SCalc8 1.5.001. (3354117) (Remove) -- view 2m

Explain what is meant by the equation

$$\lim_{x \to 2} f(x) = 3.$$

Is it possible for this statement to be true and yet $f(2) = 4$? Explain.

This says I can make f(x) as close to y = 3 as I desire, by taking x sufficiently close to 2.

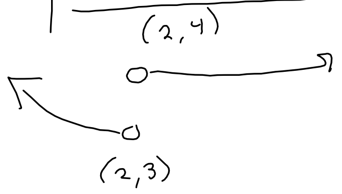
| f(x) - 3 | < small by making | x - 2 | < some other sufficiently small.

Silent on "f(2)"

SCalc8 1.5.002. (3354190) (Remove) -- view

Explain what it means to say that

$$\lim_{x \to 2^-} f(x) = 3 \quad \text{and} \quad \lim_{x \to 2^+} f(x) = 4.$$



In this situation is it possible that $\lim_{x \to 2} f(x)$ exists? Explain.

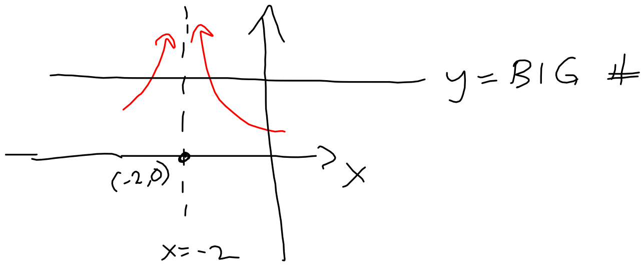
No. For the (2-sided) limit to exist, we need the left and right limits to agree!

SCalc8 1.5.003. (3413006) (Add) -- view

Explain the meaning of each of the following.

(a) $\lim_{x \to -2} f(x) = \infty$

(b) $\lim_{x \to 3^+} f(x) = -\infty$

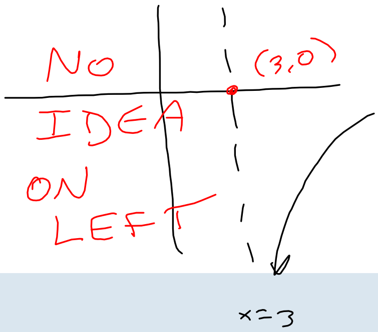


(a) $\lim_{x \to -2} f(x) = \infty$ means that I can make f(x) arbitrarily large by taking x arbitrarily close to -2. Give me a big positive number. I can take x close enough to x = -2 to make f(x) BIGGER than your positive number AND REMAIN bigger, anywhere closer to x = -2.

(b) means that I can make f(x) come in BELOW any negative number by taking x sufficiently close to x = 3, coming in from the right.

Negative # with BIG absolute value.

(Large negative number).

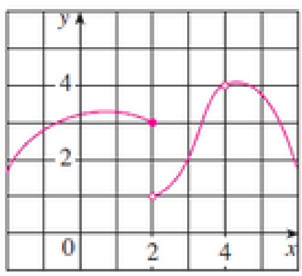


SCalc8 1.5.004. (3354294) (Add) -- view

Comment: slightly modified, not randomized

3m

Use the given graph of f to state the value of each quantity, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim_{x \to 2^-} f(x) = 3$

(b) $\lim_{x \to 2^+} f(x) = 1$

(c) $\lim_{x \to 2} f(x)$ $\nexists$

(d) $f(2) = 3$

(e) $\lim_{x \to 4} f(x) = 4$

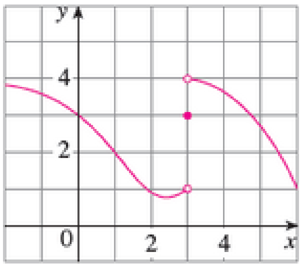
(f) $f(4)$ $\nexists$

SCalc8 1.5.005. (3354534) (Add) -- view

Comment: slightly modified, not randomized

2m

For the function f whose graph is given, state the value of each quantity, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim_{x \to 1} f(x) = 2$

(b) $\lim_{x \to 3^-} f(x) = 1$

(c) $\lim_{x \to 3^+} f(x) = 4$

(d) $\lim_{x \to 3} f(x)$ $\nexists$ (Jump Discontinuity.)

