1 (a) 5pts $\lim_{x\to 5^-} \frac{(x-5)(x+3)}{|x-5|} = \lim_{x\to 5^-} f(x)$

$= \lim_{x\to 5^-} \frac{(x-5)(x+3)}{-(x-5)} = \lim_{x\to 5^-}(-(x+3)) = \boxed{-8 = \lim_{x\to 5^-} f}$

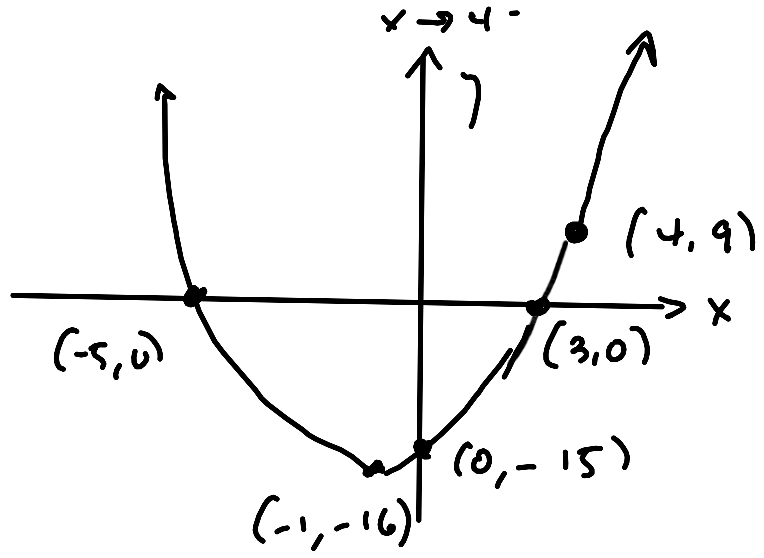
(b) 5pts $\lim_{x\to 5^+} \frac{(x-5)(x+3)}{|x-5|} = \lim_{x\to 5^+}(x+3) = \boxed{8 = \lim_{x\to 5^+} f}$

(c) 5pts $\lim_{x\to 5} \frac{(x-5)(x+3)}{|x-5|}$ $\boxed{\nexists \text{ b/c } \lim_{x\to 5^-} f \neq \lim_{x\to 5^+} f.}$

2 $f(x) = \begin{cases} x^2+2x-15 & \text{if } x<4 \\ 10x-31 & \text{if } x\geq 4 \end{cases}$

$(x+5)(x-3)$

$\lim_{x\to 4^-} f(x) = (9)(1) = 9$ $\qquad \lim_{x\to 4^+} f(x) = 40-31 = 9$




$(-1+5)(-1-3) = 4(-4) = -16$

(b) 5pts f is cnt's on each piece b/c they're polynomials.

$\lim_{x\to 4} f(x) = 9 = f(4)$ ⟶ cnt's at the suture point.

$\therefore$ f is cnt's $\forall x \in \mathbb{R}$

(b) 5pts f is difbl on both pieces. and

$f'(x) = \begin{cases} 2x+2 & \text{if } x<4 \\ 10 & \text{if } x>4 \end{cases}$

we need to check the suture point:

$2(4)+2 = 10 =$

$\lim_{h\to 0^-} \frac{f(4+h)-f(4)}{h} = 2(4)+2 = 10$

$\lim_{h\to 0^+} \frac{f(4+h)-f(4)}{h} = 10$

These limits agree ⟶ f is difbl @ x=4

⟶ $\boxed{f \text{ difbl } \forall x \in \mathbb{R}}$

3) 5 pts $f(x) = x^2 - 5x + 8 \longrightarrow$

$$\frac{f(x) - f(c)}{x-c} = \frac{x^2 - 5x + 8 - c^2 + 5c - 8}{x-c} = \frac{x^2 - c^2 - 5(x-c)}{x-c}$$

$$= \frac{(x-c)(x+c) - 5(x-c)}{x-c} = \frac{(x-c)(x+c-5)}{(x-c)} \xrightarrow{x \to c} \boxed{2c - 5 = f'(c)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 8 - x^2 + 5x - 8}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$= \frac{2xh + h^2 - 5h}{h} = \frac{h(2x + h - 5)}{h} \xrightarrow{h \to 0} \boxed{2x - 5 = f'(x)}$$

