$2\sin(2x)\sin(x) = 2\cos(x)$

$2(2\sin(x)\cos(x))\sin(x) =$

$= 4\sin^2(x)\cos(x) = 4(1-\cos^2(x))\cos(x)$

$= 4\cos(x) - 4\cos^3(x) = 2\cos(x)$

$\Longrightarrow 2\cos(x) - 4\cos^3(x) = -4\cos^3(x) + 2\cos(x)$

$= \cos(x)\left(4\cos^2(x) + 2\right) = 0 \longrightarrow$

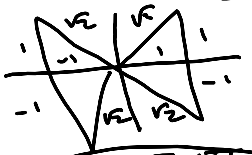
$\cos(x) = 0$

$\frac{\pi}{2}, \frac{3\pi}{2} = x$

OR

$4\cos^2(x) + 2 = 0$

$\cos(x) = \pm\frac{\sqrt{2}}{2}$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

2410 Mills

1) 10pts

$f(\theta) = 4\sin^2\theta - 3$

$f'(\theta) = 8\sin\theta\cos\theta = 4\sin(2\theta)$

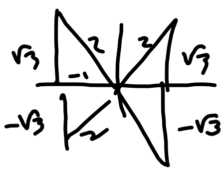
$f''(\theta) = 8\cos^2\theta - 8\sin^2\theta$

$= 8\cos(2\theta)$

$f(\theta) = 0$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$



$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$f'(\theta) = 8\sin\theta\cos\theta = 0$

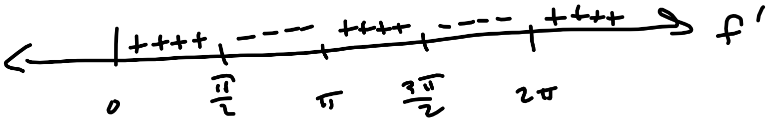
$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

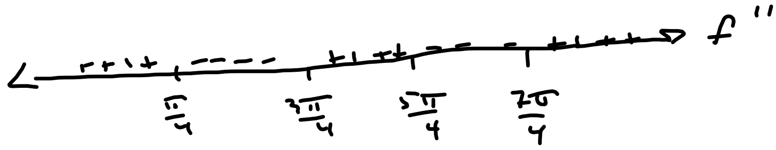
$f(\theta) = 4\sin(2\theta) \stackrel{set}{=} 0$