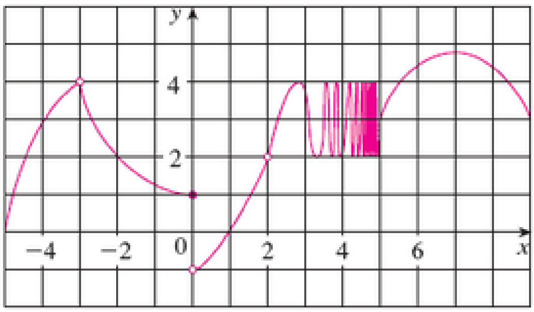
(e) $f(3) = 3$

SCalc8 1.5.006. (3354147) (Add) -- view

Comment: not randomized

5m

For the function h whose graph is given, state the value of each quantity, if it exists. (If an answer does not exist, enter DNE.)



(a) $\lim_{x \to -3^-} h(x) = 4$

(b) $\lim_{x \to -3^+} h(x) = 4$

(c) $\lim_{x \to -3} h(x) = 4$

(d) $h(-3)$ $\nexists$

(e) $\lim_{x \to 0^-} h(x) = 1$

(f) $\lim_{x \to 0^+} h(x) = -1$

(g) $\lim_{x \to 0} h(x)$ $\nexists$

(h) $h(0) = 1$

(i) $\lim_{x \to 2} h(x) = 2$

(j) $h(2)$ $\nexists$

(k) $\lim_{x \to 5^+} h(x) = 3$

(l) $\lim_{x \to 5^-} h(x)$ $\nexists$

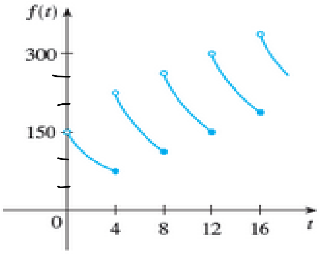
$h(x) = \begin{cases} \sin\left(\frac{\pi}{x-5}\right)+3 & 2<x<5 \end{cases}$

This behavior near $x=5$ coming in from the left

Infinite # of waves packed in close to $x=5$, from the left.

SCalc8 1.5.010.MI. (3354114) (Add) -- view

A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours.



Find $\lim_{t \to 4^-} f(t)$ and $\lim_{t \to 4^+} f(t)$.

$\lim_{t \to 4^-} f(t) =$ 75 ish

$\lim_{t \to 4^+} f(t) =$ 225 ish

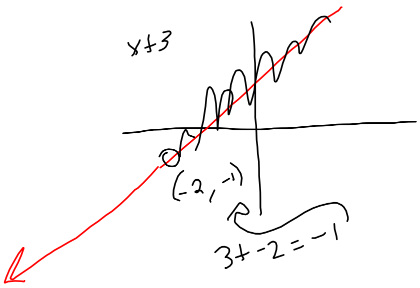
SCalc8 1.5.011. (3354152) (Add) -- view

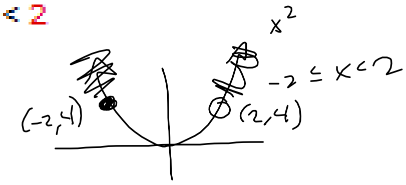
Sketch the graph of the function.

