1 a 5pts $\lim_{x\to5^-} \frac{(x-5)(x+3)}{|x-5|} = \lim_{x\to5^-} f(x)$

$= \lim_{x\to5^-} \frac{(x-5)(x+3)}{-(x-5)} = \lim_{x\to5^-}(-(x+3)) = \boxed{-8 = \lim_{x\to5^-} f}$

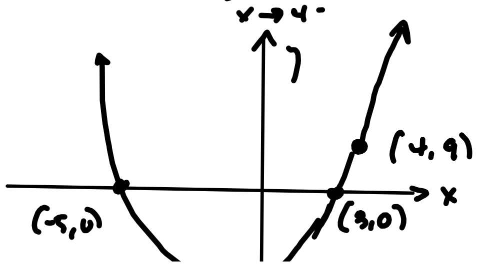
b 5pts $\lim_{x\to5^+} \frac{(x-5)(x+3)}{|x-5|} = \lim_{x\to5^+}(x+3) = \boxed{8 = \lim_{x\to5^+} f}$

c 5pts $\lim_{x\to5} \frac{(x-5)(x+3)}{|x-5|}$ $\boxed{\nexists \text{ b/c } \lim_{x\to5^-} f \neq \lim_{x\to5^+} f.}$

2 $f(x) = \begin{cases} x^2+2x-15 & \text{if } x<4 \\ 10x-31 & \text{if } x\geq 4 \end{cases}$

$(x+5)(x-3)$

$\lim_{x\to4^-} f(x) = (9)(1) = 9$ $\lim_{x\to4^+} f(x) = 40-31 = 9$



3) 5pts $f(x) = x^2 - 5x + 8 \longrightarrow$

$$\frac{f(x) - f(c)}{x-c} = \frac{x^2 - 5x + 8 - c^2 + 5c - 8}{x-c} = \frac{x^2 - c^2 - 5(x-c)}{x-c}$$

$$= \frac{(x-c)(x+c) - 5(x-c)}{x-c} = \frac{(x-c)(x+c-5)}{(x-c)} \xrightarrow{x \to c} \boxed{2c - 5 = f'(c)}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5(x+h) + 8 - x^2 + 5x - 8}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$= \frac{2xh + h^2 - 5h}{h} = \frac{h(2x+h-5)}{h} \xrightarrow{h \to 0} \boxed{2x - 5 = f'(x)}$$

4) $P(5, -14)$ is on $f(x) = x^2 - 5x - 14$

a) 5pts $f'(x) = 2x - 5 \longrightarrow f'(5) = 2(5) - 5 = 5 = f'(5)$

$$\longrightarrow L_5(x) = f'(5)(x-5) + f(5)$$

$$= \boxed{5(x-5) - 14 = L_5(x)}$$

5 5pts $\lim_{x\to 5}(-3x+6) = -9$

Proof

Let $\varepsilon>0$ be given. Define $\delta = \frac{\varepsilon}{3}$. Then, if $0<|x-5|<\delta$,

$|-3x+6-(-9)| = |-3x+15| = |-3(x-5)| =$

$= 3|x-5| < 3\delta = 3\cdot\frac{\varepsilon}{3} = \varepsilon$ ■

6 5pts $f(x) = x^4-9x^3+21x^2+x-30$ has a zero in $(4,6)$

Proof

$f(4)$:

$$\begin{array}{r|rrrrr} 4 & 1 & -9 & 21 & 1 & -30 \\ & & 4 & -20 & 4 & 20 \\ \hline & 1 & -5 & 1 & 5 & -10 = f(4) \end{array}$$

$f(6)$

$$\begin{array}{r|rrrrr} 6 & 1 & -9 & 21 & 1 & -30 \\ & & 6 & -18 & 18 & 114 \\ \hline & 1 & -3 & 3 & 19 & 84 = f(6) \end{array}$$

