1.1 EXERCISES

1. If $f(x) = x + \sqrt{2 - x}$ and $g(u) = u + \sqrt{2 - u}$, is it true that $f = g$?

$g(x) = x + \sqrt{2-x}$ Same

2. If

$$f(x) = \frac{x^2 - x}{x - 1} \quad \text{and} \quad g(x) = x$$

is it true that $f = g$?

$\mathcal{D}(g) = \mathbb{R} = (-\infty, \infty)$

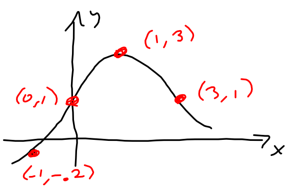
$f(x) = \dfrac{x\cancel{(x-1)}}{\cancel{x-1}} = x, \; x \neq 1$

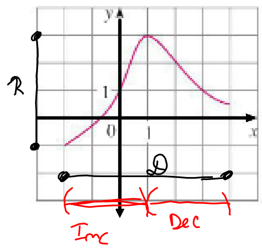
$f(1) \nexists$

$1 \notin \mathcal{D}(f)$

$f(x) = \begin{cases} x & x \neq 1 \\ \text{NADA} & x = 1 \end{cases}$

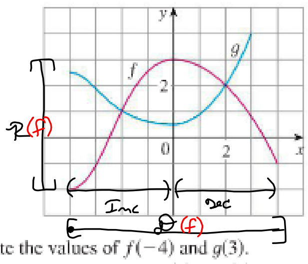
3. The graph of a function f is given.
(a) State the value of $f(1)$.
(b) Estimate the value of $f(-1)$.
(c) For what values of x is $f(x) = 1$?
(d) Estimate the value of x such that $f(x) = 0$.
(e) State the domain and range of f.
(f) On what interval is f increasing?



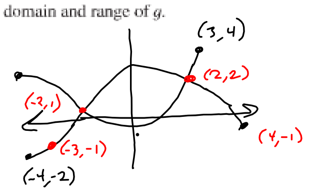


(a) $f(1) = 3$
(b) $f(-1) = -.2$
(c) $f(x) = 1 \Rightarrow \boxed{x \approx 0, 3}$
(d) $\boxed{x \approx -.6} \Rightarrow f(x) = 0$
(e) $\mathcal{D} = [-2, 4]$
$\mathcal{R} = [-1, 3]$
(f) f inc. on $(-2, 1)$
(from $y = -1$ to $y = 3$)

4. The graphs of f and g are given.



(a) State the values of $f(-4)$ and $g(3)$.
(b) For what values of x is $f(x) = g(x)$?
(c) Estimate the solution of the equation $f(x) = -1$.
(d) On what interval is f decreasing?
(e) State the domain and range of f.
(f) State the domain and range of g.



(a) $f(-4) = -2, \; g(3) = 4$
(b) $f(x) = g(x) \Rightarrow \underline{x = -2, 2}$ → $x \in \{-2, 2\}$ Alternate
(c) $f(x) = -1 \rightarrow x \in \{-3, 4\}$
(d) f dec on $(0, 4)$
(e) $\mathcal{D}(f) = [-4, 4]$, $\mathcal{R}(f) = [-2, 3]$
(f) $\mathcal{D}(g) = [-4, 3]$, $\mathcal{R}(g) = [.5, 4]$

5. Figure 1 was recorded by an instrument operated by the California Department of Mines and Geology at the University Hospital of the University of Southern California in Los Angeles. Use it to estimate the range of the vertical ground acceleration function at USC during the Northridge earthquake.

