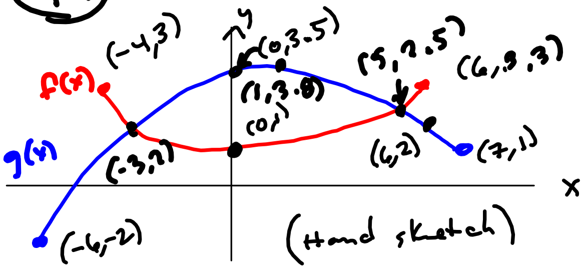
1 (5pts)



(a) $f(0) = 1$

(b) $g(6) = 2$

(c) $g(4) > f(4)$
g(4) is larger than f(4)

(d) $f(x) = g(x)$ @
$x = -3$ and $x = 6$

(e) $f(x) < g(x)$ on $(-3, 5)$

(f) g is increasing on $[-6, 1]$

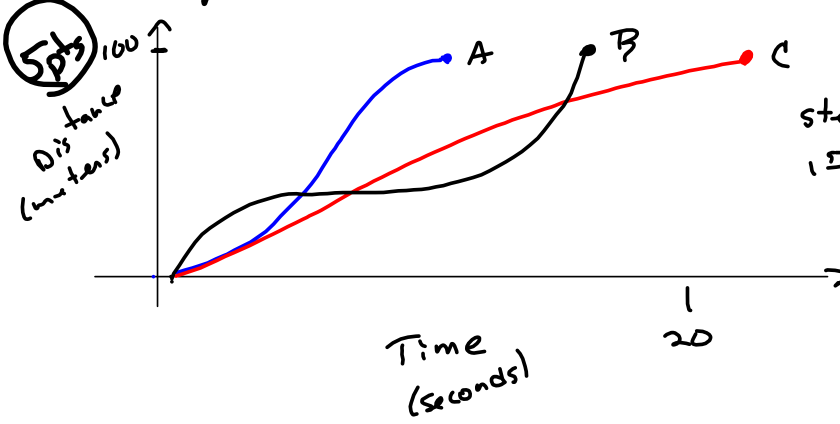
(g) g is decreasing on $[1, 7]$

(h) $\mathcal{D}(f)$ = Domain of f $= \{x \mid f(x) \text{ is defined}\}$
$= [-4, 6] = \mathcal{D}(f)$
$\mathcal{R}(f) = [1, 3.3]$ = Range of f $= \{y \mid y = f(x) \text{ for some } x \in \mathcal{D}(f)\}$

(i) $\mathcal{D}(g) = [-6, 7]$ & $\mathcal{R}(g) = [-2, 3.8]$. (ish)

2) The graph describes distance as a function of time.

5pts



Runner A ran a steady race, and finished 1^{st}.

Runner B was fastest off the blocks, but must have fallen, b/c he stopped moving for a while, finishing 2^{nd}

Runner C was slow, and finished last, in over 20 seconds.

3) $f(x) = x^2 - 3x + 2$

a) 2.5 pts

$$\frac{f(2+h)-f(2)}{h} = \frac{(2+h)^2 - 3(2+h) + 2 - (2^2 - 3(2) + 2)}{h}$$

$$= \frac{4 + 4h + h^2 - 6 - 3h + 2 - 4 + 6 - 2}{h}$$

$$= \frac{4h + h^2 - 3h}{h} = \frac{h + h^2}{h} = 1 + h \xrightarrow{h \to 0} 1$$

$(h \neq 0)$

b) 2.5 pts

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - 3(x+h) + 2 - (x^2 - 3x + 2)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h} = \frac{2xh + h^2 - 3h}{h}$$

$$= 2x + h - 3 \xrightarrow{h \to 0} \boxed{2x - 3 = f'(x)}$$

$(h \neq 0)$

2410 Mills, Harry

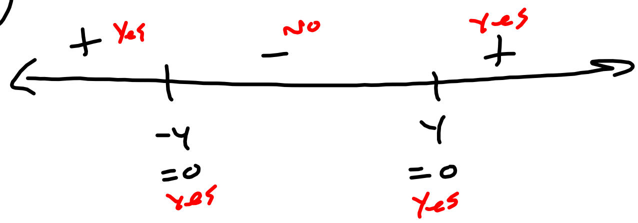
(4) $f(x) = \frac{x-3}{x^3-27} = \frac{x-3}{(x-3)(x^2+3x+9)} = \frac{1}{x^2+3x+9}$ $(x \neq 3)$

