MAT 2410 Week 3 Written Assignment
25 points Covers Section 1.7, 1.8
1.7 – Precise definition of "limit."
1.8 – Continuity and continuous functions.

Be sure to follow College Algebra formatting guidelines in your work. They're the same for us as they are for College Algebra, except we're "2410" and not "1340," so "2410" in the top left corner of your work, not "1340."

Based on early returns, students are not leaving large enough margins left, right, top and bottom. The header I ask for at least leaves a little room at the top, but you *want* room within and before and after every problem, for my annotations. This makes my assessments (annotations) a better formative exercise (helping you improve) rather than just a summative (for points) assessment.

I'm all about formative assessments. The summative assessment is necessary, but not nearly as much fun for you or for me!

1. (5 pts) Suppose we're making water tanks in the shape of a cube. We want the containers to weigh 30 kg when full. If 1 kg of water is 1000 cubic centimeters (cm), then 30 kg of water is 30,000 cubic cm $\left(\text{cm}^3\right)$. Thus, a cube of water that weighs 30 kg will have sides of length $\sqrt[3]{30000}$ cm.

   If we want to be within 10 cm^3 (basically, 10 millileters) of the target volume, how precise to we need to be, i.e., how close to $\sqrt[3]{30000}$ cm do the sides have to be, in order to keep the volume within 10 cm^3 of the 30,000 cm^3 we want? Round your answer to 5 decimal places. Round DOWN to be safe, i.e., 10.23569 rounds to 10.2356. We always want the δ to *guarantee* we don't slop out the top or bottom of the "$\varepsilon -$tube."

2. (5 pts) Prove that $\lim_{x\to 3}(5x+2)=17$, using the *formal definition of limit*.

3. (5 pts) Prove that $\lim_{x\to 3}(x^2+5x-7)=17$.

4. (5 pts) Is $f(x)=\begin{cases} x^2-4x & \text{if } x<3 \\ -2x+4 & \text{if } x\geq 3 \end{cases}$ a continuous function? Explain.

5. (5 pts) Prove $f(x)=x\sin\left(\frac{\pi}{x}\right)$ has a *removable discontinuity* at $x = 0$, namely,

   $g(x)=\begin{cases} x\sin\left(\frac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ is a continuous function that agrees with f everywhere except at one point, but by filling in that one point, we remove the hole in f. This is not an $\varepsilon-\delta$ proof. Instead, you may use facts about continuity of polynomials, rational functions, and trigonometric functions for most of it. The trick part will require the application of **The Squeeze Theorm**.