MAT 2410 | Written Final | Name_____________________
160 Points | Concentration on Chapters 4 – 6

NOTE TO PROCTORS: The time control is 2 hours. Please encourage students to write on only one side of each sheet of paper. This way they can refer to previous work for current work.

Materials Permitted: Pencil/Pen, Scientific Calculator (not graphing), Straight Edge, 2-page cheat sheet (either one sheet, 2 sided or 2-page, 1-sided). Include student cheat sheets in scans. Place them at the end.

TO STUDENTS: Show all work. That means scratch work goes with the problem, and not on a separate sheet. **Do your own work**. Leave at least ½-inch margins around each sheet. If you're taking the test in Horizon Hall, this has already been done for you, with a border.

You may work up to 4 bonus problems, at your discretion. There's something of everything on this test.

Remember #1 is above #2 is above #3a is above #3b is above #3c, ... If your work is in two columns, with #5 to the right of #3, you won't receive any points for #5. Writing to communicate is a requirement. If I can't read your work, I can't give you a passing grade.

Leave plenty of space. If it's too cramped to efficiently award partial credit, there will be no partial credit. If you get the answer right, but I can't understand what you did, you will get ½-credit, at most. I need to see the support. I need to see all the scratch work for each problem WITH that problem, not on a separate sheet.

Turn in your test sheets, your work, and your cheat sheet. Test sheets on top. Your work, next, and Cheat Sheet at the bottom.

Draw Pictures! Fare well!

1. Let $f(x) = x^2 + 4x$. We're gonna use it a *lot*.

   a. (5 pts) Use the limit definition of the derivative to find $f'(x)$

   b. (5 pts) Use the limit definition of the definite integral to evaluate $\int_{-2}^{2} f(x)\,dx$.

2. Consider the region bounded by $y = x^2 + 4x$, the x-axis, $x = -2$, and $x = 2$. We will find the area of this region in two ways.

   a. (5 pts) Sketch the region.

   b. (5 pts) Write the area of the region as an integral or sum of integrals with respect to *x*. Draw a representative rectangle or rectangles on the sketch from part a.

   c. (5 pts) Evaluate the integral from part b.

   d. (5 pts) Is your answer to part *c* the same as your answer to #1*b*? If not, where do you think you went wrong?

e. (5 pts) Sketch the region, again.

f. (5 pts) Write the area as an integral or a sum of integrals with respect to y. Draw a representative rectangle or rectangles on the sketch from part *d*. Do not evaluate the integral.

g. (**Bonus** 5 pts) Evaluate your answer to #2*f*.

3. Let $f(x) = x^2 + 4x$. Consider the region bounded by $f(x)$, the x-axis, $x = 0$, and $x = 2$. This

a. (5 pts) Suppose we rotated the region about the line $y = -2$. Sketch the graph of the solid of revolution obtained. Include a representative disk or washer.

b. (5 pts) Write the integral representing the volume of the solid of revolution in part *a*.

c. (**Bonus** 5 pts) Evaluate the integral you wrote for part *b*.

d. (5 pts) Write the integral for revolving the region about the line $x = -2$.

e. (**Bonus** 5 pts) Evaluate the integral in part *d*.

4. Differentiate the following with respect to x. **Do not simplify**:

a. (5 pts) $y = 5 \cdot 2^{\sin(x)}$

b. (5 pts) $y = \ln\left(\dfrac{\left(x^2 - 7x\right)^2}{\sqrt[3]{\sin^2(x)}}\right)$

c. (5 pts) $y = \log_7\left(x \sin(5x)\right)$

d. (5 pts) $y = \left[x^2 + 5x\right]^{\tan(x)}$

e. (5 pts) $y = \displaystyle\int_0^x \frac{\left(t^2 - 7t\right)^2}{\sqrt[3]{\sin^2(t)}}\,dt$

f. (5 pts) $y = \displaystyle\int_0^{\tan(x)} \frac{\left(t^2 - 7t\right)^2}{\sqrt[3]{\sin^2(t)}}\,dt$

5. Evaluate the indefinite integrals. **Do Not Simplify**.

   a. (5 pts) $\int \frac{\sin(x)}{\cos(x)} dx$

   b. (5 pts) $\int \frac{\cos(x)}{\sin^5(x)} dx$

   c. (5 pts) $\int x^3 (3x-5)^7 \, dx$

   d. (5 pts) $\int_0^{\sqrt{\pi}} \left(x\cos\left(x^2\right) \right) dx$

   e. (5 pts) $\int e^{\sin(x)} \cos(x) \, dx$

6. Let f be a smooth function such that $f'(x) = x^2 + 4x$

   a. (5 pts) What's the net change in f over the interval $[-2,2]$?

   b. (5 pts) What's the *total* change in f over the interval $[-2,2]$?

7. The function $f(x) = x^2 + 4x$ is 1-to-1 on $[-2,\infty)$.

   a. (5 pts) Find the inverse function $f^{-1}(x)$.

   b. (5 pts) Find $\left(f^{-1}\right)'(12)$, directly, by differentiating your answer for part b.

   c. (5 pts) Find $\left(f^{-1}\right)(12)$

   d. (5 pts) Find $\left(f^{-1}\right)'(12)$ by applying a theorem regarding derivatives of inverse functions.

8. (5 pts) A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

9. (5 pts) Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \to 3} (7x+11) = 32$.

10. The equation $x^2 + 2xy + 4y^2 = 12$ describes an ellipse.

a. (5 pts) Find an expression for $y' = \frac{dy}{dx}$.

b. (5 pts) Find an equation of the tangent line to the ellipse at the point $(2,1)$.

BONUS SECTION

1. (**Bonus** 5 pts) Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \to 3}(x^2 - 3) = 6$

2. (**Bonus** 5 pts) Use logarithmic differentiation to find $y' = \frac{dy}{dx}$ for $y = \frac{(x-3)^3 (x+2)^{\frac{3}{5}}}{(x+5)(x-7)^5}$. Do not simplify!

3. (**Bonus** 5 pts) Sketch a complete graph of $R(x) = \frac{(x+5)(x-3)}{x-1}$