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WRITTEN WORK WEEK 4
SOLNS

4. MILLS

① $f(x) = \sqrt{x+2}$

② (5pts) Find slope, m_{tan} , of the tangent line to f @ $x=2 \rightarrow -2$

$$\begin{aligned} \frac{f(2+h) - f(2)}{h} &= \left(\frac{\sqrt{2+h+2} - \sqrt{2+2}}{h} \right) \left(\frac{\sqrt{2+h+2} + \sqrt{2+2}}{\sqrt{2+h+2} + \sqrt{2+2}} \right) \\ &= \frac{1}{h} \left(\frac{2+h+2 - (2+2)}{\sqrt{2+h+2} + \sqrt{2+2}} \right) = \frac{h}{h(\sqrt{2+h+2} + \sqrt{2+2})} \\ &= \frac{1}{\sqrt{2+h+2} + \sqrt{2+2}} \quad \xrightarrow{h \rightarrow 0} \frac{1}{\sqrt{2+2} + \sqrt{2+2}} \end{aligned}$$

$$= \boxed{\frac{1}{2\sqrt{2+2}} = m_{\text{tan}} = f'(2)}$$

b) $L(x) = y = f'(2)(x-2) + f(2)$ is tangent line to f @ 2. Let $a=2$.

$$L(x) = f'(2)(x-2) + f(2)$$

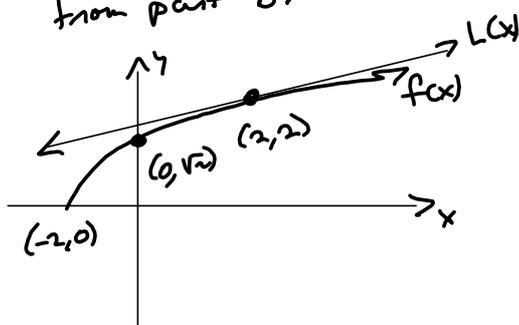
$$\left(\begin{array}{l} \text{Scratch: } f(2) = \sqrt{2+2} = \sqrt{4} = 2 = f(2) \\ f'(2) = \frac{1}{2\sqrt{2+2}} = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \boxed{\frac{1}{4} = f'(2)} \end{array} \right)$$

$$= \boxed{\frac{1}{4}(x-2) + 2 = L(x)}$$

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MILLS

(c) (5pts) We sketch $f(x)$ and the tangent line from part b.



(d) (2pts)

$$\begin{aligned}
 f(2.1) &\approx L(2.1) = \frac{1}{4}(2.1 - 2) + 2 \\
 &= \frac{1}{4}(.1) + 2 \\
 &= .25(.1) + 2 \\
 &= .025 + 2
 \end{aligned}$$

$$\begin{aligned}
 &= 2.025 = L(2.1) \approx f(2.1) \\
 &\approx \sqrt{2.1}
 \end{aligned}$$

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- ③ The cost $C(x)$, in dollars,
of producing x units of a commodity, is given by

$$C(x) = 0.1x^2 + 11x + 7000$$

- ② (5pts) The average rate of change of cost, $C(x)$ on
the interval $[100, 101]$ is

$$\frac{C(101) - C(100)}{101 - 100} =$$

$$\frac{.1(101)^2 + 11(101) + 7000 - (.1(100)^2 + 11(100) + 7000)}{1}$$

$$= .1(101)^2 + 11(101) - .1(100)^2 - 11(100)$$

$$= .1(101^2 - 100^2) + 11(101 - 100)$$

$$= .1(101 + 100)(101 - 100) + 11$$

$$= .1(201)(1) + 11 = 20.1 + 11 = 31.1 \frac{\$}{\text{unit produced}}$$

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(b) 5pts Find $C'(100)$ = the instantaneous rate of change of cost of producing x units, when $x=100$.

$$\frac{C(x) - C(100)}{x - 100} = \frac{.1x^2 + 11x + 7000 - (.1(100)^2 + 11(100) + 7000)}{x - 100}$$

$$= \frac{.1x^2 + 11x - .1(100)^2 - 11(100)}{x - 100} = \frac{.1(x^2 - 100^2) + 11(x - 100)}{x - 100}$$

$$= \frac{.1(x - 100)(x + 100) + 11(x - 100)}{x - 100}$$

$$= \frac{(x - 100)(.1(x + 100) + 11)}{x - 100}$$

$$= .1(x + 100) + 11 \xrightarrow{x \rightarrow 100} .1(200) + 11 = 20 + 11 = 31 \left(\frac{\$}{\text{unit}} \right)$$

$= C'(100)$

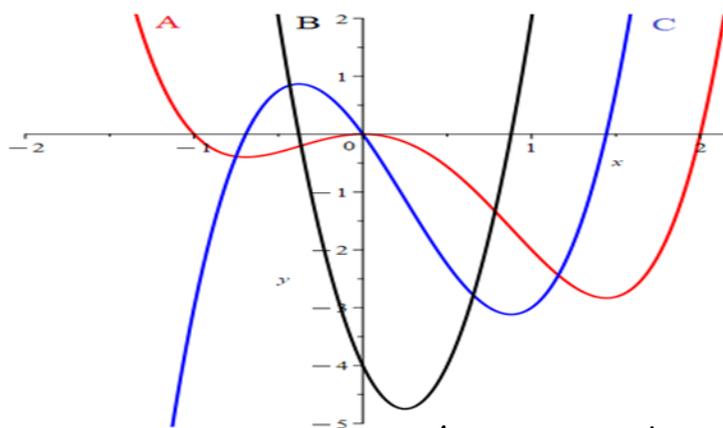
(c) 5pts

Since the average rate and the instantaneous rate are 31.1 \$/unit and 31 \$/unit, respectively, it would seem that $C'(x)$ is a reasonable approximation for marginal cost, and it's not unusual to see it be *the definition* of marginal cost.

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MILCS

(4) Spts The graphs of $A, B,$ & C are the graphs of $f, f',$ and f'' ,
not necessarily in that order



Just looking for zeros
in one, where the other
has a max/min...

$$C = A'$$

$$B = C'$$

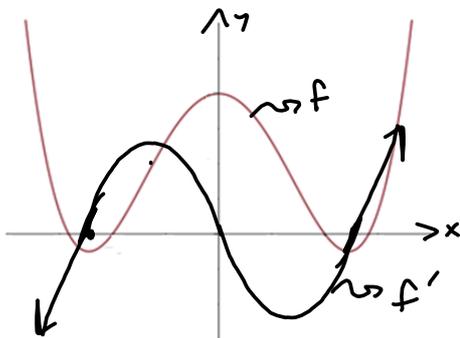
implying
 $A=f, C=f', B=f''$

Continuing

$A' < 0$	is when	$C < 0$	
$A' > 0$	" "	$C > 0$	✓
$C' > 0$	" "	$B > 0$	✓
$C' < 0$	" "	$B < 0$	

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 (5) (5pts) We sketch f' based on the graph of, below.

MILLS



(6) (5pts) Let $g(x) = \begin{cases} x \sin(\frac{\pi}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Then $g'(0) \nexists$.

$$\text{Proof } \lim_{h \rightarrow 0} \frac{g(0+h) - g(0)}{h} = \lim_{h \rightarrow 0} \frac{h \sin(\frac{\pi}{h}) - 0}{h} =$$

$= \lim_{h \rightarrow 0} \sin(\frac{\pi}{h}) \nexists$, by previous work.
 $\sin(\frac{\pi}{x})$ oscillates between $y=1$ & $y=-1$ infinitely many times on any interval containing $x=0$.