

SCalc8 1.6.001. (3554511) (Remove) -- view 4m

Given that $\lim_{x \rightarrow 2} f(x) = 9$, $\lim_{x \rightarrow 2} g(x) = -2$, $\lim_{x \rightarrow 2} h(x) = 0$, find the limits, if they exist. (If an answer does not exist, enter DNE.)

(a) $\lim_{x \rightarrow 2} [f(x) + 4g(x)] = 9 + 4(-2) = 1$

(b) $\lim_{x \rightarrow 2} [g(x)] = -2$

(c) $\lim_{x \rightarrow 2} \sqrt{f(x)^3} = (9)^{\frac{3}{2}} = 3^3 = 27$

(d) $\lim_{x \rightarrow 2} \frac{2f(x)}{g(x)} = \frac{2(9)}{-2} = -9$

(e) $\lim_{x \rightarrow 2} \frac{g(x)}{h(x)}$ ~~exists~~ b/c $\lim h = 0$

Informally: Assume $\lim f$ & $\lim g$ are real #s.

$\lim(f \pm g) = \lim f \pm \lim g$

$\lim(fg) = (\lim f)(\lim g)$

$\lim\left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}$ (if $\lim g \neq 0$)

$\lim(f^n) = (\lim f)^n$ $n \in \mathbb{Q}$

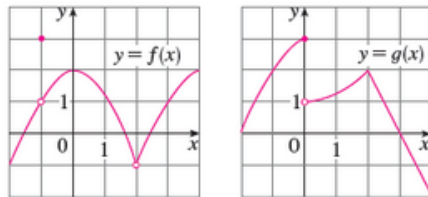
↓
Fraction

$\lim(f^{\frac{p}{q}}) = (\lim f)^{\frac{p}{q}}$

2. Question Details

SCalc8 1.6.002

The graphs of f and g are given. Use them to evaluate each limit, if it exists. (If an answer does not exist, enter DNE.)



- (a) $\lim_{x \rightarrow 2} [f(x) + g(x)] = -1 + 2 = 1 = \lim_{x \rightarrow 2} (f+g)$
- (b) $\lim_{x \rightarrow 0} [f(x) - g(x)]$ ~~exists~~
- (c) $\lim_{x \rightarrow -1} [f(x)g(x)] = (1)(2) = 2 = \lim_{x \rightarrow -1} (fg)$
- (d) $\lim_{x \rightarrow 3} \frac{f(x)}{g(x)}$ ~~exists~~
- (e) $\lim_{x \rightarrow 2} [x^2 f(x)] = 2^2(-1) = -4 \neq 0 = \lim_{x \rightarrow 2} (x^2) \lim_{x \rightarrow 2} f(x)$
- (f) $f(-1) + \lim_{x \rightarrow -1} g(x) = 3 + 2 = 5$

FACT

$\lim_{x \rightarrow c} x = c$

$\lim_{x \rightarrow c} x^n = c^n, \forall n \in \mathbb{Q}$, with caveat for negative powers & $c=0$.

3. Question Details

SCalc8 1.6.003

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\begin{aligned} \lim_{x \rightarrow 4} (5x^3 - 4x^2 + x - 2) \\ &= \lim 5x^3 - \lim 4x^2 + \lim x - \lim 2 \\ &= 5 \lim x^3 - 4 \lim x^2 + \lim x - \lim 2 \\ &= 5(4)^3 - 4(4)^2 + 4 - 2 \\ &= 320 - 64 + 2 = 258? \end{aligned}$$

*c is constant, then $\lim (cf) = c \lim f$
 $\lim c = c$*

$$\begin{array}{r} 320 \\ -64 \\ \hline 256 \end{array}$$

4. Question Details

SCalc8 1.6.005

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\begin{aligned} \lim_{t \rightarrow -2} \frac{t^4 - 4}{2t^2 - 3t + 4} &= \frac{(-2)^4 - 4}{2(-2)^2 - 3(-2) + 4} \\ &= \frac{16 - 4}{8 + 6 + 4} = \frac{12}{18} = \boxed{\frac{2}{3}} \end{aligned}$$

If you CAN plug in the #, directly, DO IT! Concern, here, was denominator = 0, but that didn't happen, so plug right in!