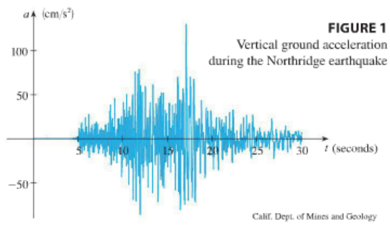


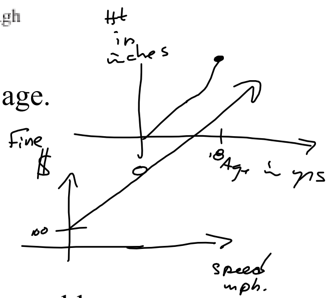
FIGURE 1 Vertical ground acceleration during the Northridge earthquake

$R \approx [-87.5, 131]$

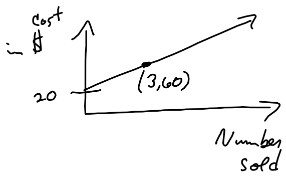
6. In this section we discussed examples of ordinary, everyday functions: Population is a function of time, postage cost is a function of weight, water temperature is a function of time. Give three other examples of functions from everyday life that are described verbally. What can you say about the domain and range of each of your functions? If possible, sketch a rough graph of each function.

Child's height as a function of age.

Fine as a function of speed.

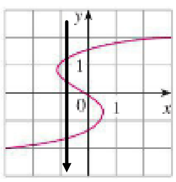


Cost as a function of the number sold.



7–10 Determine whether the curve is the graph of a function of x. If it is, state the domain and range of the function.

7.

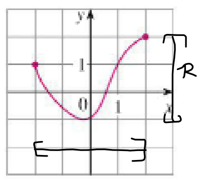


NOT

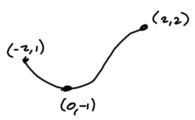
All are relations.

Not all are $y = f(x)$

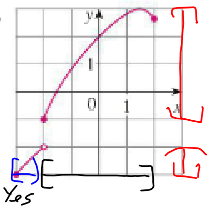
8.



Yes. $D = [-2,2]$, $R = [-1,2]$



9.

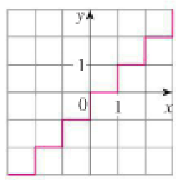


Yes

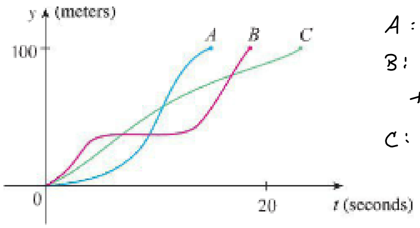
$D: [-2,2]$

$R = [-3,-2) \cup [-1,3]$

10.



14. Three runners compete in a 100-meter race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race?



A: Won the race

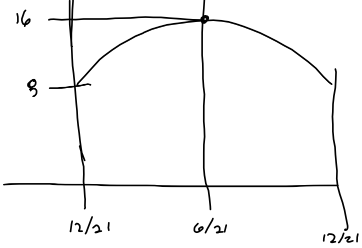
B: Was winning, Fell down, then came in 2nd

C: Slow, but steady. Came in 3rd

16. Sketch a rough graph of the number of hours of daylight as a function of the time of year.

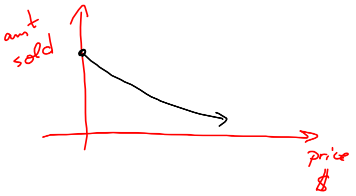
winter Solstice shortest

Summer Solstice Longest

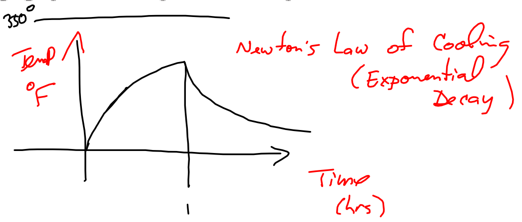


18. Sketch a rough graph of the market value of a new car as a function of time for a period of 20 years. Assume the car is well maintained.

19. Sketch the graph of the amount of a particular brand of coffee sold by a store as a function of the price of the coffee.



20. You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time.



25. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a+1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a+h)$.