4) $P(5, -14)$ is on $f(x) = x^2 - 5x - 14$

a) 5 pts $f'(x) = 2x - 5 \longrightarrow f'(5) = 2(5) - 5 = 5 = f'(5)$

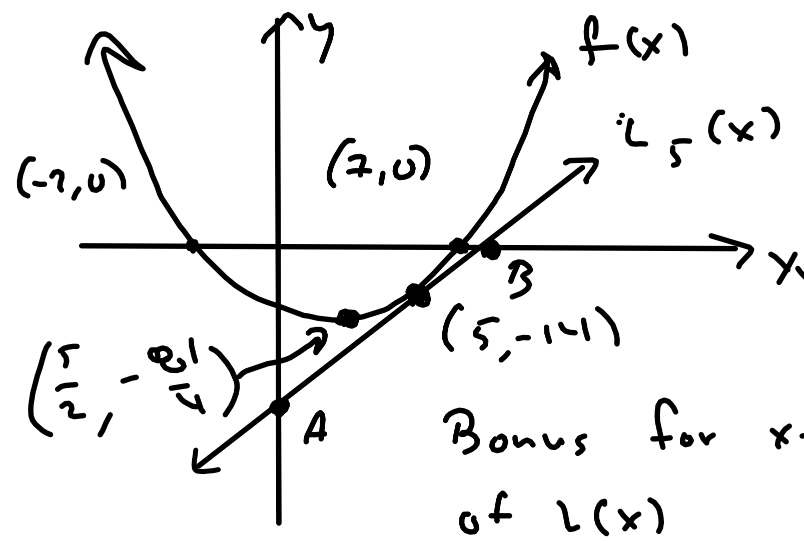
$$\longrightarrow L_5(x) = f'(5)(x-5) + f(5)$$

$$= \boxed{5(x-5) - 14 = L_5(x)}$$

b) 5 pts

$$x^2 - 5x - 14 = (x-7)(x+2) = x^2 - 5x + \left(\frac{5}{2}\right)^2 - \frac{25}{4} - \frac{14}{1} \cdot \frac{4}{4}$$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{81}{4}$$




$$\boxed{\begin{array}{l} A = (0, -39) \\ B = \left(\frac{39}{5}, 0\right) \end{array}}$$

Bonus for x- & y-ints of $L(x)$

$$y = 5(x-5) - 14 = 0 \longrightarrow$$

$$5(x-5) = 14$$

$$x - 5 = \frac{14}{5}$$

$$x = \frac{25 + 14}{5} = \frac{39}{5} \rightsquigarrow \boxed{\left(\frac{39}{5}, 0\right) = B}$$

$$y = 5(-5) - 14 = -25 - 14 = -39$$

$$\rightsquigarrow \boxed{(0, -39) = A}$$

5 5pts $\lim_{x \to 5} (-3x+6) = -9$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{3}$. Then, if $0 < |x-5| < \delta$,

$|-3x+6-(-9)| = |-3x+15| = |-3(x-5)| =$

$= 3|x-5| < 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon$ $\blacksquare$

6 5pts $f(x) = x^4 - 9x^3 + 21x^2 + x - 30$ has a zero in $(4,6)$

Proof

$f(4)$:

$$\begin{array}{r|rrrrr} 4 & 1 & -9 & 21 & 1 & -30 \\ & & 4 & -20 & 4 & 20 \\ \hline & 1 & -5 & 1 & 5 & -10 = f(4) \end{array}$$

$f(6)$

$$\begin{array}{r|rrrrr} 6 & 1 & -9 & 21 & 1 & -30 \\ & & 6 & -18 & 18 & 114 \\ \hline & 1 & -3 & 3 & 19 & 84 = f(6) \end{array}$$

Since $f$ is a polynomial, it's cnts on $(-\infty, \infty)$, and, since $-10 = f(4) < 0 < f(6) = 84$, $\exists\ c \in (4,6)$ $\ni f(c) = 0$, by the Intermediate-Value Theorem.

7 a 5pts $f(x) = x^{\frac{8}{5}} - 5x^{\frac{4}{7}} - 11x^{-\frac{2}{3}} \longrightarrow$

$$\boxed{f'(x) = \frac{8}{5}x^{\frac{3}{5}} - \frac{20}{7}x^{-\frac{3}{7}} + \frac{22}{3}x^{-\frac{5}{3}}}$$

b 5pts $g(x) = \sec(4x)\sin(4x) = \tan(4x)$

$$\longrightarrow \boxed{g'(x) = 4\sec^2(4x)}$$

OR $\boxed{g'(x) = 4\sec(4x)\tan(4x)\sin(4x) + \sec(4x)(4\cos(4x))}$

$$= 4\tan^2(4x) + 1 = 4\sec^2(4x)$$

c 5pts $h(\theta) = \frac{\cot(\theta)}{\cos\theta} = \csc\theta$

$$\longrightarrow \boxed{h'(\theta) = -\csc\theta\cot\theta}$$

OR $\boxed{h'(\theta) = \frac{-\csc^2\theta\cos\theta - \cot(\theta)(-\sin\theta)}{\cos^2\theta}}$