5. Question Details

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{u \rightarrow -1} \sqrt{u^4 + 3u + 3} = \sqrt{(-1)^4 + 3(-1) + 3} = \sqrt{1 - 3 + 3} = \sqrt{1} = \boxed{1}$$

6. Question Details

SCalc8 1.6.007.

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 8} \left[3 + \sqrt[3]{x} \right] (5 - 5x^2 + x^3) \quad \text{needs brackets.}$$

$$= [(3+2)(5 - (5)(8)^2 + 8^3)]$$

$$= (5)(5 - 320 + 512) = 5(197) = \boxed{985}$$

$$\begin{array}{r} 3 \ 6 \ 4 \\ \underline{5 \ 1 \ 2} \\ - \ 3 \ 2 \ 0 \\ \hline 1 \ 9 \ 2 \\ + \ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \ 1 \ 9 \ 7 \\ \underline{ \ 5} \\ 9 \ 8 \ 5 \end{array}$$

7. Question Details

SCalc8 1.6.008.

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{t \rightarrow 2} \left(\frac{t^2 - 7}{t^3 - 4t + 6} \right)^2 \quad \because \left(\frac{2^2 - 7}{2^3 - 4(2) + 6} \right)^2 = \left(\frac{-3}{6} \right)^2 = \left(-\frac{1}{2} \right)^2 = \boxed{\frac{1}{4}}$$

Plug it in if it's
in the domain!

8. Question Details

SCalc8 1.6.009.

Evaluate the limit using the appropriate Limit Law(s). (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 2} \sqrt{\frac{5x^2 + 5}{5x - 1}} = \sqrt{\frac{5(2)^2 + 5}{5(2) - 1}} = \sqrt{\frac{25}{9}} = \boxed{\frac{5}{3}}$$

9. Question Details

SCalc8 1.6.010

(a) What is wrong with the following equation?

$$f(x) = \frac{x^2 + x - 20}{x - 4} = x + 5$$

$f(x) \neq x + 5$ at $x = 4$. $x + 5$ does.

(b) In view of part (a), explain why the equation

$$\lim_{x \rightarrow 4} \frac{x^2 + x - 20}{x - 4} = \lim_{x \rightarrow 4} (x + 5)$$

is correct.

They agree everywhere, except $x = 4$,

$$\frac{x^2 + x - 20}{x - 4} = \frac{(x + 5)(x - 4)}{x - 4} = x + 5, \text{ if } x \neq 4$$

$$\lim_{x \rightarrow 4} \frac{x^2 + x - 20}{x - 4} = 4 + 5 = 9 = \lim_{x \rightarrow 4} f(x)$$

10. Question Details

SCalc8 1.6.013

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 3} \frac{x^2 - 7x + 6}{x - 3} = \lim_{x \rightarrow 3} \frac{(x - 6)(x - 1)}{x - 3} \neq$$

11. Question Details

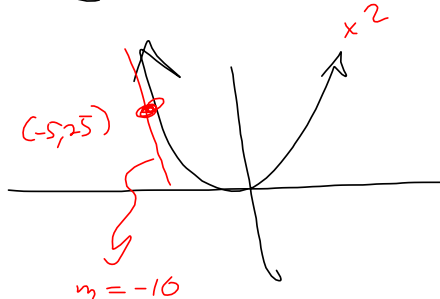
SCalc8 1.6.017

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \rightarrow 0} \frac{(-5 + h)^2 - 25}{h} = \lim_{h \rightarrow 0} \frac{(-5)^2 + (2)(-5)(h) + h^2 - 25}{h}$$

$$= \lim_{h \rightarrow 0} \frac{25 - 10h + h^2 - 25}{h} = \lim_{h \rightarrow 0} \frac{-10h + h^2}{h} = \lim_{h \rightarrow 0} \frac{h(-10 + h)}{h} = \lim_{h \rightarrow 0} (-10 + h) = -10$$

Slope of x^2 at $x = -5$ is $m_{tan} = -10!$



12. Question Details

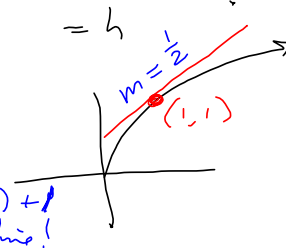
S Calc8 1.6.020.