$\mathcal{D}(f) = \mathbb{R} \setminus \{3\}$

x^3-27 has root $\sqrt[3]{27} = 3$

(5) $f(x) = \sqrt{x^2-16} \longrightarrow$

2.5 pts (a) $\mathcal{D}(f) = \{x \mid x^2-16 \geq 0\} = \{x \mid (x-4)(x+4) \geq 0\}$



want ≥ 0, i.e., "+" or "= 0"

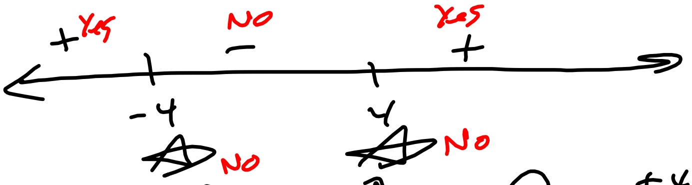
Sign pattern for x^2-16

$\Rightarrow \mathcal{D}(f) = (-\infty, -4] \cup [4, \infty)$

(b) 2.5 pts $g(x) = \frac{1}{\sqrt{x^2-16}} \longrightarrow \mathcal{D}(g)$ is same as $\mathcal{D}(f)$, almost.

Not quite, b/c we also need $x^2-16 \neq 0$, i.e.,

$x^2-16 > 0$:



Blows up @ $x = \pm 4$

$\therefore \boxed{\mathcal{D}(g) = (-\infty, -4) \cup (4, \infty)}$

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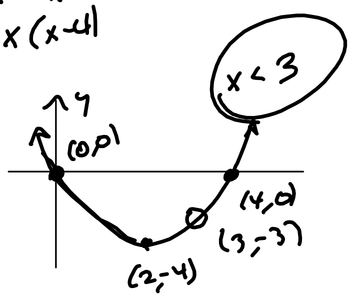
MILLS, HARRY

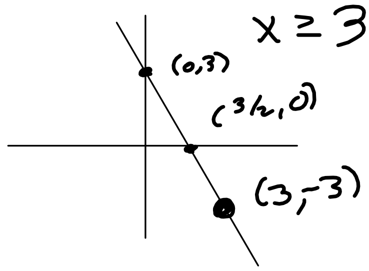
(6) we sketch $f(x) = \begin{cases} x^2-4x & \text{if } x<3 \\ -2x+3 & \text{if } x \geq 3 \end{cases}$

x^2-4x
$= x(x-4)$

$y=-2x+3$

x	y
0	3
3/2	0



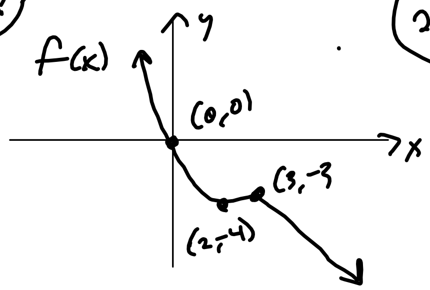


$-2(3)+3=-3$

$f(2)= 2^2-4(2)=4-8=-4$

$f(3)= 3^2-4(3) = 9-12=-3$

(a) 2.5 pts



(b) cool! they connect @ the suture point x=3.!
So f is cts!

2.5 pts

(7) Find a linear equation that models temperature T (in degrees Fahrenheit, as a function of N = the # of times a cricket chirps per minute if

$(N_1, T_1) = (115, 72)$ & $(N_2, T_2) = (180, 81)$, i.e.,

115 Chirps/min $\rightsquigarrow 72^\circ$ &

180 Chirps/min $\rightsquigarrow 81^\circ$

Lines are easier w/ x's & y's:

$y = T$

$x = N$

5pts

$(x_1, y_1) = (N_1, T_1) = (115, 72)$

$(x_2, y_2) = (N_2, T_2) = (180, 81)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{81 - 72}{180 - 115} = \frac{9}{65} = m \longrightarrow$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{9}{65}(x - 115) + 72 \text{, i.e.,}$$

$$\boxed{T = \frac{9}{65}(N - 115) + 72}$$

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(8) To graph $-3f(-2x-14)+11$ from $f(x)$,
I'd factor out the "-2" inside, and get
$-3f(-2(x+7))+11$ and I would:

5 pts

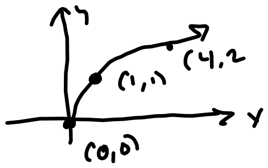
Flip vertically and stretch vertically by a factor of -3, i.e., $y \longmapsto -3y$, $(-3f(x))$

Then I'd shrink towards y-axis by a factor of 2 and flip about the y-axis $(-3f(-2x))$. $x \mapsto -\frac{1}{2}x$

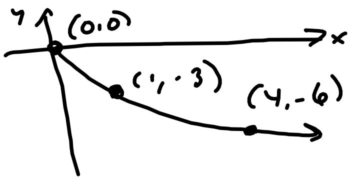
Then I'd shift left 7: $(-3f(-2(x+7)))$. $x \mapsto x-7$

Finally, I'd move it all up 11 units. $y \mapsto y+11$

(0) $f(x)=\sqrt{x}$



(1) $\therefore f(x)=-3\sqrt{x}$ $y \mapsto -$



(2) $-3f(-2x)=-3\sqrt{-2x}$ $x \mapsto -\frac{1}{2}x$

NOT ASKED.