$$= \frac{-\csc^2\theta\cos\theta + \cos\theta}{\cos^2\theta} = \frac{-\cos\theta(\csc^2\theta - 1)}{\cos^2\theta}$$

$$= \frac{-\cot^2\theta}{\cos\theta} = -\frac{\cos\theta}{\sin^2\theta} = -\csc\theta\cot\theta$$

d 5pts $r(w) = (w^2+11w)^{-5}(w^3+w^{-1})^{13} \longrightarrow$

$$\boxed{r'(w) = -5(w^2+11w)^{-6}(w^3+w^{-1})^{13} + (w^2+11w)^{-5}\left(13(w^3+w^{-1})^{12}(2w+2)\right)}$$

e $G(\tau) = \sqrt{\sin\left(\frac{\pi}{6}\tau^3\right)} \longrightarrow$

$$\boxed{G'(\tau) = \frac{1}{2}\left(\sin\left(\frac{\pi}{6}\tau^3\right)\right)^{-\frac{1}{2}}\left(\cos\left(\frac{\pi}{6}\tau^3\right)\right)\left(\frac{3\pi}{6}\tau^2\right)}$$

8) $\sin(x^2+y^2) = \sqrt{\frac{2}{\pi}}\, x + 2y$

a) 5pts $(\cos(x^2+y^2))(2x + 2yy') = \sqrt{\frac{2}{\pi}} + 2y'$

$\Longrightarrow 2x\cos(x^2+y^2) + 2yy'\cos(x^2+y^2 = \sqrt{\frac{2}{\pi}} + 2y'$

$\Longrightarrow 2yy'\cos(x^2+y^2) - 2y'' = \sqrt{\frac{2}{\pi}} - 2x\cos(x^2+y^2)$

$\Longrightarrow \boxed{y' = \dfrac{\sqrt{\frac{2}{\pi}} - 2x\cos(x^2+y^2)}{2y\cos(x^2+y^2) - 2}}$

b) 5pts $(x_0, y_0) = \left(\sqrt{\frac{\pi}{2}}, 0\right)$

$$y'\Big|_{(x_0,y_0)} = \frac{\sqrt{\frac{2}{\pi}} - 2\sqrt{\frac{\pi}{2}}\cos\left(\sqrt{\frac{\pi}{2}}^2 + 0^2\right)}{2(0)\cos\left(\sqrt{\frac{\pi}{2}}^2 + 0^2\right) - 2} = \frac{\sqrt{\frac{2}{\pi}}}{-2} = -\frac{1}{2}\sqrt{\frac{2}{\pi}}$$

$\Longrightarrow \boxed{L(x) = -\frac{1}{2}\sqrt{\frac{2}{\pi}}\left(x - \sqrt{\frac{\pi}{2}}\right)}$

8 $\sin(x^2+y^2) = x+2y \longrightarrow$

a $\cos(x^2+y^2)(2x+2yy') = 1+2y' \longrightarrow$

5pts $2x\cos(x^2+y^2)+(2y\cos(x^2+y^2))y' = 1+2y'$

$\rightarrow (2y\cos(x^2+y^2)-2)y' = 1-2x\cos(x^2+y^2)$

$$\rightarrow \boxed{y' = \frac{1-2x\cos(x^2+y^2)}{2y\cos(x^2+y^2)-2}}$$

$x^2 = \frac{\pi}{2}$

$x = \frac{\sqrt{2}\sqrt{\pi}}{2} = \frac{\sqrt{2\pi}}{2}$

$\sin\left(\left(\frac{\sqrt{2\pi}}{2}\right)^2+0^2\right) \overset{?}{=} \sqrt{\frac{2}{\pi}}\left(\sqrt{\frac{\pi}{2}}\right)+0$

$1 = 1$ ✓

$ax+2y$

$x = \frac{\sqrt{2\pi}}{2} = \sqrt{\frac{\pi}{2}}$

$ax = 1$

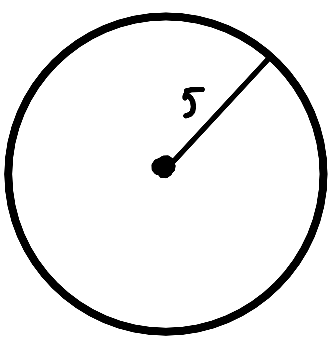
$x = \frac{1}{a} \rightarrow a = \frac{1}{x}$, duh

$\rightarrow a = \frac{\sqrt{2}}{\sqrt{\pi}}$

9) 10 pts

$\text{Area} = \frac{1}{2} r^2 \theta$, $r$ = radius, $\theta$ = angle

$t$ = time, in hours



$\longrightarrow \frac{d\,\text{Area}}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$ ($r = 5$ is constant)