$f(x) = 3x^2 - x + 2 \Rightarrow$

$f(2) = 3(2)^2 - 2 + 2 = \boxed{12 = f(2)}$,

$f(-2) = 3(-2)^2 - (-2) + 2 = \boxed{16 = f(-2)}$,

$\boxed{f(a) = 3a^2 - a + 2}$,

$f(-a)\ 3(-a)^2 - (-a) + 2 = \boxed{3a^2 + a + 2 = f(-a)}$,

$f(a+1) = 3(a+1)^2 - (a+1) + 2$

$= 3(a+1)^2 - a - 1 + 2$

$= 3(a+1)^2 - a + 1 = 3(a^2 + 2a + 1) - a + 1$

$= 3a^2 + 6a + 3 - a + 1$

$= \boxed{3a^2 + 5a + 4 = f(a+1)}$,

$2f(a) = 2(3a^2 - a + 2)$

$= \boxed{6a^2 - 2a + 4 = 2f(a)}$,

$f(x) = 3x^2 - x + 2$

$f(2a) = 3(2a)^2 - (2a) + 2$

$= 3(4a^2) - 2a + 2$

$= \boxed{12a^2 - 2a + 2 = f(2a)}$,

$f(a^2) = 3(a^2)^2 - a^2 + 2$

$= \boxed{3a^4 - a^2 + 2 = f(a^2)}$,

$[f(a)]^2 = (3a^2 - a + 2)^2 = (3a^2 - a + 2)(3a^2 - a + 2)$

$= 9a^4 - 3a^3 + 6a^2$

$-3a^3 + a^2 - 2a$

$6a^2 - 2a + 4$

$\boxed{9a^4 - 6a^3 + 3a^2 - 4a + 4 = [f(a)]^2}$,

AND, $f(a+h) = 3\boxed{a+h}^2 - \boxed{a+h} + 2$

$= 3(a+h)^2 - (a+h) + 2$

$= 3(a^2 + 2ah + h^2) - a - h + 2$

$= \boxed{3a^2 + 6ah + 3h^2 - a - h + 2 = f(a+h)}$.

The most important one, here.

26. A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 1$ inches.

New air added = New Volume − Old Volume

$$= \frac{4}{3}\pi\left[r+1\right]^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi\left[r^3+3r^2+3r+1-r^3\right]$$

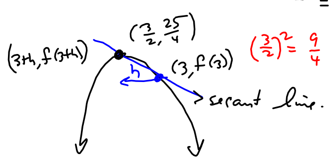
$$\boxed{= \frac{4}{3}\pi\left[3r^2+3r+1\right] = 4\pi r^2 + 4\pi r + \frac{4}{3}\pi}$$

1
1 1
1 2 1
1 3 3 1

27–30 Evaluate the difference quotient for the given function. Simplify your answer.

27. $f(x) = 4 + 3x - x^2$, $\quad \dfrac{f(3+h) - f(3)}{h} = m_{sec}$

To see it, complete the square or find intercepts, etc.



$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$$-x^2+3x+4$$
$$= -\left(x^2-3x+\left(\tfrac{3}{2}\right)^2\right)+4+\frac{9}{4}$$
$$= -\left(x-\tfrac{3}{2}\right)^2+\frac{25}{4} = a(x-h)^2+k$$
$$(h,k) = \left(\tfrac{3}{2}, \tfrac{25}{4}\right)$$

$$\frac{f(3+h)-f(3)}{h} = \frac{4+3(3+h)-(3+h)^2-\left[4+3(3)-3^2\right]}{h}$$

$$= \frac{4+9+3h-\left[3^2+6h+h^2\right]-(4)}{h} = \frac{13+3h-9-6h-h^2-4}{h}$$

$$= \frac{3h-6h-h^2}{h} = \frac{-3h-h^2}{h} = \frac{h[-3-h]}{h}$$

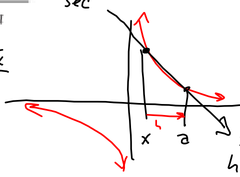
$$= \boxed{-3-h = \frac{f(3+h)-f(3)}{h}}$$