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{h}$$
 STYLE: Leave $\lim_{h \rightarrow 0}$ for last step; $(a-b)(a+b) = a^2 - b^2$

$$\frac{\sqrt{1+h} - 1}{h} = \frac{(\sqrt{1+h} - 1)(\sqrt{1+h} + 1)}{h(\sqrt{1+h} + 1)} = \frac{1+h-1}{h(\sqrt{1+h} + 1)} = \frac{h}{h(\sqrt{1+h} + 1)} = \frac{1}{\sqrt{1+h} + 1}$$

$$\xrightarrow{h \rightarrow 0} \frac{1}{\sqrt{1+0} + 1} = \frac{1}{1+1} = \frac{1}{2}$$



$y = \frac{1}{2}(x-1) + 1$
tangent line!

13. Question Details

S Calc8 1.6.024.

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

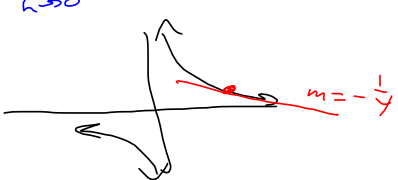
$$\lim_{h \rightarrow 0} \frac{(2+h)^{-1} - 2^{-1}}{h}$$

$$\frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \frac{\frac{2}{2(2+h)} - \frac{2+h}{2(2+h)}}{h} = \frac{1}{h} \cdot \frac{2 - (2+h)}{2(2+h)} = \frac{1}{h} \cdot \frac{-h}{2(2+h)}$$

$$\xrightarrow{h \rightarrow 0} \frac{-1}{2(2+0)} = \frac{-1}{4}$$

The slope of $f(x) = \frac{1}{x}$ @ $x=2$!

$$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \# 13!$$



$m = -\frac{1}{4}$

14. Question Details

S Calc8 1.6.026.

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{t \rightarrow 0} \left(\frac{8}{t} - \frac{8}{t^2 + t} \right)$$

$$\left(\frac{8}{t} \right) \left(\frac{t+1}{t+1} \right) - \frac{8}{LCD}$$

$$= \frac{8t + 8 - 8}{LCD} = \frac{8t}{t(t+1)} = \frac{8}{t+1} \xrightarrow{t \rightarrow 0} \frac{8}{1} = 8$$

$t^2 + t = t(t+1) = LCD$

15. Question Details SCalc8 1.6.027

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 25} \frac{5 - \sqrt{x}}{25x - x^2}$$

$$\left(\frac{5 - \sqrt{x}}{25x - x^2} \right) \left(\frac{5 + \sqrt{x}}{5 + \sqrt{x}} \right) = \frac{25 - x}{x(25 - x)(5 + \sqrt{x})} = \frac{1}{x(\sqrt{x} + 5)}$$

$$\xrightarrow{x \rightarrow 25} \frac{1}{25(5 + 5)} = \frac{1}{625}$$

16. Question Details SCalc8 1.6.031

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$$

$$\xrightarrow{h \rightarrow 0} 3x^2$$

(Binomial expansion)

$$(x+h)^3 = 1x^3h^0 + 3x^2h + 3xh^2 + 1x^0h^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

17. Question Details SCalc8 1.6.033. [3354477]

A graphing calculator is recommended.

(a) Estimate the value of

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1}$$

by graphing the function $f(x) = x/(\sqrt{1+3x} - 1)$. (Round your answer to one decimal place.)

(b) Make a table of values of $f(x)$ for x close to 0 and guess the value of the limit. (Round your answer to six decimal places.)

(c) Use the Limit Laws to find the exact value of $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1}$.

Analytic Way

$$\frac{x}{\sqrt{1+3x} - 1} \cdot \frac{\sqrt{1+3x} + 1}{\sqrt{1+3x} + 1} = \frac{x(\sqrt{1+3x} + 1)}{(3x+1) - 1} = \frac{x(\sqrt{1+3x} + 1)}{3x}$$

$$\xrightarrow{x \rightarrow 0} \frac{\sqrt{1} + 1}{3} = \frac{2}{3}$$

x	f(x)
1	1
0.5	0.86038
0.25	0.774292
0.125	0.724201
0.0625	0.696575
0.03125	0.681942
0.015625	0.67439
0.007813	0.67055
0.003906	0.668614
0.001953	0.667642
0.000977	0.667155
0.000488	0.666911
0.000244	0.666789
0.000122	0.666728

$$f(x) = \frac{x}{\sqrt{1+3x} - 1}$$

Browser tabs: Aims Community College | Index of /201/videos/chapter-... | plot(x/(sqrt(3x+1)-1) from ...