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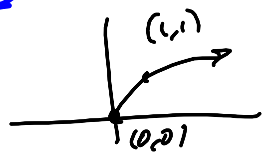
(9) $g(x) = -2\sqrt{5x-20} + 8 = -2\sqrt{5(x-4)} + 8$

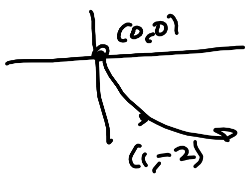
5pts

(1) $f(x) = \sqrt{x}$



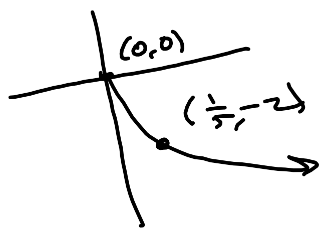
(2) $-2f(x)$

$= -2\sqrt{x}$



(3) $-2f(5x)$

$= -2\sqrt{5x}$

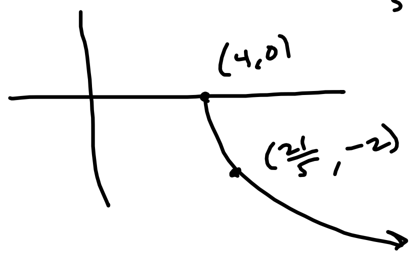


(4) $-2f(5(x-4))$

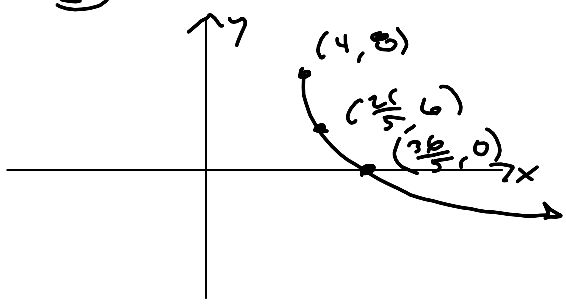
$-2\sqrt{5(x-4)}$

$x \mapsto x+4$

$\frac{1}{5} + 4 = \frac{1+20}{5} = \frac{21}{5}$



(5) $g(x) = -2f(5(x-4)) + 8$



x-int:

$g(x) = 0$

$-2\sqrt{5x-20} + 8 = 0$

$-2\sqrt{5x-20} = -8$

$\sqrt{5x-20} = 4$

$5x - 20 = 4^2 = 16$

$5x = 36$

$x = \frac{36}{5} \rightsquigarrow \left(\frac{36}{5}, 0\right)$

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MILLS, HARRY

(10) $f(x) = x^2 - 2x - 3$ & $g(x) = 2x^2 - 3x - 5$ $\longrightarrow$

(a) $(f+g)(x) = f(x) + g(x) = x^2 - 2x - 3 + 2x^2 - 3x - 5$

2.5 pts

$= \boxed{3x^2 - 5x - 8 = f(x) + g(x)}$ $\boxed{\mathcal{D}(f+g) = \mathbb{R}}$

(b) $R(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 - 2x - 3}{2x^2 - 3x - 5} = \dfrac{(x-3)(x+1)}{(2x-5)(x+1)} = \dfrac{x-3}{2x-5}$ $(x \neq -1)$

2.5 pts

$\mathcal{D}(R) = \mathbb{R} \setminus \{\frac{5}{2}, -1\} = (-\infty, -1) \cup (-1, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

$= \mathbb{R} \setminus (\{-1\} \cup \{\frac{5}{2}\}) = \{x \mid x \neq \frac{5}{2} \text{ and } x \neq -1\} = \mathbb{R} \setminus \{-1\} \cap \mathbb{R} \setminus \{\frac{5}{2}\}$

$= \{x \mid x \neq \frac{5}{2} \text{ or } -1\} = \{x \mid x \notin \{-1, \frac{5}{2}\}\}$

$= \{x \mid x \notin \{-1\} \cup \{\frac{5}{2}\}\}$