimal places.)

[ ] cm $\pi r^2 = 1100 \Rightarrow r^2 = \frac{1100}{\pi} \Rightarrow r \approx \boxed{18.7121 \text{ cm}}$

(b) If the machinist is allowed an error tolerance of ±7 cm^2 in the area of the disk, how close to the ideal radius in part (a) must the machinist control the radius? (Round your answers to four decimal places.)

[ ] cm < r < [ ] cm

(c) In terms of the ε, δ definition of $\lim_{x \to a} f(x) = L$, what is x? RADIUS

$evalf\left(\frac{1100}{\text{Pi}}\right) \approx 350.1408747$

$\text{sqrt}(\%) \approx 18.71205159$

What is $f(x)$? Area

What is a? 18.7121 cm

What is L? 1100 cm^2

What value of ε is given? 7 cm

$\lim_{x \to 18.7121} \pi x^2 = 1100$

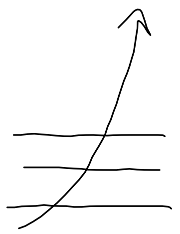
What is the corresponding value of δ? (Round your answer to four decimal places.) $\boxed{\delta = .0594}$

(b) ±.7 cm^2 is tolerance, so want $f(x)$ between 1093 & 1107 cm2

$\varepsilon = 7$. want δ.

$\pi r^2 = 1107$ & $\pi r^2 = 1093$

Then find how far each is from $r = 18.7121$



0.05939552

0.05968178

$\boxed{\delta = .0594}$

8. Question Details SCalc8 1.7.012. [3354405]

A graphing calculator is recommended.

A crystal growth furnace is used in research to determine how best to manufacture crystals used in electric components for the space shuttle. For proper growth of the crystal, the temperature must be controlled accurately by adjusting the input power. Suppose the relationship is given by

$$T(w) = 0.1w^2 + 2.151w + 20$$

where T is the temperature in degrees Celsius and w is the power input in watts.

(a) How much power is needed to maintain the temperature at 203°C? (Round your answer to two decimal places.)

33.35 watts

$\varepsilon = 1$

(b) If the temperature is allowed to vary from 203°C by up to ±1°C, what range of wattage is allowed for the input power? (Round your answers to two decimal places.)

33.24 watts $< w <$ 33.47 watts

Solve $T(w) = 202°, 204°$

(c) In terms of the ε, δ definition of $\lim_{x \to a} f(x) = L$, what is x?

What is $f(x)$? T

What is a? 33.35 watts

What is L? 203°

What value of ε is given? $\varepsilon = 1$

What is the corresponding value of δ? (Round your answer to four decimal places.) Nutty

$$\begin{array}{r} 33.35 \\ 33.25 \\ \hline .10 \end{array} \qquad \begin{array}{r} 33.47 \\ -\,33.35 \\ \hline .12 \end{array}$$

$\boxed{\delta = .10}$ Previous figures to 2 places. The heck with re-doing all that.

Aug 30-2:58 PM

9. Question Details

Find the largest number δ such that if $|x - 2| < \delta$, then $|4x - 8| < \varepsilon$, where $\varepsilon = 0.5$.

$\delta \le$ .125

Repeat and determine δ with $\varepsilon = 0.05$.

$\delta \le$ .0125

$\varepsilon = .5 \Rightarrow$

$\delta = \frac{\varepsilon}{4} = \frac{\frac{1}{2}}{4} = \frac{1}{8}$

Want $|4x - 8| < \varepsilon$

$4|x-2| < \varepsilon$

$|x-2| < \frac{\varepsilon}{4} = \delta$

$8\overline{)1.0000}$ = .125

$\delta = \frac{\varepsilon}{4} = \frac{.05}{4}$

$= \frac{5}{100}\left(\frac{1}{4}\right) = \frac{5}{400} = \frac{1}{80}$

10. Question Details SCalc8 1.7.014. [3354510]

Given that $\lim_{x \to 3} (4x - 10) = 2$, illustrate Definition 2 by finding values of δ that correspond to $\varepsilon = 0.5$, $\varepsilon = 0.1$, and $\varepsilon = 0.05$.

$\varepsilon = 0.5$ $\delta \le$ .125

$\varepsilon = 0.1$ $\delta \le$ .025

$\varepsilon = 0.05$ $\delta \le$ .0125

$m = 4$, again. $\delta = \frac{\varepsilon}{4}$, again.