Address bar: www.wolframalpha.com/input/?i=plot(x%2F(sqrt(3x%2B1)-1))+from+-1+to+5

Search: plot(x/(sqrt(3x+1)-1) from -1 to 5

Input: plot $\frac{x}{\sqrt{3x+1}-1}$ $x = -1$ to 5

Plot:

20. Question Details

SCalc8 1.6.039

Evaluate the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 0} x^2 \cos\left(\frac{3}{x}\right) = 0, \text{ by Squeeze.}$$

$$-x^2 \leq x^2 \cos\left(\frac{3}{x}\right) \leq x^2$$

$\cos\left(\frac{3}{x}\right)$ approaches Nothing as $x \rightarrow 0$, but the x^2 drags it down to zero.
KEY: $\cos\left(\frac{3}{x}\right)$ is bounded above & below by ± 1

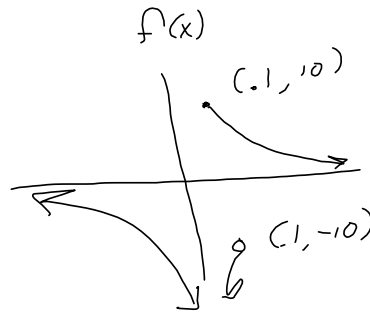
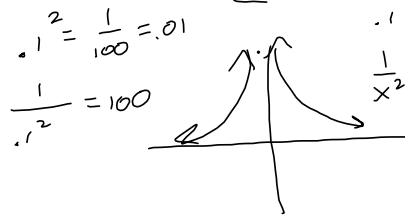
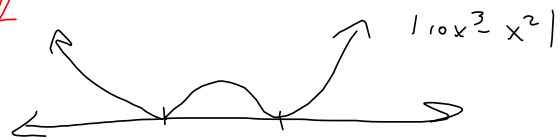
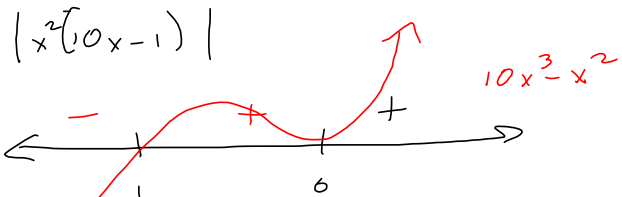
Question Details

SCalc8 1.6.043

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

$$\lim_{x \rightarrow 0.1^-} \frac{10x - 1}{|10x^3 - x^2|}$$

Quick analysis of $|10x^3 - x^2|$



$$\frac{10x - 1}{|x^2(10x - 1)|} =$$

$$\begin{cases} \frac{10x - 1}{x^2(10x - 1)} & \text{if } x \geq .1 \\ -\frac{10x - 1}{x^2(10x - 1)} & \text{if } x < .1 \end{cases}$$

$$= \begin{cases} \frac{1}{x^2} & \text{if } x \geq .1 \\ -\frac{1}{x^2} & \text{if } x < .1 \end{cases}$$

$$x \rightarrow .1^- \rightarrow \boxed{-10}$$

2. Question Details

SCalc8 1.6.045

Find the limit, if it exists. (If an answer does not exist, enter DNE.)

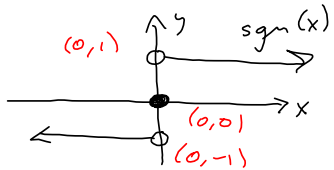
$$\lim_{x \rightarrow 0^-} \left(\frac{2}{x} - \frac{2}{|x|} \right) = \lim_{x \rightarrow 0^-} f(x) = \frac{2}{x} - \frac{2}{|x|} = \begin{cases} \frac{2}{x} - \frac{2}{x} & \text{if } x \geq 0 \\ \frac{2}{x} - \frac{2}{-x} & \text{if } x < 0 \end{cases} = \begin{cases} 0 & \text{if } x \geq 0 \\ \frac{4}{x} & \text{if } x < 0 \end{cases}$$

So, $\lim_{x \rightarrow 0^-} f(x) = -\infty$ (or just say \nexists)

23. Question Details SCalc8 1.6.047

The signum (or sign) function, denoted by sgn , is defined by

$$\text{sgn } x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0. \\ 1 & \text{if } x > 0 \end{cases}$$



(a) Sketch the graph of this function.