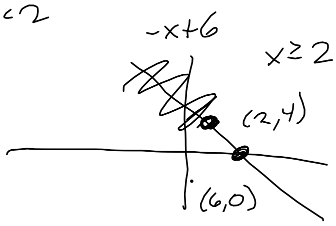
$$f(x) = \begin{cases} 3 + x & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x < 2 \\ 6 - x & \text{if } x \ge 2 \end{cases}$$

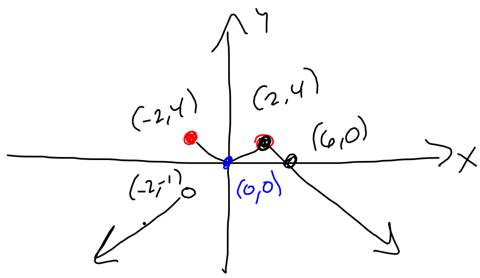
Graph separately.

Manage Suture Points & domains









Good 'Nuff!

SCalc8 1.5.013. (3354374) (Remove) -- view

5m

Use the graph of the function f to state the value of each limit, if it exists. (If an answer does not exist, enter DNE.)

$$f(x) = \frac{5}{1 + 2^{1/x}}$$

(a) $\lim_{x \to 0^-} f(x)$

5 | 5

(b) $\lim_{x \to 0^+} f(x)$

0 | 0

(c) $\lim_{x \to 0} f(x)$

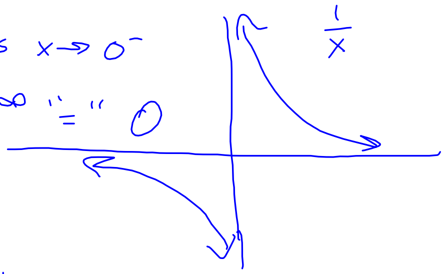
~~3~~ | DNE

b/c $5 \neq 0$.

$\frac{1}{x} \to -\infty$ as $x \to 0^-$

So, $2^{\frac{1}{x}} \to 2^{-\infty}$ "=" 0

So $\frac{5}{1+0} = 5$



as $x \to 0^+$, $\frac{1}{x} \to +\infty$

So $2^{\frac{1}{x}} \to 2^{\infty}$ "=" ∞

& $\frac{5}{1+\infty}$ is where it's headed.

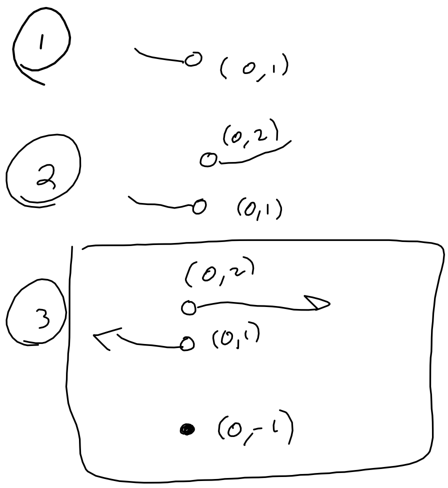
"=" 0.

SCalc8 1.5.015. (3354148) (Add) -- view

Sketch the graph of an example of a function f that satisfies all of the given conditions.

$\lim_{x \to 0^-} f(x) = 1$, $\lim_{x \to 0^+} f(x) = 2$, $f(0) = -1$

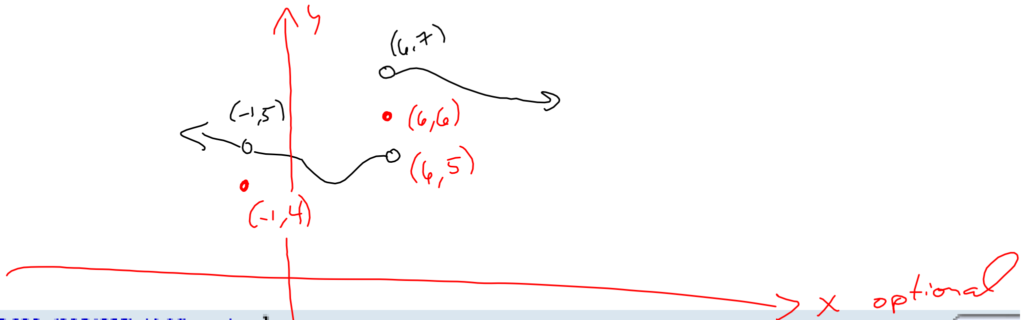
$\underbrace{\textcircled{1}\quad\textcircled{2}}_{\textcircled{3}}$



SCalc8 1.5.017. (3354192) (Remove) -- view

Sketch the graph of an example of a function f that satisfies all of the given conditions.