29. $f(x) = \dfrac{1}{x}$, $\quad \dfrac{f(x)-f(a)}{x-a} = m_{sec}$

$$m_{sec} = \frac{\frac{1}{x}-\frac{1}{a}}{x-a} = \frac{\frac{a}{ax}-\frac{x}{ax}}{x-a}$$

$$= \frac{\frac{a-x}{ax}}{\frac{x-a}{1}} = \frac{a-x}{ax}\cdot\frac{1}{x-a} =$$

$$\frac{-(x-a)}{ax(x-a)} = \boxed{-\frac{1}{ax} = m_{sec}}$$



secant line has slope m_{sec}

30. $f(x) = \dfrac{x+3}{x+1}$, $\Rightarrow \dfrac{f(x)-f(1)}{x-1} = \dfrac{f(x)-f(a)}{x-a} = \dfrac{f(x+h)-f(x)}{h}$

$$= \frac{\frac{x+3}{x+1}-\frac{1+3}{1+1}}{x-1} = \frac{\frac{x+3}{x+1}-\frac{4}{2}}{x-1} = \frac{\frac{x+3}{x+1}-2\left(\frac{x+1}{x+1}\right)}{x-1}$$

$$= \frac{\frac{x+3}{x+1}-\left(\frac{2x+2}{x+1}\right)}{x-1} = \frac{\frac{x+3-2x-2}{x+1}}{x-1} = \frac{\frac{-x+1}{x+1}}{x-1} = \frac{1-x}{(x+1)(x-1)}$$

$$= \frac{-(x-1)}{(x+1)(x-1)} = \boxed{\frac{-1}{x+1} = m_{sec}}$$

31–37 Find the domain of the function.

31. $f(x) = \dfrac{x+4}{x^2-9}$

Can't divide by 0.

$x^2-9 \neq 0$

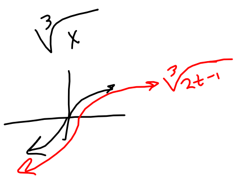
$(x-3)(x+3) \neq 0$

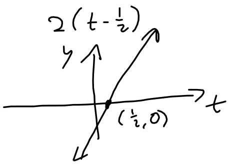
$x \notin \{-3,3\}$, so $\mathcal{D} = \{x \mid x \neq \pm 3\}$

$= \mathbb{R} \setminus \{-3,3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$

33. $f(t) = \sqrt[3]{2t-1}$

$\Rightarrow \mathcal{D}(f) = \mathbb{R}$!





35. $h(x) = \dfrac{1}{\sqrt[4]{x^2 - 5x}}$

$\dfrac{\text{stuff}}{0}$, $\sqrt[2n]{\text{negative}}$

≥ 0 & $\neq 0 \Rightarrow > 0$

Need $x^2 - 5x > 0$

$x(x-5) > 0$

>0

$+ \quad - \quad +$

Y N Y

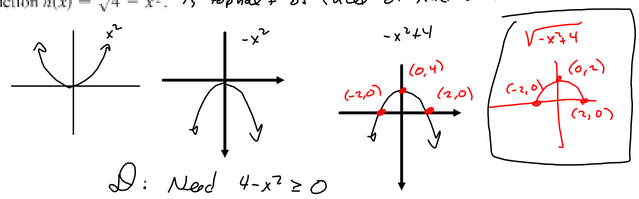
0 5

=0 =0

N N

$\boxed{(-\infty,0)\cup(5,\infty) = \mathcal{D}(h)}$

38. Find the domain and range and sketch the graph of the function $h(x) = \sqrt{4 - x^2}$. is top half of circle of radius 2.



$\mathcal{D}$: Need $4 - x^2 \geq 0$

$\Rightarrow x^2 - 4 \leq 0$

$x^2 - 4$

-2 2

$+ \quad - \quad +$ ≤ 0

-2 2

N Y Y Y N

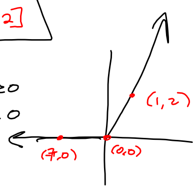
$(x-2)^1(x+2)^1$

$\boxed{\mathcal{D} = [-2,2] \quad \mathcal{R} = [0,2]}$

45–50 Sketch the graph of the function.

45. $f(x) = x + |x|$

$= \begin{cases} x + x & \text{if } x \geq 0 \\ x - x & \text{if } x < 0 \end{cases} = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$



49. $f(x) = \begin{cases} |x| & \text{if } |x| \leqslant 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

$|x| \leq 1$

$\{x \mid x \leq 1 \text{ AND } x \geq -1\}$ AND

-1 1

$= [-1,1]$

$-1 \leq x \leq 1$

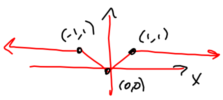
-1 0 1

$|x| > 1$

$\{x \mid x > 1 \text{ OR } x < -1\}$

-1 1

$= (-\infty,-1)\cup(1,\infty)$



51–56 Find an expression for the function whose graph is the given curve.