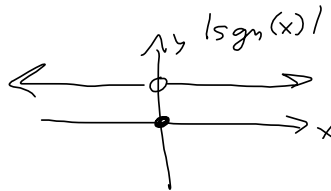
(b) Find each of the following limits. (If an answer does not exist, enter DNE.)

(i) $\lim_{x \rightarrow 0^+} \text{sgn } x = ($

(ii) $\lim_{x \rightarrow 0^-} \text{sgn } x = -1$

(iii) $\lim_{x \rightarrow 0} \text{sgn } x = \text{DNE}$

(iv) $\lim_{x \rightarrow 0} |\text{sgn } x| = ($



24. Question Details SCalc8 1.6.050

Let

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < 1 \\ (x-3)^2 & \text{if } x \geq 1 \end{cases}$$

(a) Find the following limits. (If an answer does not exist, enter DNE.)

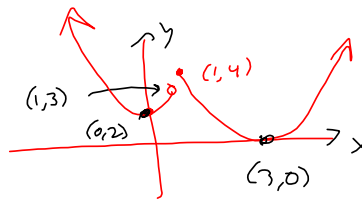
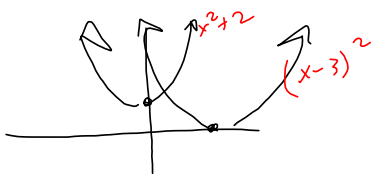
$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2 + 2) = 1^2 + 2 = 3$

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x-3)^2 = (1-3)^2 = (-2)^2 = 4$ $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 1 \\ (x-3)^2 & \text{if } x \geq 1 \end{cases}$

(b) Does $\lim_{x \rightarrow 1} f(x)$ exist? No. $\lim_{x \rightarrow 1^-} f(x) = 3 \neq 4 = \lim_{x \rightarrow 1^+} f(x)$

"Left- & right-hand limits disagree!"

(c) Sketch the graph of f .



25. Question Details

SCalc8 1.6.052

Let

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 5 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 1 & \text{if } x > 2 \end{cases}$$

(a) Evaluate each of the following, if it exists. (If an answer does not exist, enter DNE.)

(i) $\lim_{x \rightarrow 1^-} g(x) = 1$

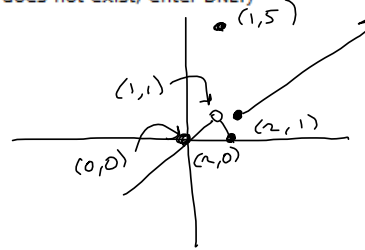
(ii) $\lim_{x \rightarrow 1^+} g(x) = 2 - 1^2 = 1$

(iii) $g(1) = 5$!?

(iv) $\lim_{x \rightarrow 2^-} g(x) = 2 - 2^2 = 0$

(v) $\lim_{x \rightarrow 2^+} g(x) = 2 - 1 = 1$

(vi) $\lim_{x \rightarrow 2} g(x)$ \nexists . Left & right disagree.



(b) Sketch the graph of g.

26. Question Details

SCalc8 1.6.053. [33916]

(a) If the symbol $\llbracket x \rrbracket$ denotes the greatest integer function defined in this example, evaluate the following. (If an answer does not exist, enter DNE.)

(i) $\lim_{x \rightarrow -3^+} \llbracket x \rrbracket = -3$

(ii) $\lim_{x \rightarrow -3} \llbracket x \rrbracket$ \nexists

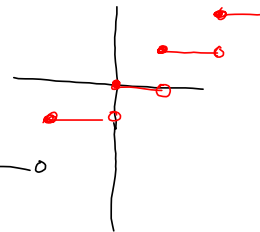
(iii) $\lim_{x \rightarrow -3.4} \llbracket x \rrbracket = -4$

$\llbracket x \rrbracket = \text{greatest integer } \leq x$
 $\llbracket 5.2 \rrbracket = 5$

(b) If n is an integer, evaluate the following.