$\lim_{x \to 6^+} f(x) = 7$, $\lim_{x \to 6^-} f(x) = 5$, $\lim_{x \to -1} f(x) = 5$, $f(6) = 6$, $f(-1) = 4$



SCalc8 1.5.019. (3354098) (Add) -- view

11m

Evaluate the function $f(x)$ at the given numbers (correct to six decimal places).

$f(x) = \frac{x^2 - 5x}{x^2 - 25}$, $= \frac{x(x-5)}{(x-5)(x+5)} = \frac{x}{x+5}$ if $x \neq 5$

Hole @ $(5, \frac{1}{2})$!

$x = 5.1, 5.05, 5.01, 5.001, 5.0001,$
$4.9, 4.95, 4.99, 4.999, 4.9999$

x	$f(x)$	x	$f(x)$
5.1	0.504950	4.9	0.494949
5.05	0.502488	4.95	0.497487
5.01	0.500500	4.99	0.499499
5.001	0.500050	4.999	0.499950
5.0001	0.500005	4.9999	0.499995

Guess the value of the limit (correct to six decimal places). (If an answer does not exist, enter DNE.)

$$\lim_{x \to 5} \frac{x^2 - 5x}{x^2 - 25}$$

#19	x	f(x)
	5.1	0.50495
	5.05	0.502488
	5.01	0.5005
	5.001	0.50005
	5.0001	0.500005
	4.9	0.494949
	4.95	0.497487
	4.99	0.499499
	4.999	0.49995
	4.9999	0.499995

$$f(x) = \frac{x^2 - 5}{x^2 - 25}$$

$\lim_{x \to 5} f(x) = .5$

#19	x	f(x)
	5.1	=(B3^2-5*B3)/(B3^2-25)
	5.05	0.502488

B3-CELL

$D = \mathbb{R} \setminus \{\pm 5\}$

V.A.: $x = -5$, ~~$x = 5$~~

Cancelled $(x-5)$'s. So HOLE @ $x = 5$

$$\frac{x^2 - 5x}{x^2 - 25} = \frac{x\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} = \frac{x}{x+5} \text{ if } x \neq 5$$

$f^*(x)$, after cancelling

$f^*(5) = \frac{5}{5+5}$

$= \frac{5}{10} = \frac{1}{2} = .5$

SCalc8 1.5.022. (3354484) (Add) -- view

Evaluate the function $f(h)$ at the given numbers (correct to six decimal places).

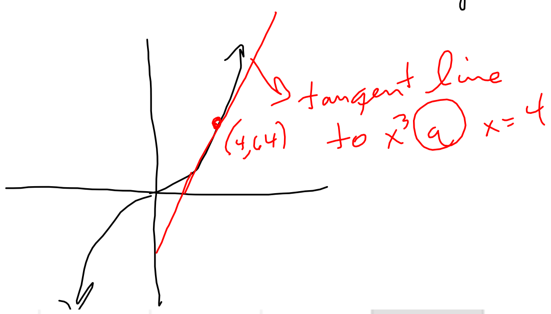
$$f(h) = \frac{(4+h)^3 - 64}{h},$$

$h = \pm0.5, \pm0.1, \pm0.01, \pm0.001, \pm0.0001$

h	$f(h)$	h	$f(h)$
0.5	54.250000	−0.5	42.250000
0.1	49.210000	−0.1	46.810000
0.01	48.120100	−0.01	47.880100
0.001	48.012001	−0.001	47.988001
0.0001	48.001200	−0.0001	47.998800

Guess the value of the limit (correct to six decimal places). (If an answer does not exist, enter DNE.)