$\frac{.1}{4} = \frac{\frac{1}{10}}{4} = \frac{1}{40}$

$40\overline{)1.0000}$

11. Question Details SCalc8 1.7.015. [3354236]

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 5} \left(3 + \frac{1}{5}x\right) = 4$$

Given $\varepsilon > 0$, we need δ [---Select---] such that if $0 < |x - 5| < \delta$, then $\left|\left(3 + \frac{1}{5}x\right) - 4\right|$ [---Select---]. But $\left|\left(3 + \frac{1}{5}x\right) - 4\right| < \varepsilon \Leftrightarrow \left|\frac{1}{5}x - 1\right| < \varepsilon \Leftrightarrow \left|\frac{1}{5}\right| |x - 5| < \varepsilon \Leftrightarrow |x - 5| <$ [---Select---]. So if we choose $\delta =$ [---Select---] then $0 < |x - 5| < \delta \Rightarrow \left|\left(3 + \frac{1}{5}x\right) - 4\right| < \varepsilon$. Thus, $\lim_{x \to 5} \left(3 + \frac{1}{5}x\right) = 4$ by the definition of a limit.

Illustrate with a diagram.

Scratch: work backwards from what you want!

Want $\left|3 + \frac{1}{5}x - 4\right| < \varepsilon$ Try to get an $|x-5|$ out!

$\left|\frac{1}{5}x - 1\right| < \varepsilon$

$\frac{1}{5}|x - 5| < \varepsilon$

$|x-5| < \boxed{5\varepsilon = \delta}$

OOps! Didn't do the diagram!

Write the Proof:

Let $\varepsilon > 0$ be given. Define $\delta = 5\varepsilon$. Then, whenever $0 < |x-5| < \delta$, we have

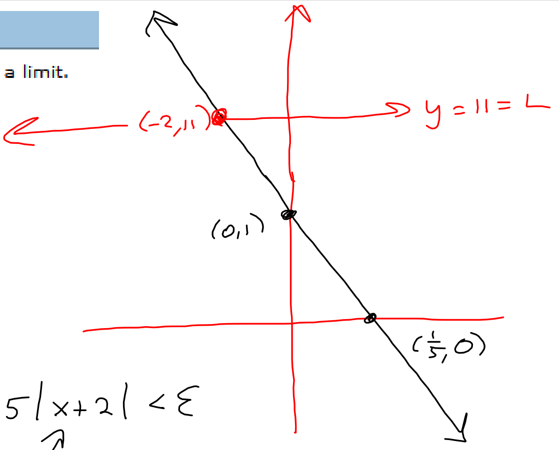
$$\left|\frac{1}{5}x + 3 - 4\right| = \left|\frac{1}{5}x - 1\right| = \frac{1}{5}|x - 5| < \frac{1}{5}\delta = \frac{1}{5} \cdot 5\varepsilon = \varepsilon$$

12. Question Details

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to -2} (1 - 5x) = 11$$

Illustrate with a diagram. → ? Not sure.



Scratch: $\delta = \frac{1}{5}\varepsilon$

$|1 - 5x - 11| < \varepsilon$

$|-5x - 10| = 5|-x-2| = 5\underbrace{|x+2|}_{|x-(-2)|} < \varepsilon$

$\Rightarrow |x+2| < \frac{\varepsilon}{5} \equiv \delta$

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{5}$. Then $0 < |x-(-2)| < \delta$

$\Rightarrow |1 - 5x - 11| = |-5x - 10| = 5|-x-2| = 5|x+2|$

$= 5|x-(-2)| < 5\delta = 5 \cdot \frac{\varepsilon}{5} = \varepsilon$. ■

13. Question Details

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 1} \frac{11 + 4x}{5} = 3$$

$\frac{11}{5} + \frac{4}{5}x$ Growth rate $m = \frac{4}{5}$, so

$\delta \equiv \frac{\varepsilon}{\frac{4}{5}} = \frac{5\varepsilon}{4}$.

Proof:

Let $\varepsilon > 0$ be given. Define $\delta = \frac{5\varepsilon}{4}$.