(i) $\lim_{x \rightarrow n^-} \llbracket x \rrbracket = n - 1$

(ii) $\lim_{x \rightarrow n^+} \llbracket x \rrbracket = n$



(c) For what values of a does $\lim_{x \rightarrow a} \llbracket x \rrbracket$ exist?

$\forall a \notin \mathbb{Z}$

For all a that are not in the integers.

- Integers $\mathbb{Z} = \{ \dots, -2, -1, 0, 1, 2, \dots \}$
- Naturals $\mathbb{N} = \{ 1, 2, 3, \dots \}$
- Rationals $\mathbb{Q} = \{ \frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0 \}$
- Reals $\mathbb{R} = \text{All reals}$
- Imationals $\mathbb{R} \setminus \mathbb{Q} = \text{Imationals!}$

27. Question Details

SCalc8 1.6.056. [3354232]

In the theory of relativity, the Lorentz contraction formula

$$L = L_0 \sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find $\lim_{v \rightarrow c^-} L$.

Why is a left-hand limit necessary?

Speed of light's the ceiling for velocity for any body with mass.
Mathematically

$\sqrt{1 - v^2/c^2} \notin \mathbb{R}$ if $v > c$,
so $v \rightarrow c^-$ is only thing that makes sense, physically & by our model.

28. Question Details

SCalc8 1.6.059.

If $\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1} = 10$, evaluate $\lim_{x \rightarrow 1} f(x)$.

$f(x) \xrightarrow{x \rightarrow 1} 8$ for $\lim_{x \rightarrow 1} \frac{f(x) - 8}{x - 1}$ to converge.

For it to equal 10 says, in some sense, $f(x) \xrightarrow{x \rightarrow 1} 8$
10 times faster than $x \rightarrow 1$

29. Question Details

SCalc8 1.6.060.

If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 8$, evaluate the following limits.

(a) $\lim_{x \rightarrow 0} f(x) = 0$

(b) $\lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{x}{x} \cdot \frac{f(x)}{x} = \lim_{x \rightarrow 0} \frac{x f(x)}{x^2} = \lim_{x \rightarrow 0} x \cdot \lim_{x \rightarrow 0} \frac{f(x)}{x^2}$
 $= 0!$

30. Question Details

SCalc8 1.6.062

Show by means of an example that $\lim_{x \rightarrow a} [f(x) + g(x)]$ may exist even though neither $\lim_{x \rightarrow a} f(x)$ nor $\lim_{x \rightarrow a} g(x)$ exists.

$$a=0, f(x) = \frac{1}{x}, g(x) = -\frac{1}{x}$$

$$\lim_{x \rightarrow 0} f(x) \nexists, \lim_{x \rightarrow 0} g(x) \nexists, \text{ but}$$

$$\lim_{x \rightarrow 0} (f(x) + g(x)) = \lim_{x \rightarrow 0} \left[\frac{1}{x} - \frac{1}{x} \right] = \lim_{x \rightarrow 0} [0] = 0.$$

31. Question Details

SCalc8 1.6.065

Find the number a such that the limit exists.

$$\lim_{x \rightarrow -2} \frac{4x^2 + ax + a + 12}{x^2 + x - 2}$$

Find the value of the limit.

$$x^2 + x - 2 = (x+2)(x-1) \xrightarrow{x \rightarrow -2} 0$$

$$\text{So, need } 4x^2 + ax + a + 12 \xrightarrow{x \rightarrow -2} 0$$

Other way

$$4(-2)^2 + a(-2) + a + 12$$

set = 0

$$16 - 2a + a + 12 = 0 \Rightarrow a = 28$$

$$\begin{array}{r} -2 \overline{) 4 \quad a \quad a+12} \\ \underline{-8 \quad -2a+16} \\ 4 \quad a-8 \quad -2+28 \end{array} \begin{array}{l} \text{SET} \\ = 0 \\ a = 28! \end{array}$$

$$\begin{aligned} &4x^2 + 28x + 28 + 12 \\ &= 4x^2 + 28x + 40 \\ &= x^2 + 7x + 10 \\ &= (x+5)(x+2) \Rightarrow \end{aligned}$$

$$f(x) = \frac{(x+5)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)} = \frac{x+5}{x-1} \xrightarrow{x \rightarrow -2} \frac{-2+5}{-2-1} = \frac{3}{-3} = -1$$