$\sin(2\theta) = 0$

$2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$ ✓

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$



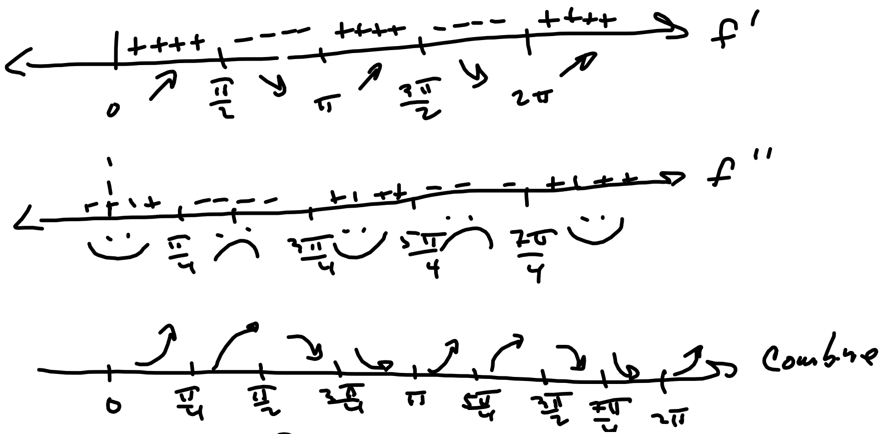


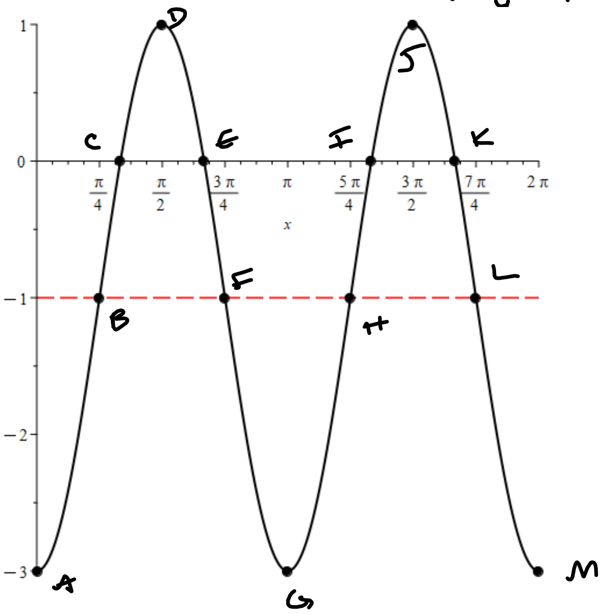
$f''(\theta) = 8\cos(2\theta) = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

2410 **Mills**





$A = (0, -3)$ MIN
$B = (\frac{\pi}{4}, -1)$ IP
$C = (\frac{\pi}{3}, 0)$ x-int
$D = (\frac{\pi}{2}, 1)$ MAX
$E = (\frac{2\pi}{3}, 0)$ x-int
$F = (\frac{3\pi}{4}, -1)$ IP
$G = (\pi, -3)$ MIN
$H = (\frac{5\pi}{4}, -1)$ IP
$I = (\frac{4\pi}{3}, 0)$ x-int
$J = (\frac{3\pi}{2}, 1)$ MAX
$K = (\frac{5\pi}{3}, 0)$ x-int
$L = (\frac{7\pi}{4}, -1)$ IP
$M = (2\pi, -3)$ MIN

2410 Mills

#s 2 - 4: Sketch a complete graph of the given function, using all your calculus skills.

2 10pts

$$R(x) = \frac{3x^2+6x-24}{4x^2+27x+18} = \frac{3(x^2+2x-8)}{4x^2+24x+3x+18}$$

$2\cdot2\cdot2\cdot3\cdot3$

$4x(x+6)+3(x+6)$

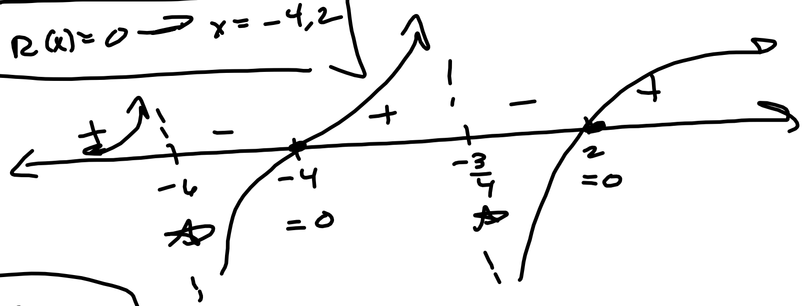
$=(4x+3)(x+6)$

$$= \frac{3(x+4)(x-2)}{(4x+3)(x+6)}$$

$D = \mathbb{R} \setminus \{-6, -\frac{3}{4}\}$

$\boxed{\text{V.A. } x=-6,\ x=-\frac{3}{4}}$

$\boxed{R(x)=0 \rightarrow x=-4, 2}$



$\boxed{\text{H.A.: } y=\frac{3}{4}}$

$-\frac{24}{18} = -\frac{4}{3}$

$y = \frac{3}{4}$

College Algebra Graph.

College Algebra can't find the inflection point that *must* be there, in between $x = -6$ and $x = -3/4$.

2410 Mills

$$\frac{3x^2+6x-24}{4x^2+27x+18} = \frac{3(x^2+2x-8)}{4x^2+24x+3x+18}$$

$$R'(x) = 3\left(\frac{(2x+2)(4x^2+27x+18)-(x^2+2x-8)(8x+27)}{(4x^2+27x+18)^2}\right)$$

$$=3\left(\frac{\cancel{8x^3}+54x^2+36x+8x^2+54x+36-(\cancel{8x^3}+16x^2-64x+27x^2+54x-216)}{(\qquad)^2}\right)$$