$r = 5 \longrightarrow \frac{d\,\text{Area}}{dt} = \frac{1}{2}(5^2)\frac{d\theta}{dt}$

For angular velocity, $\frac{d\theta}{dt}$, we have

$$\left(\frac{1 \text{ rev}}{\text{min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right) = 120\pi \frac{\text{radians}}{\text{hr}} \longrightarrow$$

$$\frac{d\,\text{Area}}{dt} = \left(\frac{25}{2}\right)(120\pi) = \boxed{1250\pi \frac{\text{cm}^2}{\text{hr}}}$$

(10) Diameter $= 50$ ft $\Rightarrow$ radius $= 25$ ft

Radius increases from 25 ft to 25 ft $+ (.005 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)$

r = radius in ft.

$$((.005) \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = \frac{.005}{12} = \Delta r = dr$$

$$\left(\frac{5}{1000}\right)\left(\frac{1}{12}\right) = \left(\frac{1}{200}\right)\left(\frac{1}{12}\right) = \frac{1}{2400}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow$$

$$\Delta V \approx dV = 4\pi r^2 dr = 4\pi (25)^2\left(\frac{.005}{12}\right)$$

$$= 4(625)\left(\frac{1}{2400}\right)\pi$$

$$= \pi \frac{625}{600} = \frac{25}{24}\pi = \frac{25\pi}{24} \text{ ft}^3$$

$$= \left(\frac{25\pi}{24}\right)(\text{ft}^3)\left(7.48052 \frac{\text{gallons}}{\text{ft}^3}\right) \approx 24.47994446 \text{ gallons.}$$

$$\approx \boxed{24.4799 \text{ gallons}}$$

(b) 5pts $\Delta V = V(25 + \Delta r) - V(25)$

$$= V\left(25 + \frac{.005}{12}\right) - V(25)$$

$$\approx 65453.11951 - 65449.84695$$

$$\approx (3.27254690 \text{ ft}^3)\left(7.48052 \frac{\text{gal}}{\text{ft}^3}\right) \approx 24.48035246$$

B1 5pts $\lim_{x\to 3}(3x^2-2x-1)=20$

Scratch

want $|3x^2-2x-1-20|<\varepsilon$

$|3x^2-2x-21| = |3x+7||x-3|$

Assume $\delta \le 1$

Then $2 < x < 4$

$6 < 3x < 12$

$13 < 3x+7 < 19$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{19}\}$. Then

$0<|x-3|<\delta \Longrightarrow |3x^2-2x-1-20| = |3x^2-2x-21|$

$= |3x+7||x-3| < 19|x-3| < 19\delta \le 19\cdot\frac{\varepsilon}{19} = \varepsilon$ ∎

B2 5pts $\left(\frac{x^{\frac{2}{3}}-c^{\frac{2}{3}}}{x-c}\right)\left(\frac{x^{\frac{4}{3}}+x^{\frac{2}{3}}c^{\frac{2}{3}}+c^{\frac{4}{3}}}{x^{\frac{4}{3}}+x^{\frac{2}{3}}c^{\frac{2}{3}}+c^{\frac{4}{3}}}\right)$

$= \frac{x^2-c^2}{(x-c)(x^{\frac{4}{3}}+x^{\frac{2}{3}}c^{\frac{2}{3}}+c^{\frac{4}{3}})} = \frac{\cancel{(x-c)}(x+c)}{\cancel{(x-c)}(x^{\frac{4}{3}}+x^{2/3}c^{2/3}+c^{4/3})}$

$\xrightarrow{x\to c} \frac{c+c}{c^{4/3}+c^{4/3}+c^{4/3}} = \frac{2c}{3c^{4/3}} = \boxed{\frac{2}{3}c^{-\frac{1}{3}} = f'(c)}$

B3 5 pts $f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is difbl on $(-\infty, \infty)$

Pf Clearly, $f$ is difbl $\forall x \neq 0$.

Now $\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \frac{h^2 \sin\left(\frac{\pi}{h}\right) - 0}{h}$

$= h \sin\left(\frac{\pi}{h}\right) \xrightarrow{h \to 0} 0$, since
$(h \neq 0)$

$-1 \leq \sin\left(\frac{\pi}{h}\right) \leq 1 \quad \forall h \neq 0$

Now, $h > 0 \longrightarrow$
$-h \leq h \sin\left(\frac{\pi}{h}\right) \leq h$

SQUEEZE THEOREM

& $h < 0 \longrightarrow$
$-h \geq h \sin\left(\frac{\pi}{h}\right) \geq h$ is the same bound on $h \sin\left(\frac{\pi}{h}\right)$. As $h \to 0$, we obtain
$h \sin\left(\frac{\pi}{h}\right) \longrightarrow 0$ and so
$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$ $\exists$ and we're done