51. The line segment joining the points $(1, -3)$ and $(5, 7)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7+3}{5-1} = \dfrac{10}{4} = \dfrac{5}{2} = m$

$y = m(x - x_1) + y_1$

$y = \frac{5}{2}(x-1) - 3$ for $1 \leq x \leq 5$

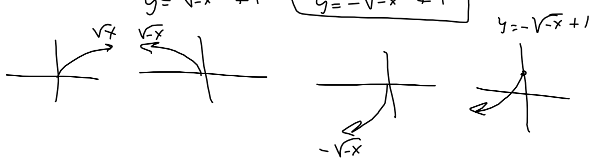
53. The bottom half of the parabola $x + (y - 1)^2 = 0$ Solve for y.

$(y-1)^2 = -x$

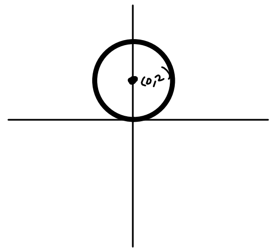
$\sqrt{(y-1)^2} = |y-1| = \sqrt{-x}$

$y - 1 = \sqrt{-x}$ OR $y - 1 = -\sqrt{-x}$ is the bottom half

$y = \sqrt{-x} + 1$ $\boxed{y = -\sqrt{-x} + 1}$



54. The top half of the circle $x^2 + (y-2)^2 = 4$



Solve for y:

$(y-2)^2 = 4-x^2$

$|y-2| = \sqrt{4-x^2}$

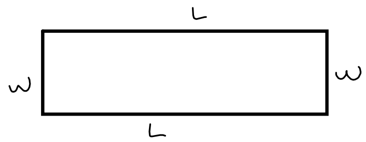
$y-2 = \pm\sqrt{4-x^2}$

$y = 2 \pm \sqrt{4-x^2}$

$\boxed{y = 2+\sqrt{4-x^2}}$ is the top!

57–61 Find a formula for the described function and state its domain.

57. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.



$P = 2L + 2W = 20$

$A = LW$

$\to L + W = 10$

$W = 10 - L$

$\Longrightarrow \boxed{A = L(10-L)}$

Could have degenerate case $\to 0 \le L \le 10$

$0 < L < 10$

58. A rectangle has area 16 m². Express the perimeter of the rectangle as a function of the length of one of its sides.

$LW = 16$

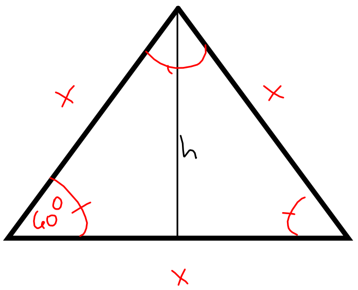
$P = 2L + 2W$

$\to W = \frac{16}{L}$

$\Longrightarrow P = 2L + 2\left(\frac{16}{L}\right)$

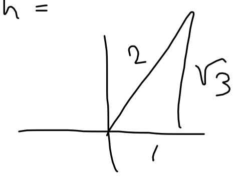
$= \boxed{2L + \frac{8}{L} = P(L)}$

59. Express the area of an equilateral triangle as a function of the length of a side.



$\text{Area} = \frac{1}{2} b h = \frac{1}{2} x h =$

$\frac{h}{x} = \sin 60^\circ = \frac{\sqrt{3}}{2}$



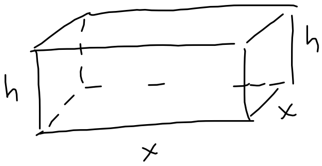
$\Rightarrow h = \frac{\sqrt{3}}{2} x \rightarrow$

$A = \left(\frac{1}{2} x\right)\left(\frac{\sqrt{3}}{2} x\right) = \boxed{\frac{\sqrt{3}}{4} x^2 = A(x)}$

61. An open rectangular box with volume 2 m³ has a square base. Express the surface area of the box as a function of the length of a side of the base.

x = length of side of base (m)

h = height (m)



S = Surface Area (m^2)

Given: $x^2 h = 2$ = Volume

Want

Surface area as function of x.