$$=3\left(\frac{62x^2-43x^2+100x+252}{(4x^2+27x+18)^2}\right)$$

$$=3\left(\frac{19x^2+100x+252}{(4x^2+27x+18)^2}\right)$$

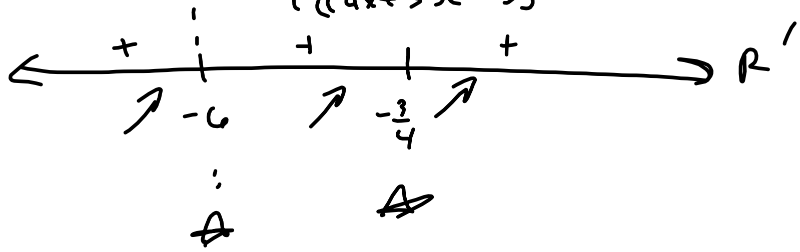
$$\begin{array}{r} 252 \\ \times\ 76 \\ \hline 1512 \\ 17640 \\ \hline 19152 \end{array}$$

$100^2-4(19)(252) = 10000-19152$

$= -9152 < 0$

No real zeros.

so $3\left(\frac{19x^2+100x+252}{((4x+3)(x+6))^2}\right)$



$$R''(x) = \frac{-24(19x^3 + 150x^2 + 756x + 1476)}{(4x+3)^3(x+6)^3} \quad \overset{\text{SET}}{=} 0 \longrightarrow$$

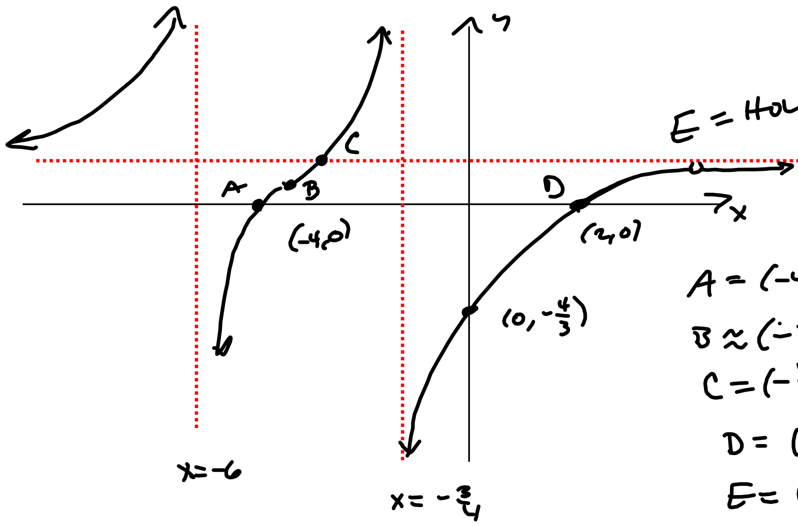
$x \approx -3.121026351$

R'': $+$ on $(-\infty, -6)$; $-$ on $(-6, -3.12)$; $+$ on $(-3.12, -\frac{3}{4})$; $-$ on $(-\frac{3}{4}, \infty)$

-6: ∄ $\quad -3.12$: $=0$ $\quad -\frac{3}{4}$: ∄

R'

This locates our inflection point for us



E = HOLE for #3

$A = (-4, 0)$

$B \approx (-3.12, .494)$ IP

$C = (-\frac{50}{19}, \frac{3}{4})$ BONUS

$D = (2, 0)$

$E = (8, \frac{108}{245})$

2410 Mills

(3) $\hat{R}(x) = R(x)$ with a hole. Find the hole

$3x^3 - 18x^2 - 72x + 192$

$$\begin{array}{r|rrrr} 2 & 3 & -18 & -72 & 192 \\ & & 6 & -24 & -192 \\ \hline -4 & 3 & -12 & -96 & 0 \\ & & -12 & +96 & \\ \hline & 3 & -24 & 0 & \end{array}$$

$$= \frac{3(x+4)(x-2)(x-c)}{(4x+3)(x+6)(x-c}$$

$3x - 24 = 3(x-8)$

$x = 8$ is the hole

$$R(8) = \frac{3(8+4)(8-2)}{(4(8)+3)(8+6)} = \frac{3(12)(6)}{(35)(14)} = \frac{3(36)}{35(7)} = \frac{108}{245}$$

Hole @ $x = 8$, $\boxed{y = \frac{108}{245}}$ =

2410 Mills

(4) (10 pts) $T(x)=\dfrac{3x^3-18x^2-72x+192}{4x^2+27x+18}$

$$=\frac{3(x+4)(x-2)(x-8)}{(4x+3)(x+6)}$$

$\boxed{\begin{array}{l}\text{V.A.: } x=-\frac{3}{4},\ x=-6 \\ \text{x-int: } x=-4,2,8\end{array}}$

