MAT 2410-R11 Written Midterm, Spring, 2025 (5 pts) Name _____
125 Points Covers Chapters 1 – 3

Note to the Proctor: Please scan the written pages and the front page of this test (with their name on it).

Please make sure that the student turns in their cheat sheet and all paper with any writing on it. I want to see all of it.

Do all your work and circle all your final answers on the blank paper provided. Do your own work.

You don't need to write out the questions, but you do need to write out your answers as completely as possible.

Spend no more than 5 minutes on any one problem before moving on to the next. If you get stuck, start the next problem on a fresh sheet of paper.

Be sure to submit all the problems in the order in which they appear on the test, itself. If you don't, that's 10% off the top.

Remember that partial credit is awarded liberally. Final answers are important, but most of the points are in the supporting work.

1. Evaluate the following limits, if they exist. If one does not exist, explain why.
   a. (5 pts) $\lim\limits_{x\to 2^-} \dfrac{x^2-7x+10}{|x-2|}$
   b. (5 pts) $\lim\limits_{x\to 2^+} \dfrac{x^2-7x+10}{|x-2|}$
   c. (5 pts) $\lim\limits_{x\to 2} \dfrac{x^2-7x+10}{|x-2|}$

2. Consider the piecewise-defined function $f(x)=\begin{cases} x^2+3x-18 & \text{if } x<4 \\ 2x+2 & \text{if } x\geq 4 \end{cases}$.
   a. (5 pts) Sketch the graph of $f(x)$. Label the *x*- and *y*-intercepts, the suture point(s), and the vertex of the quadratic piece, if it's in the picture. When I say "Label," I mean an ordered pair, like (0, 5), next to the point.
   b. (5 pts) On what interval(s) is $f(x)$ continuous? Explain.
   c. (5 pts) On what interval(s) is $f(x)$ differentiable? Explain.

3. (5 pts) Simplify the limit of the difference quotient $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-3x-5$.

4. The point $P(4,10)$ lies on the graph of $f(x)=x^2+3x-18$.

a. (5 pts) Write the equation of the tangent line $L_4(x)$ to $f(x)$ at $x = 4$.

b. (5 pts) Sketch a graph of $f(x)$ and $L_4(x)$ on the same set of coordinate axes.

5. (5 pts) Prove that $\lim_{x\to 5}(2x-3) = 7$, using the $\varepsilon - \delta$ definition of limit.

6. (5 pts) Prove that the equation $f(x) = x^4 - 9x^3 + 13x^2 + 9x - 14$ has a root in the interval $(0,8)$, but *do not solve*!

7. Differentiate the following with respect to the indicated independent variable. **Do not simplify!**

a. (5 pts) $f(x) = \sqrt[4]{x^3} - 5x^{\frac{4}{7}} - \dfrac{11}{x^{2/3}}$; x.

b. (5 pts) $g(x) = \tan(4x)\csc(2x)$; x.

c. (5 pts) $h(\beta) = \dfrac{\tan\theta}{\theta^3 + 2\theta}$; θ.

d. (5 pts) $r(w) = (w^2 + 11w + 5)^4 (\cot(2w))^3$; w.

8. Consider the relation $\sin(x+y) = 2x - 2y$.

a. (5 pts) Use implicit differentiation to find $y' = \dfrac{dy}{dx}$

b. (5 pts) Find an equation of the tangent line to the curve at the point (π, π).

9. (10 pts) A street light is mounted on top of a 15-foot pole. A man who is 6 feet tall is walking away from the pole at a rate of 5 feet per second on a straight path. At what rate is the tip of his shadow moving when he is 40 feet from the pole?

10. (10 pts) If the minute hand of a clock has length r (in centimeters), find the rate at which it sweeps out area as a function of r.

11. The radius of a sphere was measured to be 2 cm, with a possible error of 0.1 cm.

a. (5 pts) Use a differential to estimate the error in the calculation of the surface area of the sphere.
Hint: The surface area of a sphere is $4\pi r^2$.

b. (5 pts) Use a differential to estimate the error in the calculation of the volume of the sphere.
Hint: The volume of a sphere is $\dfrac{4}{3}\pi r^3$

BONUS:

1. (5 pts) Prove that $\lim_{x\to 2}\left(2x^2-3x-1\right)=1$, using the $\varepsilon-\delta$ definition of limit.

2. (5 pts) Compute the derivative of $f\left(x\right)=x^{\frac{1}{3}}$ by the limit definition of the derivative.

3. (5 pts) See if you can *squeeze* out a *convincing* argument to support the statement

$$f\left(x\right)=\begin{cases} x^2\sin\left(\dfrac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$$ is continuous on $\left(-\infty,\infty\right)$.