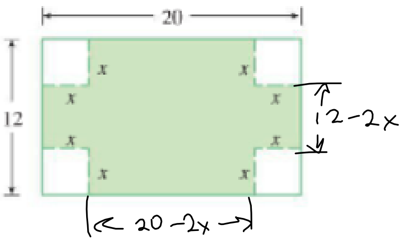
$x^2 + 4xh = S$

$x^2 h = 2 \Rightarrow$

$h = \frac{2}{x^2} \Rightarrow S = S(x) = x^2 + 4x\left(\frac{2}{x^2}\right)$

$= \boxed{x^2 + \frac{8}{x} = S(x)}$

63. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



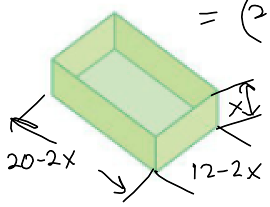
$V = \text{Volume } (\text{in}^3) = l \cdot w \cdot h$

$= (20-2x)(12-2x)x$

$= (240 - 40x - 24x + 4x^2)x$

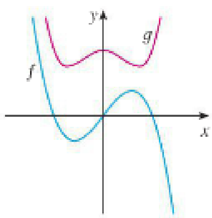
$= (240 - 64x + 4x^2)x$

$= \boxed{4x^3 - 64x^2 + 240x = V(x)}$



69–70 Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

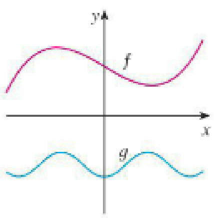
69.



f is odd

g is even

70.



f neither

g is even

Even: $f(-x) = f(x)$

Odd: $f(-x) = -f(x)$

71. (a) If the point (5, 3) is on the graph of an even function, what other point must also be on the graph?

(b) If the point (5, 3) is on the graph of an odd function, what other point must also be on the graph?

(a) $(x, f(x)) = (5,3)$ & Even $\Rightarrow$

$(-x, f(-x)) = \boxed{(-x, f(x)) = (-5, 3)}$

(b) Same, but ODD $\rightarrow$

$(-x, f(-x)) = \boxed{(-x, -f(x)) = (-5, -3)}$

Combining Even & ODD Functions

$(-)(-) = +$

$(-f(x))(-g(x)) = +f(x)g(x)$

$(\text{odd})(\text{odd}) = $ Even Func.

$(+)(+) = +$

$(f)(g) = fg$

$(\text{even})(\text{even}) = \text{even}$

$(+)(-) = -$

$(f)(-g) = -fg$

fg is odd

Use is "$-$" for odd
"$+$" for even.

Stand even funcs.
x^2, constant, x^{2n}

Stand. odd funcs
$x, x^3, x^5, \ldots, x^{2n+1}$

Adding / Subtracting:

odd + odd = odd

$f(x) = x + x^3$; $(-x) + (-x)^3$

$= -x - x^3 = -(x + x^3) = -f(x) = $ ODD

even + even = even

even + odd = neither!

Aug 23-10:59 AM

73–78 Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

73. $f(x) = \dfrac{x}{x^2+1} = \dfrac{\text{odd}}{\text{even}} = \dfrac{-}{+} = - = \text{odd}$

74. $f(x) = \dfrac{x^2}{x^4+1} = \dfrac{+}{+} = + = \boxed{\text{Even}}$

75. $f(x) = \dfrac{x}{x+1} = \dfrac{-}{\text{Nope}} = \boxed{\text{Nope.}}$

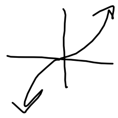
$|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

76. $f(x) = x|x|$

$= \begin{cases} x \cdot x & \text{if } x \ge 0 \\ x \cdot (-x) & \text{if } x < 0 \end{cases}$

$= \begin{cases} x^2 & \text{if } x \ge 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

(ODD)



Confirm:

$f(3) = 9$

$f(-3) = -9$

77. $f(x) = 1 + 3x^2 - x^4$

78. $f(x) = 1 + 3x^3 - x^5$

Aug 22-2:05 PM