MAT 2410 Week 4 Written Assignment
60 points Covers Section 2.1, 2.2
2.1 – Derivatives and Rate of Change.
2.2 – The Derivative as a Function

Be sure to follow College Algebra formatting guidelines in your work. They're the same for us as they are for College Algebra, except we're "2410" and not "1340," so "2410" in the top left corner of your work, not "1340."

1. Let $f(x)=\sqrt{x+2}$.

   a. (5 pts) Find the slope $m_{\tan}$ of the tangent line to f at $x=a>-2$.

   b. (5 pts) Find the quation of the tangent line (or "linearization") $y=L(x)=m_{\tan}(x-a)+f(a)$ to $f(x)$ at $a=2$.

   c. (5 pts) Sketch the graph of $f(x)$ and the tangent line you found in part b on the same set of axes.

   d. **Bonus** (5 pts) Use $L(x)$ to approximate $f(2.1)$.

2. Assume the function $f(x)$ is differentiable at the point $P(3,2)=P(3,f(3))$ on the graph of $f(x)$.

   a. (5 pts) Write an expression for the slope m_{sec} of the secant line through the points P and $Q(x,f(x))$.

   b. (5 pts) Write an expression for the slope $m_{\tan}=f'(3)$ of the tangent line through P.

   c. (5 pts) Write an expression for the equation of the tangent line at P for $f(x)$.

3. The cost (in dollars) of producing x units of a certain commodity is $C(x)=0.1x^2+11x+7000$.

   a. (5 pts) Find the average rate of change (in \$/unit) of C with respect to x when the production level is changed from x = 100 to x = 101

   b. (5 pts) Find the instantaneous rate of change (in \$ per unit) of C with respect to x when x = 100.

   c. (5 pts) The *marginal cost* of producing x units of a product is defined to be the cost of producing one additional unit of the product at a given production level x units produced. Based on your work, would you say that $C'(x)$ is a reasonable measurement of marginal cost? Why or why not?

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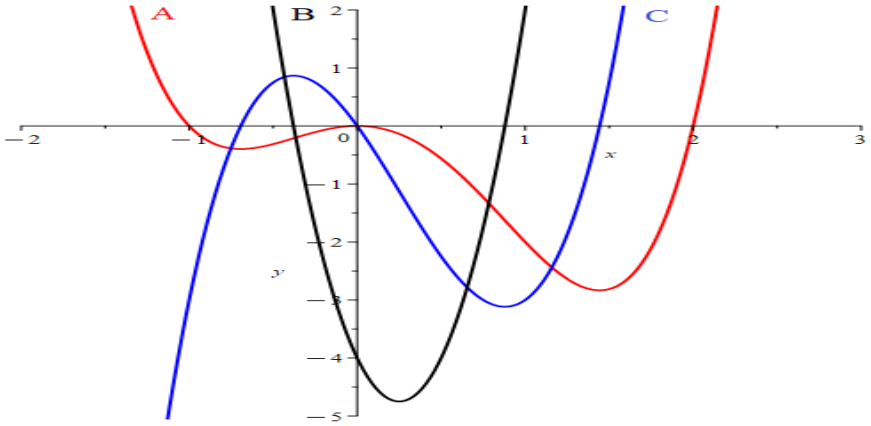
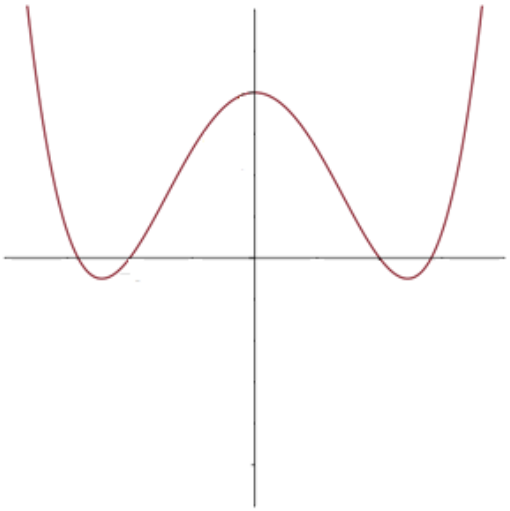
4. (5 pts) The graphs of functions, *A*, *B*, and *C* are actually the graphs of a function *f* and its derivatives *f* ' and *f* ". Identify which of *A*, *B*, and *C* are *f*, *f* ', and *f* ". Explain your reasoning. On your homework sheet, you may copy-paste the graph, below, trace it, or do the best possible job you can of copying it, with its essential features intact.



5. (5 pts) Sketch the graph of f' based on the graph of *f*, below. You may copy-paste or trace the graph below, or give a decent hand sketch of it on your homework sheet.



6. (5 pts) Recall, the function *g*(*x*) from previous work:

$$g(x)=\begin{cases} x\sin\left(\dfrac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$$

We showed that *g* is a continuous function in last week's written assignment, after some fiddling about with the question statement, itself, thanks to terrible editing. Now, tell me whether or not $g'(0)$ exists. If it does, then what is it? If it doesn't, explain why it doesn't.

We will see in the sequel that $h(x)=\begin{cases} x^2\sin\left(\dfrac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ *is* differentiable at $x=0$.