Then $0 < |x-1| < \delta \Rightarrow \left|\frac{11+4x}{5} - 3\right| = \left|\frac{11+4x-15}{5}\right| = \left|\frac{4x-4}{5}\right| = \frac{4}{5}|x-1|$

$< \frac{4}{5}\delta = \frac{4}{5} \cdot \frac{5\varepsilon}{4} = \varepsilon$ ■

14. Question Details

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x\to 6} \frac{x^2 - 4x - 12}{x - 6} = 8$$

The $x-6$ cancels, so linear method will work

$$\frac{x^2-4x-12}{x-6} = \frac{(x-6)(x+2)}{x-6} = x+2$$

We're really showing $\lim_{x\to 6}(x+2) = 8$. $m = 1$. $\delta = \frac{\varepsilon}{1} = \varepsilon$

Proof Let $\varepsilon > 0$. Define $\delta = \varepsilon$. Then $0 < |x-6| < \delta \Longrightarrow$

$$\left|\frac{x^2-4x-12}{x-6} - 8\right| = |x+2-8| = |x-6| < \delta = \varepsilon \;\blacksquare$$

See Scratch!

15. Question Details

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x\to a} x = a$$

Too Easy.

Proof Let $\varepsilon > 0$. Define $\delta = \varepsilon$. Then $0 < |x-a| < \delta \Longrightarrow$

$$|x-a| < \delta = \varepsilon \;\blacksquare$$

16. Question Details

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 0} x^2 = 0$$

$\delta = \varepsilon$

$|x-0| < \delta \Rightarrow |x^2 - 0| = |x||x| < \varepsilon^2 < \varepsilon$ if $\varepsilon < 1$

Let $\varepsilon > 0$. Define $\delta = \min\{1, \varepsilon\}$. Then $0 < |x-0| < \delta \Rightarrow$

$|x^2 - 0| = |x||x| \le 1|x| < \delta \le \varepsilon$ ∎

7. Question Details

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to 5} (x^2 - 10x + 29) = 4$$

You'll always have an $|x-a| = |x-5|$ to factor out, if it's legit polynomial.

Scratch:

$|x^2 - 10x + 29 - 4| = |x^2 - 10x + 25| = |x-5|^2 < \varepsilon$

$|x-5||x-5| < \varepsilon$ standard move for these higher powers is "assume $\delta \le 1$."

$< \delta$

Now, $\delta \le 1$ & $\delta \to 5$, says x's in here!

So $x-5$ is between

So $|x-5| \le 1$

So $|x-5||x-5| \le |x-5| < \varepsilon \equiv \delta$

Bounded by 1.

Proof

Let $\varepsilon > 0$. Define $\delta = \min\{1, \varepsilon\}$. Then $0 < |x-5| < \delta \Rightarrow$

$|x^2 - 10x + 29 - 4| = |x^2 - 10x + 25| = |x-5|^2 \le |x-5| < \delta \le \varepsilon$ ∎

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18. Question Details

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \to -4} (x^2 - 5) = 11$$

Scratch:

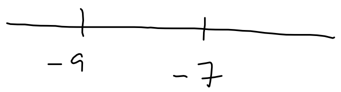
$$|x^2-5-11| = |x^2-16| = |x+4||x-4| < \varepsilon$$

want; $<\delta$

Assume $\delta \le 1 \Rightarrow -5 < x < -3$

$-9 < x-4 < -7$

$|x-4|$ So $|x-4| < 9$



So $|x-4||x+4|$
$\le 9|x+4|$
$< 9\delta$

Proof: Let $\varepsilon > 0$. Define $\delta = \min\{1, \frac{\varepsilon}{9}\}$. Then $0 < |x-(-4)| < \delta \Rightarrow$

$|x^2-5-11| = |x^2-16|$
$= |x+4||x-4|$
$< 9\delta \le 9 \cdot \frac{\varepsilon}{9}$
$= \varepsilon$ ▨

19. Question Details SCalc8 1.7.035. [335

(a) For the limit $\lim_{x \to 1} (x^3 + x + 4) = 6$, use a graph to find the largest possible value of δ that corresponds to $\varepsilon = 0.4$. (Round your answer down to three decimal places.)

(b) By using a computer algebra system to solve the cubic equation $x^3 + x + 4 = 6 + \varepsilon$, find the largest possible value of δ that works for any given $\varepsilon > 0$.

(c) Put $\varepsilon = 0.4$ in your answer to part (b). (Round your answer down to three decimal places.)

How does $\delta(0.4)$ compare with your answer to part (a)?

The heck with this! Just prove the limit is correct.

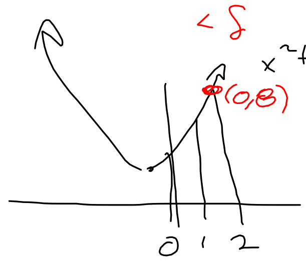
$\lim_{x \to 1} (x^3+x+4) = 6$ ↳ checks.