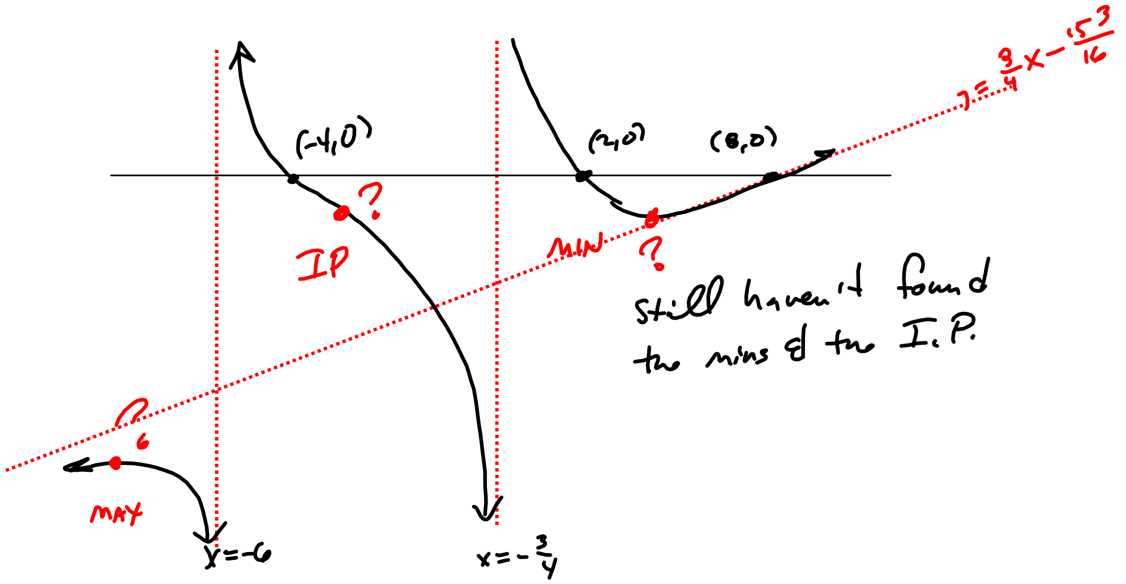
y-int: $\dfrac{192}{18}=\dfrac{64}{6}=\dfrac{32}{3}$

$$\begin{array}{r l}
 & \boxed{\frac{3}{4}x-\frac{153}{16}=\text{Slant As.}} \\
4x^2+27x+18 \big) & 3x^3-18x^2-72x+192 \\
 & -\left(3x^3+\frac{81}{4}x^2\right) \\ \hline
 & -\frac{153}{4}x^2
\end{array}$$

$-18-\dfrac{81}{4}$

$=\dfrac{-72-81}{4}=\dfrac{-153}{4}$

$T(x)=\dfrac{3x^3-18x^2-72x+192}{4x^2+27x+18}$



still haven't found the mins & the I.P.

2410 Mills

$$T(x) = \frac{3(x+4)(x-2)(x-8)}{(4x+3)(x+6)}$$

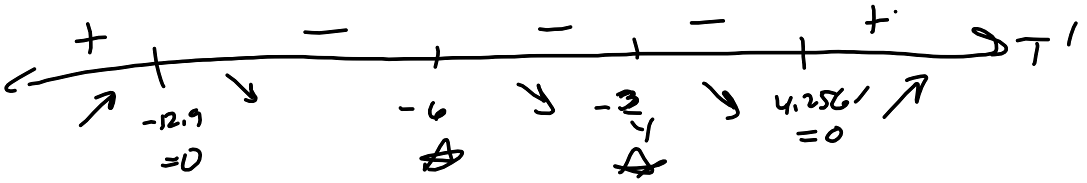
$$T'(x) = 3\left[\frac{\big((x-2)(x-8) + (x+4)(x-8) + (x+4)(x-2)\big)(4x+3)(x+6) - (x+4)(x-2)(x-8)\big(4(x+6)+(4x+3)\big)}{(\quad)^2}\right]$$

$$= 3\Big[(2x+2)(x-8)$$

Wolfram Alpha ?