Since f is a polynomial, it's cts on $(-\infty,\infty)$, and

Since

7 a 5pts $f(x) = x^{\frac{8}{5}} - 5x^{\frac{4}{7}} - 11x^{-\frac{2}{3}} \longrightarrow$

$$\boxed{f'(x) = \frac{8}{5}x^{\frac{3}{5}} - \frac{20}{7}x^{-\frac{3}{7}} + \frac{22}{3}x^{-\frac{5}{3}}}$$

b 5pts $g(x) = \sec(4x)\sin(4x) = \tan(4x)$

$$\longrightarrow \boxed{g'(x) = 4\sec^2(4x)}$$

or $\boxed{g'(x) = 4\sec(4x)\tan(4x)\sin(4x) + \sec(4x)(4\cos(4x))}$

$= 4\tan^2(4x) + 1 = 4\sec^2(4x)$ ✓

c 5pts $h(\theta) = \frac{\cot(\theta)}{\cos\theta} = \csc\theta$

$$\longrightarrow \boxed{h'(\theta) = -\csc\theta\cot\theta}$$

OR $\boxed{h'(\theta) = \frac{-\csc^2\theta\cos\theta - \cot(\theta)(-\sin\theta)}{\cos^2\theta}}$

$- \csc^2\theta\cos\theta + \cos\theta \quad -\cos\theta(\csc^2\theta - 1)$

8) $\sin(x^2+y^2) = \sqrt{\frac{2}{\pi}}\, x + 2y$

a) 5pts $$(\cos(x^2+y^2))(2x+2yy') = \sqrt{\frac{2}{\pi}} + 2y'$$

$$\Longrightarrow 2x\cos(x^2+y^2) + 2yy'\cos(x^2+y^2 = \sqrt{\frac{2}{\pi}} + 2y'$$

$$\Longrightarrow 2yy'\cos(x^2+y^2) - 2y' = \sqrt{\frac{2}{\pi}} - 2x\cos(x^2+y^2)$$

$$\Longrightarrow \boxed{y' = \frac{\sqrt{\frac{2}{\pi}} - 2x\cos(x^2+y^2)}{2y\cos(x^2+y^2) - 2}}$$

b) 5pts $(x_0, y_0) = \left(\sqrt{\frac{\pi}{2}}, 0\right)$

$$y'\Big|_{(x_0,y_0)} = \frac{\sqrt{\frac{2}{\pi}} - 2\sqrt{\frac{\pi}{2}}\cos\left(\sqrt{\frac{\pi}{2}}^2 + 0^2\right)}{2(0)\cos\left(\sqrt{\frac{\pi}{2}}^2 + 0^2\right) - 2} = \frac{\sqrt{\frac{2}{\pi}}}{-2} = -\frac{1}{2}\sqrt{\frac{2}{\pi}}$$

$$\Longrightarrow \boxed{L(x) = -\frac{1}{2}\sqrt{\frac{2}{\pi}}\left(x - \sqrt{\frac{\pi}{2}}\right)}$$

8 $\sin(x^2+y^2) = x+2y \longrightarrow$

a $\cos(x^2+y^2)(2x+2yy') = 1+2y' \longrightarrow$

5pts $2x\cos(x^2+y^2) + (2y\cos(x^2+y^2))y' = 1+2y'$

$\longrightarrow (2y\cos(x^2+y^2)-2)y' = 1-2x\cos(x^2+y^2)$

$\longrightarrow \boxed{y' = \dfrac{1-2x\cos(x^2+y^2)}{2y\cos(x^2+y^2)-2}}$

$x^2 = \frac{\pi}{2}$

$x = \frac{\sqrt{2}\sqrt{\pi}}{2} = \frac{\sqrt{2\pi}}{2}$

$2x+2y$

$x = \frac{\sqrt{2\pi}}{2} = \sqrt{\frac{\pi}{2}}$

$\sin\left(\left(\frac{\sqrt{2\pi}}{2}\right)^2 + 0^2\right) \stackrel{?}{=} \sqrt{\frac{2}{\pi}}\left(\sqrt{\frac{\pi}{2}}\right) + 0$

$1 = 1$ ✓

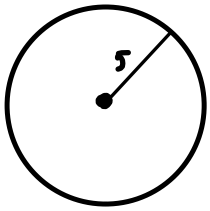
$2x = 1$

$x = \frac{1}{2} \longrightarrow 2 = \frac{1}{x}$, duh

$\longrightarrow 2 = \frac{\sqrt{2}}{\sqrt{\pi}}$

9 (10 pts) Area $= \frac{1}{2} r^2 \theta$, r = radius, θ = angle

t = time, in hours



$\longrightarrow \frac{d\,\text{Area}}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$ ($r = 5$ is constant)