Scratch: $|x^3+x+4-6| = |x^3+x-2|$ So will have $|x-1|$ in it!

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

This says $x^3+x-2 = (x-1)(x^2+x+2)$, $(x-1) < \delta$

$x^2+x+2 = x^2+x+\left(\frac{1}{2}\right)^2 - \frac{1}{4} + 2$
$\left(x+\frac{1}{2}\right)^2 + \frac{7}{4}$



$x^2+x+2 = g(x)$
$g(0) = 2$
$g(2) = 8$

So $|x^2+x+2| < 8$ if $\delta \le 1$

$x \to 1$

$0 < x < 2$

Proof: Let $\varepsilon > 0$. Define $\delta = \min\{1, \frac{\varepsilon}{8}\}$. Then $0 < |x-1| < \delta \Rightarrow$

$|x^3+x+4-6| = |x^3+x-2| = |x-1||x^2+x+2| < \delta \cdot 8 \le \frac{\varepsilon}{8} \cdot 8$

$= \varepsilon$ ▨

Pretty advanced problem, here.

20. Question Details SCalc8 1.7.037. [3354142]

Prove that $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ if $a > 0$. $\left[\textit{Hint:} \text{ Use } |\sqrt{x} - \sqrt{a}| = \dfrac{|x - a|}{\sqrt{x} + \sqrt{a}}. \right]$

Given $\varepsilon > 0$, we must find δ [---Select---] such that $|\sqrt{x} - \sqrt{a}|$ [---Select---] whenever $0 < |x - a| <$ [?]. But $|\sqrt{x} - \sqrt{a}| = \dfrac{|x - a|}{\sqrt{x} + \sqrt{a}} < \varepsilon$ (from the hint). Now if we can find a positive constant C such that $\sqrt{x} + \sqrt{a}$ [?] C then $\dfrac{|x - a|}{\sqrt{x} + \sqrt{a}} < \dfrac{|x - a|}{C}$, and we can make $\dfrac{|x - a|}{C} < \varepsilon$ by taking $|x - a| < C\varepsilon$. We can find this number by restricting x to lie in some interval centered at a. If $|x - a| < \frac{1}{2}a$, then

$$-\frac{1}{2}a < x - a < \frac{1}{2}a \Rightarrow \frac{1}{2}a < x < \frac{3}{2}a \Rightarrow \sqrt{x} + \sqrt{a} \; [?] \; \sqrt{\frac{1}{2}a} + \sqrt{a},$$

and so $C = \sqrt{\frac{1}{2}a} + \sqrt{a}$ is a suitable choice for the constant. So $|x - a| < \left(\sqrt{\frac{1}{2}a} + \sqrt{a}\right)\varepsilon$. This suggests that we let $\delta = \min\left\{\frac{1}{2}a, \left(\sqrt{\frac{1}{2}a} + \sqrt{a}\right)\varepsilon\right\}$. Thus, if $0 < |x - a| <$ [?], then

$$|x - a| < \frac{1}{2}a \Rightarrow \sqrt{x} + \sqrt{a} \; [?] \; \sqrt{\frac{1}{2}a} + \sqrt{a}.$$

Also $|x - a| < \left(\sqrt{\frac{1}{2}a} + \sqrt{a}\right)\varepsilon$, so

$$|\sqrt{x} - \sqrt{a}| = \frac{|x - a|}{\sqrt{x} + \sqrt{a}} \; [?] \; \frac{(\sqrt{a/2} + \sqrt{a})\varepsilon}{(\sqrt{a/2} + \sqrt{a})} = \varepsilon.$$

Therefore, $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ by the definition of a limit.

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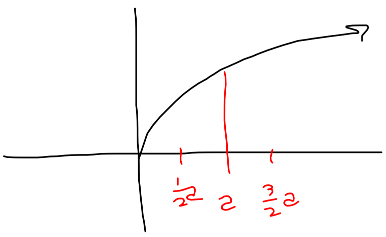
(20) $\lim_{x \to a} \sqrt{x} = \sqrt{a}$



Scratch want an $|x-a|$ to pop out.

$$|\sqrt{x} - \sqrt{a}| = \left| \left(\frac{\sqrt{x}-\sqrt{a}}{1} \right) \left(\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} \right) \right|$$

$$= \left| \frac{x-a}{\sqrt{x}+\sqrt{a}} \right| = \frac{|x-a|}{\sqrt{x}+\sqrt{a}} \underset{<}{\text{want}} \varepsilon$$

Can mak $\frac{A}{B}$ Bigger with smaller B.