$\lim_{h\to 0} \frac{(4+h)^3 - 64}{h}$ is difference quotient for $g(x) = x^3$ @ $x = 4$



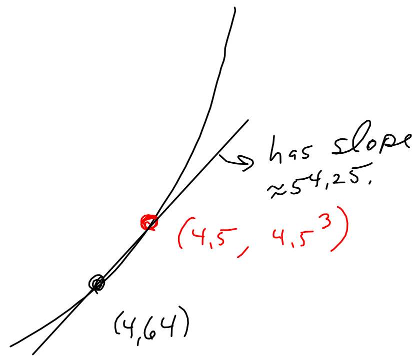
$\frac{f(x+h)-f(x)}{h}$ $\{ x = 4 \Rightarrow$

$\frac{f(4+h)-f(4)}{h} = \frac{(4+h)^3 - 4^3}{h}$

$\xrightarrow{h \to 0}$?

I	J	K	L	M	N
#22	h	f(h)			
	0.5	54.25		$\frac{(4+h)^3 - 64}{h}$	
	0.1	49.21			
	0.01	48.1201			
	0.001	48.012			
	0.0001	48.0012			
	-0.5	42.25			
	-0.1	46.81			
	-0.01	47.8801			
	-0.001	47.988			
	-0.0001	47.9988			

Trending towards 48



$$\frac{f(4+h)-f(4)}{h} = \frac{(4+h)^3 - 4^3}{h} = \frac{\cancel{64} + 48h + 12h^2 + h^3 - \cancel{64}}{h}$$

1
1 1
1 2 1
1 3 3 1

$$= \frac{\cancel{h}(48 + 12h + h^2)}{\cancel{h}} = 48 + 12h + h^2 \xrightarrow{h \to 0} \boxed{48}$$

$(4+h)^3 = 4^3 + 3 \cdot 4^2 h + 3 \cdot 4^1 h^2 + 1 \cdot 4^0 h^3$

SCalc8 1.5.023. (3354340) (Add) -- view

Comment: slightly modified

Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically. (Round your answer to two decimal places.)

$$\lim_{\theta \to 0} \frac{\sin(7\theta)}{\tan(2\theta)}$$

L'Hôpital, in future

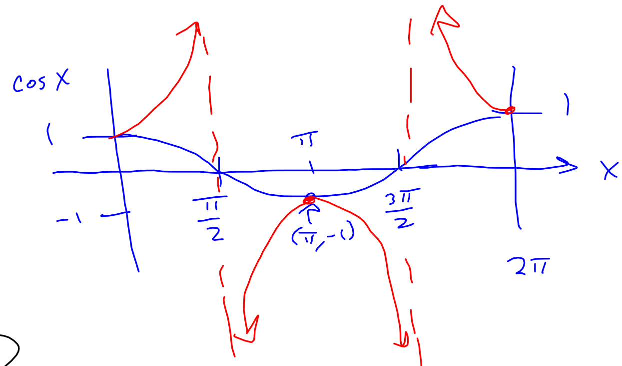
```
Y1(.1
     3.178025625
Y1(.01
     3.496676069
Y1(.00001
     3.499999997
```

$\to \frac{7}{2} = 3.5$

SCalc8 1.5.035. (3354342) (Add) -- view

Determine the infinite limit.

$$\lim_{x \to (\pi/2)^+} \frac{1}{x} \sec(x) = -\infty$$




```
        3.496676069
Y1(.00001
        3.499999997
Y1(π/2+.001)
       -636.2148515
Y1(π/2+.000001)
       -636619.3671
```

Confirms the $-\infty$, but moves nothing!

SCalc8 1.5.041. (3354111) (Add) -- view*

Evaluate the function for values of x that approach 1 from the left and from the right.