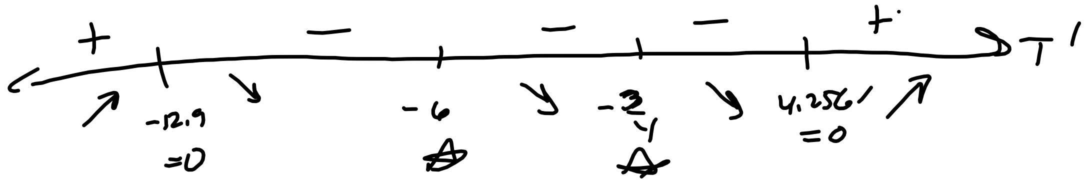
$$T'(x) = \frac{6(2x^4+27x^3-6x^2-364x-1080)}{((x+6)(4x+3))^2} \overset{SET}{=} 0$$

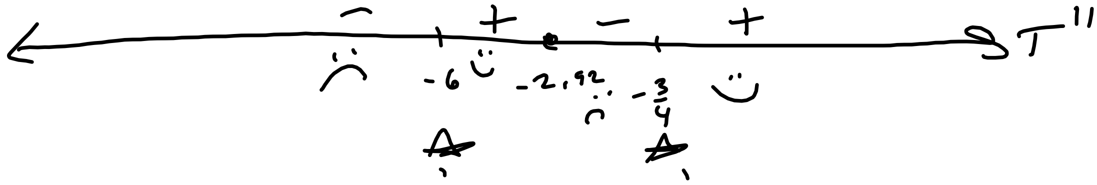
$x \approx -12.889, 4.2561$

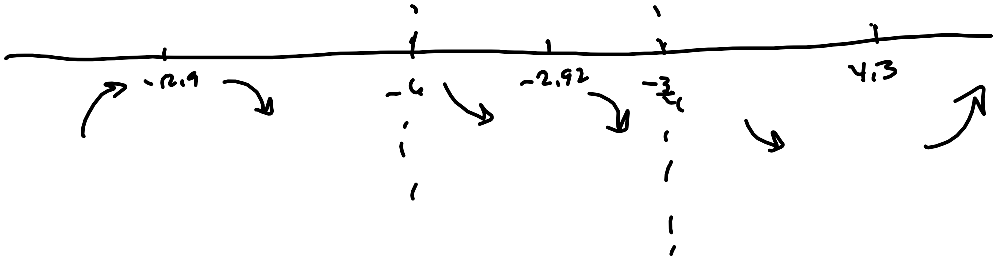


$$T''(x) = \frac{18(387x^3+1942x^2+8964x+17256)}{(4x+3)^3(x+6)^3} \overset{SET}{=} 0$$

$\Longrightarrow x \approx -2.9193$ **Done with Wolfram Alpha**







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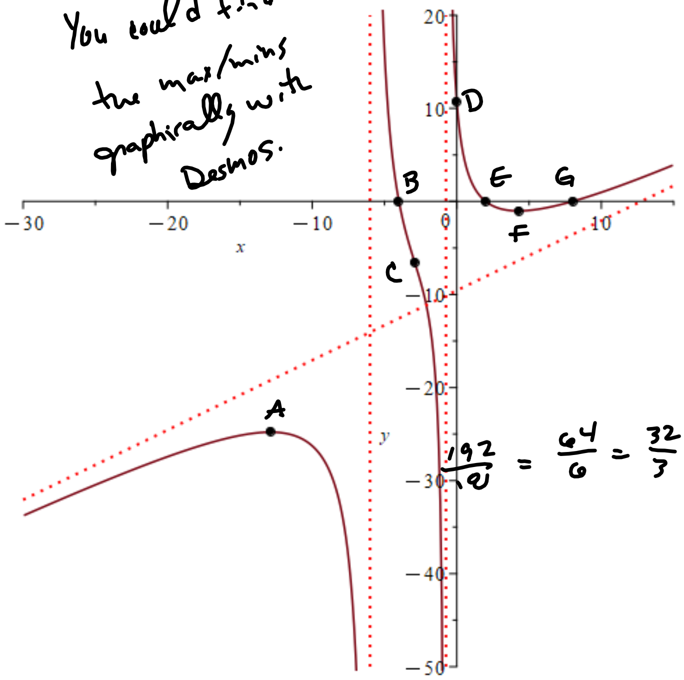
You could find the max/mins graphically with Desmos.