Assume $\frac{1}{2}a < x < \frac{3}{2}a$, i.e., $\delta \le \frac{1}{2}a$

Then $\sqrt{\frac{1}{2}a} < \sqrt{x} < \sqrt{\frac{3}{2}a}$

$$\sqrt{\tfrac{1}{2}a} + \sqrt{a} < \sqrt{x} + \sqrt{a} < \sqrt{\tfrac{3}{2}a} + \sqrt{a}$$

Proof: Let $\varepsilon > 0$. Define $\delta = \min\{\frac{1}{2}a, \varepsilon(\sqrt{\frac{1}{2}a} + \sqrt{a})\}$

Then $0 < |x-a| < \delta \implies |\sqrt{x} - \sqrt{a}| = \frac{|x-a|}{\sqrt{x}+\sqrt{a}} < \frac{\delta}{\sqrt{x}+\sqrt{a}} < \frac{\delta}{\sqrt{\frac{1}{2}a}+\sqrt{a}}$

$$\le \frac{\varepsilon(\sqrt{\frac{1}{2}a}+\sqrt{a})}{\sqrt{\frac{1}{2}a}+\sqrt{a}} = \varepsilon \quad \blacksquare$$

Sep 6-12:48 PM

21. Question Details

How close to -1 do we have to take x so that

$$\frac{1}{(x+1)^6} > 1{,}000{,}000$$

is satisfied? (Give the largest possible value.)

$$\frac{1}{(x+1)^6} > 10^6$$

$$1 > 10^6 (x+1)^6$$

$$\frac{1}{10^6} > (x+1)^6 = |x+1|^6 = |x-(-1)|^6$$

$$\sqrt[6]{\frac{1}{10^6}} > \sqrt[6]{|x+1|^6}$$

$$\boxed{\delta = \frac{1}{10}} > |x+1|$$

General: $\frac{1}{(x+1)^6} \xrightarrow{x \to -1} \infty$

Let $M > 0$ be given.

Scratch

$$\frac{1}{(x+1)^6} > M \implies 1 > M(x+1)^6 \implies \frac{1}{M} > (x+1)^6 \implies \sqrt[6]{\frac{1}{M}} > |x+1|$$

Proof Let $\varepsilon > 0$. Define $\delta = \sqrt[6]{\frac{1}{M}}$. Then

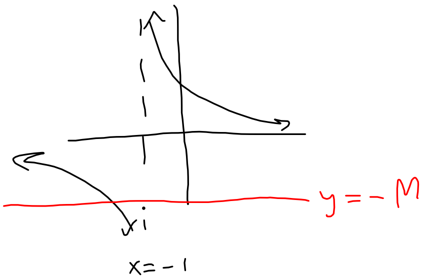
$0 < |x-(-1)| < \delta \implies \left|\frac{1}{(x+1)^6}\right| > M$ ∎

22. Question Details

Prove that $\lim_{x \to -1^-} \frac{2}{(x+1)^3} = -\infty$.

Want $\frac{2}{(x+1)^3} < -M$

$$-M > \frac{2}{(x+1)^3}$$



$$-M(x+1)^3 < 2$$

$$(x+1)^3 > -\frac{2}{M}$$

$$x+1 > \sqrt[3]{\frac{-2}{M}}$$

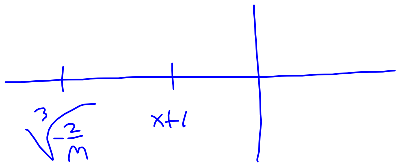
Both negative

$$\left|\frac{2}{(x+1)^3}\right| > M$$

$\frac{2}{(x+1)^3} > M$ OR $\frac{2}{(x+1)^3} < -M$

$$2 > -M(x+1)^3$$

$$-\frac{2}{M} < (x+1)^3$$



So, $|x+1| < \boxed{\sqrt[3]{\frac{2}{M}} = \delta}$

Proof Let $M > 0$ be given. We show $\frac{2}{(x+1)^3} < -M$, for x sufficiently close to $x = -1$. Define $\delta = \sqrt[3]{\frac{2}{M}}$. Then if $0 < |x-(-1)| < \delta$, we have $\frac{2}{(x+1)^3} < \frac{2}{-\delta^3} = \frac{2}{-\left(\sqrt[3]{\frac{2}{M}}\right)^3}$

$$= -\frac{2}{\frac{2}{M}} = -M$$ ∎

$$-\frac{2}{3} < -\frac{2}{5}$$

Aug 30-3:10 PM