2410 Week 13 MILLS

① f cnts, $f \geq 0$, $m = \min_{[0,5]}\{f(x)\}$, $M = \max_{[0,5]}\{f(x)\}$.

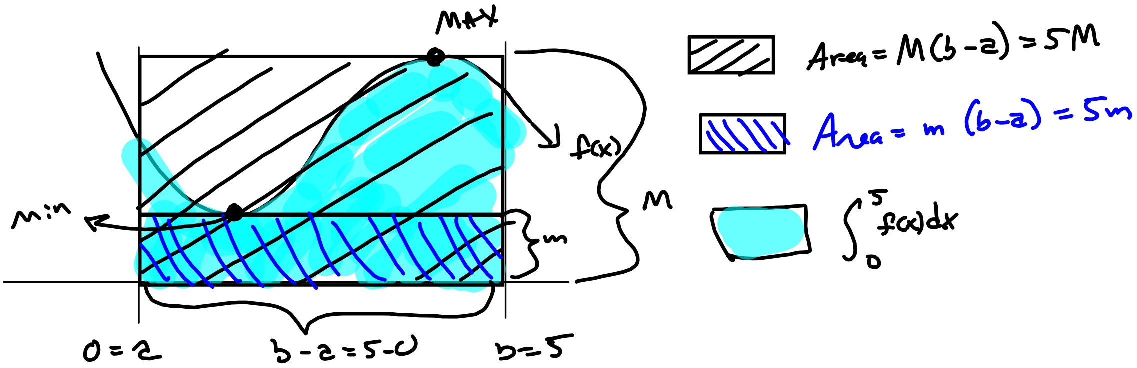
ⓐ 5pts Give an upper and lower estimate for $\int_0^5 f(x)\,dx$

$b-a = 5-0 = 5$

$m(b-a) \leq \int_0^5 f(x)\,dx \leq M(b-a)$

$$\boxed{5m \leq \int_0^5 f(x)\,dx \leq 5M}$$

ⓑ 5pts Draw a picture for this.



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(2) (10 pts) we find $\int_{-2}^{4}(3x^2+2x-5)\,dx$ by the limit definition

$$\Delta x = \frac{b-a}{n} = \frac{4-(-2)}{n} = \frac{6}{n}$$

$$x_k = a + k\Delta x = -2 + k\left(\frac{6}{n}\right) = \frac{6k}{n} - 2$$

$$\sum_{k=1}^{n} f(x_k)\,\Delta x = \Delta x \sum_{k=1}^{n} f(x_k) = \frac{6}{n}\sum_{k=1}^{n}\left(3x_k^2 + 2x_k - 5\right)$$

$$= \frac{6}{n}\sum_{k=1}^{n}\left(3\left(\frac{6k}{n}-2\right)^2 + 2\left(\frac{6k}{n}-2\right) - 5\right)$$

$$= \frac{6}{n}\sum_{k=1}^{n}\left(3\left(\frac{36k^2}{n^2} - 2\left(\frac{6k}{n}\right)(2) + 4\right) + \frac{12k}{n} - 4 - 5\right)$$

$$= \frac{6}{n}\sum_{k=1}^{n}\left(\frac{108k^2}{n^2} - 3\left(\frac{24k}{n}\right) + 12 + \frac{12k}{n} - 9\right)$$

$$= \frac{6}{n}\sum_{k=1}^{n}\left(\frac{108k^2}{n^2} - \frac{72k}{n} + \frac{12k}{n} + 3\right)$$

$$= \frac{6}{n}\sum_{k=1}^{n}\left(\frac{108k^2}{n^2} - \frac{60k}{n} + 3\right)$$

$$= \frac{6}{n}\left[\frac{108}{n^2}\sum_{k=1}^{n} k^2 - \frac{60}{n}\sum_{k=1}^{n} k + \sum_{k=1}^{n} 3\right]$$

$$= \frac{6}{n}\left[\frac{108}{n^2}\left(\frac{n^3+\text{lower}}{3}\right) - \frac{60}{n}\left(\frac{n^2+\sim}{2}\right) + 3n\right]$$

$$= \frac{2(108)(n^3+\sim)}{n^3} - \frac{3(60)}{n^2}(n^2+\sim) + \frac{6\cdot 3}{n}\cdot n$$

$$= \frac{216}{n^3}(n^3+\sim) - 180\left(\frac{n^2+\sim}{n^2}\right) + 18 \xrightarrow{n\to\infty}$$

$$216 - 180 + 18 = 234 - 180 = \boxed{54}$$

Check: $\int_{-2}^{4}(3x^2+2x-5)\,dx = x^3+x^2-5x\Big]_{-2}^{4}$

$$= 64 + 16 - 20 - (-8 + 4 + 10)$$

$$= 60 - (6) = 54$$

③ (10 pts) Evaluate $\int_1^4 \left(x^2 - \pi \sin\left(\frac{\pi}{6}x\right)\right) dx$ using FTC II.

$$= \frac{x^3}{3}\Big]_1^4 - \int_1^4 \pi \sin\left(\frac{\pi}{6}x\right) dx = \frac{4^3}{3} - \frac{1^3}{3} - \int_1^4 \sin(u)\,\frac{6\,du}{\pi}$$