$f(x) = \frac{1}{x^3 - 1}$

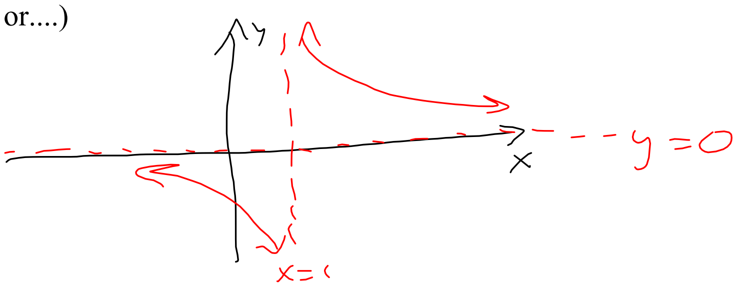
$$a^3 - b^3 = (a-b)\boxed{(a^2+ab+b^2)}$$

→ Never has real roots

$$\boxed{\begin{array}{l} \lim_{x \to 1^-} f(x) = -\infty \\ \lim_{x \to 1^+} f(x) = +\infty \end{array}}$$

Intuition is greatly helped by strong College Algebra chops. And if you work hard in this class, you will end UP with AWESOME College Algebra chops, even if you forgot a bunch of stuff since MAT 121 (or high school or....)

$$\frac{1}{x^3 - 1} = \frac{1}{(x-1)(x^2+x+1)}$$



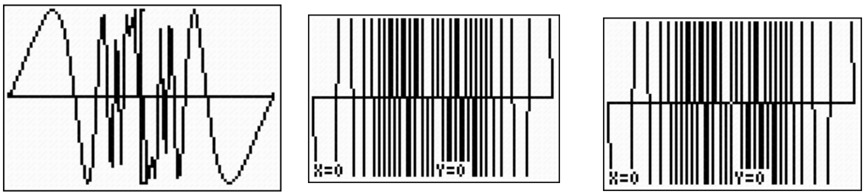
SCalc8 1.5.045. (3394372) (Remove) -- view

Comment: not randomized

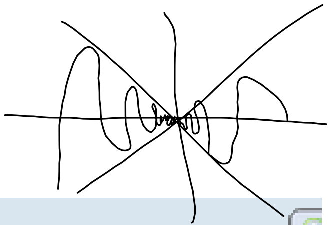
A graphing calculator is recommended.

Graph the function $f(x) = \sin(\pi/x)$ of the example in the viewing rectangle $[-1, 1]$ by $[-1, 1]$. Then zoom in toward the origin several times. Comment on the behavior of this function.

○ No matter how many times we zoom in toward the origin, the graphs of $f(x) = \sin(\pi/x)$ appear to consist of almost-vertical lines. This indicates more and more frequent oscillations as $x \to 0$.



$x \sin\left(\frac{\pi}{x}\right)$ is continuous!



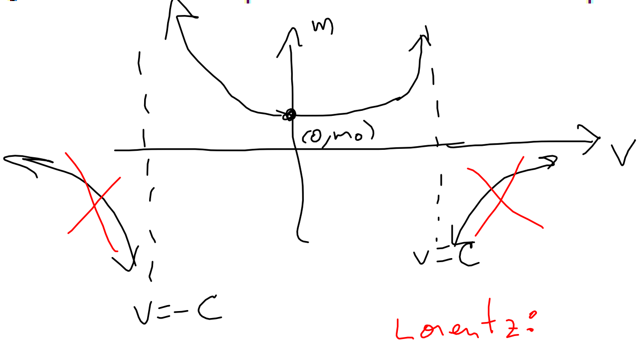
SCalc8 1.5.048. (3354355) (Add) -- view

In the theory of relativity, the mass of a particle with velocity v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

Mass increasing without bound as velocity approaches the speed of light.

where m_0 is the mass of the particle at rest and c is the speed of light. What happens as $v \to c^-$?



$$\frac{m_0}{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}$$

$$\frac{m_0}{\left(\frac{c-v}{c}\right)\left(\frac{c+v}{c}\right)}$$

$$\frac{c^2 m_0}{(c-v)(c+v)}$$

Lorentz:

Time dilation (tes)

Distance contraction (cts)

Mass "dilates"

$\sqrt{1 - v^2/c^2}$

Aug 29-10:18 AM