$\frac{192}{18} = \frac{64}{6} = \frac{32}{3}$

$A \approx (-12.889, -24.795)$ MAX

$B = (-4, 0)$ x-int

$C \approx (-2.919, -6.515)$ IP

$D = (0, \frac{32}{3}) = (0, 10.\overline{6})$ y-int

$E = (2, 0)$ x-int

$F \approx (4.256, -1.019)$ MIN

$G = (8, 0)$ x-int

2410 Mills

5 5pts Let P = profit in \$ as a function of
x = # of widgets produced be given by
$P(x) = -.01x^3 + x^2 - 3x + 300$. How many widgets will maximize the profit?

$P'(x) = -.03x^2 + 2x - 3 \overset{SET}{=} 0 \rightarrow$

$3x^2 - 200x - 300 = 0 \rightarrow$

$3\left(x^2 - \frac{200}{3}x + \left(\frac{100}{3}\right)^2\right) - 300\left(\frac{3}{3}\right) - 3\left(\frac{100^2}{3^2}\right)$

$3\left(x - \frac{100}{3}\right)^2 - \frac{900}{3} - \frac{10000}{3} = 3\left(x - \frac{100}{3}\right)^2 - \frac{10900}{3} \overset{SET}{=} 0$

$100 \overline{)10900}$ 109

$\rightarrow 3\left(x - \frac{100}{3}\right)^2 = \frac{10900}{3}$

$\rightarrow \left(x - \frac{100}{3}\right)^2 = \frac{10900}{9}$

$\rightarrow x = \frac{100 \pm \sqrt{10900}}{3} = \frac{100 \pm 10\sqrt{109}}{3}$

$\rightarrow x = \frac{100 + 10\sqrt{109}}{3} \approx 68.1343550297$

≈ 68 widgets

Profit of \$1574.89269165

$\approx P(68) \approx \$1574.89$

2410 Mills

6 we find the x-intercept, x_2, of the tangent line $L(x)$ to $f(x)$ @ the point $(x_1, f(x_1))$:

5pts

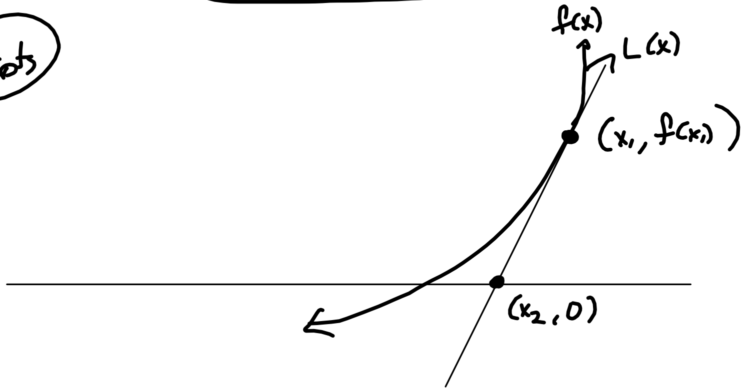
$$L(x) = f'(x_1)(x - x_1) + f(x_1) \overset{\text{SET}}{=} 0 \longrightarrow$$

$$f'(x_1)(x - x_1) = -f(x_1) \longrightarrow$$

$$x - x_1 = -\frac{f(x_1)}{f'(x_1)} \longrightarrow$$

$$\boxed{x = x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

7 5pts



2410 Mills

9 5pts Given $f(0)=f(1)=2$. Find $f(x)$ if $f''(x)=\sqrt[3]{x}-\cos(x)$

$f''(x)= x^{\frac{1}{3}} \longrightarrow$

$f'(x)=\frac{3}{4}x^{\frac{4}{3}}+C$

$f(x)=\frac{3}{4}\cdot\frac{3}{7}x^{\frac{7}{3}}+Cx+D$

$f(0)=\boxed{D=2}$

$f(1)=\frac{9}{28}(1)+C(1)+2=2 \Rightarrow$

$\frac{9}{28}+C=0 \Rightarrow$

$\boxed{C=-\frac{9}{28}}$

$$\boxed{\therefore\ f(x)=\frac{9}{28}x^{\frac{7}{3}}-\frac{9}{28}x+2}$$