$u = \frac{\pi}{6}x$ $\qquad u(1) = \frac{\pi}{6}$

$du = \frac{\pi}{6}dx$ $\qquad u(4) = \frac{\pi}{6}\cdot 4 = \frac{2\pi}{3}$

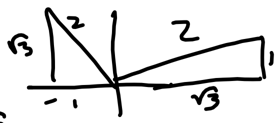
$dx = \frac{du}{\frac{\pi}{6}} = \frac{6\,du}{\pi}$

$\frac{1024-1}{3} +$

$64 \times 4 = 256$, $256 \times 4 = 1024$

$$= \frac{64-1}{3} \quad 6\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \sin(u)\,du = \frac{63}{3} - 6(-\cos(u))\Big]_{\frac{\pi}{6}}^{\frac{2\pi}{3}}$$

$$= 21 + 6\left[\cos\left(\frac{2\pi}{3}\right) - \cos\left(\frac{\pi}{6}\right)\right]$$



$$= 21 + 6\left[-\frac{1}{2} - \frac{\sqrt{3}}{2}\right] = 21 - 3 - 3\sqrt{3}$$

$$= \boxed{18 - 3\sqrt{3}}$$

(4) Use FTC I to evaluate the following

(a) 10pts $\frac{d}{dx}\left[\int_0^x \left(t^2 - \sin\left(\frac{\pi}{6}t\right)\right) dt\right]$

$= \boxed{x^2 - \sin\left(\frac{\pi}{6}x\right)}$

$\sqrt{3x} = (3x)^{\frac{1}{2}}$

$\to \frac{1}{2}(3x)^{-\frac{1}{2}}(3)$

(b) 10pts $\frac{d}{dx}\left[\int_0^{\sqrt{3x}+\sin(x)} t^2 - \pi \sin\left(\frac{\pi}{6}t\right) dt\right]$

$= \left(\left(\sqrt{3x}+\sin(x)\right)^2 - \pi \sin\left(\frac{\pi}{6}\left(\sqrt{3x}+\sin(x)\right)\right)\right)\left(\frac{1}{2}(3x)^{-\frac{1}{2}}(3) + \cos(x)\right)$

5) Evaluate the following definite and indefinite integrals.

a) 5pts $\int (2x-3)\sin(x^2-3x+5)\,dx$

$$= \int \underbrace{\sin(x^2-3x+5)}_{\sin(u)}\underbrace{(2x-3)\,dx}_{du} = \boxed{-\cos(x^2-3x+5)+C}$$

b) 5pts $\int_{-3}^{3} \frac{\sin(x)}{x^4+5x^2+1}\,dx = 0$, b/c $\frac{\sin(x)}{x^4+5x^2+1}$ is odd and $[-3,3]$ is a symmetric (about $x=0$) interval.

c) 5pts $\int_{-3}^{3} x\sqrt{x+4}\,dx = \int_1^7 (u-4)u^{\frac{1}{2}}\,du = \int_1^7 \left(u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\right)du$

$u = x+4 \implies$ $u(-3) = 1$

$du = dx$ & $x = u-4$ $u(3) = 7$

$$= \frac{2}{5}u^{\frac{5}{2}} - 4\left(\frac{2}{3}\right)u^{\frac{3}{2}}\Big]_1^7 = \frac{2}{5}\left(7^{\frac{5}{2}}\right) - \frac{8}{3}\left(7^{\frac{3}{2}}\right) - \left[\frac{2}{5} - \frac{8}{3}\right]$$

$$= \frac{2}{5}(49)\sqrt{7} - \frac{8}{3}(7)\sqrt{7} - \left[\frac{6-40}{15}\right]$$

$$= \frac{98}{5}\sqrt{7} - \frac{56}{3}\sqrt{7} + \frac{\ }{15} = \frac{98(3) - 56(5)}{15}\sqrt{7} + \frac{34}{15}$$

$$= \frac{294-280}{15}\sqrt{7} + \frac{34}{15}$$

$$= \boxed{\frac{14}{15}\sqrt{7} + \frac{34}{15}} \approx 4.736034557$$

$$\begin{array}{r} {}^{2}98 \\ \times\ 3 \\ \hline 294 \end{array} \qquad \begin{array}{r} {}^{3}56 \\ \times\ 5 \\ \hline 280 \end{array}$$

d) 5pts $\int \sec^4 x \tan x\,dx = \int \underbrace{\sec^3(x)}_{u^3}\underbrace{(\sec(x)\tan(x)\,dx)}_{du}$

$$= \boxed{\frac{\sec^4(x)}{4} + C}$$

(6) 10 pts $\int_{-5}^{5} (x+7)\sqrt{25-x^2}\,dx$

$= \int_{-5}^{5} x\sqrt{25-x^2}\,dx + 7\int_{-5}^{5}\sqrt{25-x^2}\,dx$

$= 0^{*} + \frac{7\pi(5)^2}{2} = \boxed{\frac{175\pi}{2}}$

$7(25) = 175$

$7\int_{-5}^{5}\sqrt{25-x^2}\,dx = 7\left[\underbrace{\text{area of } \tfrac{1}{2} \text{ circle, radius } r=5}_{\frac{\pi(5)^2}{2}}\right] = \boxed{\frac{175\pi}{2}}$

$y = \sqrt{25-x^2} \rightarrow$

$y^2 = 25 - x^2 \rightarrow$

$x^2 + y^2 = 25$