$r = 5 \longrightarrow \frac{d\,\text{Area}}{dt} = \frac{1}{2}(5^2)\frac{d\theta}{dt}$

For angular velocity, $\frac{d\theta}{dt}$, we have

$$\left(\frac{1 \text{ rev}}{\text{min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{2\pi \text{ radians}}{1 \text{ rev}}\right) = 120\pi \frac{\text{radians}}{\text{hr}} \longrightarrow$$

$$\frac{d\,\text{Area}}{dt} = \left(\frac{25}{2}\right)(120\pi) = \boxed{1250\pi \frac{\text{cm}^2}{\text{hr}}}$$

10 Diameter = 50 ft $\Rightarrow$ radius = 25 ft

Radius increases from 25 ft to 25 ft + $(.005\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)$

r = radius in ft.

$$((.005)\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \frac{.005}{12} = \Delta r = dr$$

$$V = \frac{4}{3}\pi r^3 \Longrightarrow \qquad \left(\frac{5}{1000}\right)\left(\frac{1}{12}\right) = \left(\frac{1}{200}\right)\left(\frac{1}{12}\right) = \frac{1}{2400}$$

$$\Delta V \approx dV = 4\pi r^2\,dr = 4\pi(25)^2\left(\frac{.005}{12}\right)$$

$$= 4(625)\left(\frac{1}{2400}\right)\pi$$

$$= \pi\frac{625}{600} = \frac{25}{24}\pi = \frac{25\pi}{24}\text{ ft}^3$$

$$= \left(\frac{25\pi}{24}\right)(\text{ft}^3)\left(7.48052\ \frac{\text{gallons}}{\text{ft}^3}\right) \approx 24.47994446 \text{ gallons.}$$

$$\approx \boxed{24.4799 \text{ gallons}}$$

B1 5pts $\lim_{x\to 3}(3x^2-2x-1) = 20$

Scratch

Want $|3x^2-2x-1-20| < \varepsilon$

$|3x^2-2x-21| = |3x+7||x-3|$

Assume $\delta \le 1$

Then $2 < x < 4$

$6 < 3x < 12$

$13 < 3x+7 < 19$

Proof

Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{19}\}$. Then

$0 < |x-3| < \delta \Longrightarrow |3x^2-2x-1-20| = |3x^2-2x-21|$

$= |3x+7||x-3| < 19|x-3| < 19\delta \le 19\cdot\frac{\varepsilon}{19} = \varepsilon$ ∎

B2 5pts $\left(\dfrac{x^{\frac{2}{3}} - c^{\frac{2}{3}}}{x-c}\right)\left(\dfrac{x^{\frac{4}{3}} + x^{\frac{2}{3}}c^{\frac{2}{3}} + c^{\frac{4}{3}}}{x^{\frac{4}{3}} + x^{\frac{2}{3}}c^{\frac{2}{3}} + c^{\frac{4}{3}}}\right)$

B3 **5pts** $f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is difbl on $(-\infty, \infty)$

$\boxed{\text{Pf}}$ Clearly f is difbl $\forall\, x \neq 0$.

Now $\lim\limits_{h \to 0} \dfrac{f(0+h) - f(0)}{h} = \dfrac{h^2 \sin\left(\frac{\pi}{h}\right) - 0}{h}$

$= \underset{(h \neq 0)}{h \sin\left(\frac{\pi}{h}\right)} \xrightarrow{h \to 0} 0$, since

$-1 \leq \sin\left(\frac{\pi}{h}\right) \leq 1 \quad \forall\, h \neq 0$

Now, $h > 0 \implies$

$-h \leq h\sin\left(\frac{\pi}{h}\right) \leq h$

SQUEEZE THEOREM

& $h < 0 \implies$

$-h \geq h\sin\left(\frac{\pi}{h}\right) \geq h$ is the same bound on $h\sin\left(\frac{\pi}{h}\right)$. As $h \to 0$, we obtain

$h\sin\left(\frac{\pi}{h}\right) \to 0$ and so

$f(0+h) - f(0)$ $\exists$ and we're done