① $\int_0^3 (x^2-2x)\,dx \qquad \Delta x = \frac{3-0}{n} = \frac{3}{n}$

$x_k = 0 + \frac{3k}{n} = \frac{3k}{n}$

$\frac{3}{n}\sum_{k=1}^{n} (x_k^2 - 2x_k) = \frac{3}{n}\sum_{k=1}^{n}\left(\left(\frac{3k}{n}\right)^2 - 2\left(\frac{3k}{n}\right)\right)$

$= \frac{3}{n}\cdot\frac{9}{n^2}\sum k^2 - \frac{3}{n}\cdot 2\cdot\frac{3}{n}\sum k$

$= \frac{27}{n^3}\cdot\frac{n^3 + \cdots}{3} - \frac{18}{n^2}\cdot\frac{n^2 + \cdots}{2}$

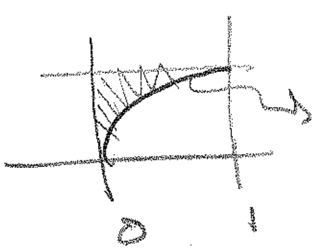
$\xrightarrow{n\to\infty} \frac{27}{3} - \frac{18}{2} = 9 - 9 = 0$!?

② $\int_0^3 (x^2-2x)\,dx = \left[\frac{x^3}{3} - x^2\right]_0^3 = \frac{27}{3} - 9 = 0$

③ $\int_0^1 y^4\,dy = \frac{y^5}{5}\Big]_0^1 = \frac{1}{5}$

$\int_0^1 \left(1 - x^{\frac{1}{4}}\right)dx = \left[x - \frac{4}{5}x^{\frac{5}{4}}\right]_0^1 = 1 - \frac{4}{5} = \frac{1}{5}$

B1 $[a,b] = [1,3] \longrightarrow \boxed{x_k = 1 + \frac{3k}{n}}$

B2 

$y = x^{\frac{1}{4}} \longrightarrow x = y^4$

$\int_0^1 \left(1 - x^{\frac{1}{4}}\right) = \int_0^1 y^4\,dy$ = Shaded area.

201 TEST 3

(4) $\int_0^6 |x-4|\,dx = \int_0^4 -(x-4)\,dx + \int_4^6 (x-4)\,dx$

$= -\left[\frac{x^2}{2}-4x\right]_0^4 + \left[\frac{x^2}{2}-4x\right]_4^6$

$= -\left[\frac{4^2}{2}-4(4)-(0-0)\right] + \left[\frac{6^2}{2}-4(6)-\left(\frac{4^2}{2}-4(4)\right)\right]$

$= -\left[\frac{16}{2}-16\right]+\left[\frac{36}{2}-24-\left(\frac{16}{2}-16\right)\right]$

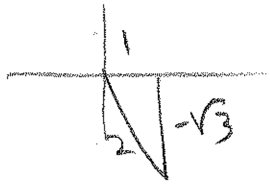
$= -[-8]+[18-24-(-8)]$

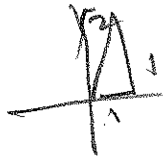
$= 8+2 = 10$

B3 $\int_{-\frac{\pi}{3}}^{\frac{\pi}{4}} |\sec x \tan x|\,dx = -\int_{-\frac{\pi}{3}}^{0} \sec x\tan x\,dx + \int_0^{\frac{\pi}{4}}\sec x \tan x\,dy$

$= -\left[\sec x\right]_{-\frac{\pi}{3}}^{0} + \left[\sec x\right]_0^{\frac{\pi}{4}} = -\left[\sec 0 - \sec\left(-\frac{\pi}{3}\right)\right] + \left[\sec\frac{\pi}{4} - \sec 0\right]$

$= -[1-2]+[\sqrt{2} - 1] = 1+\sqrt{2}-1 = \boxed{\sqrt{2}}$





201 TEST 3

(5) $\int \frac{x}{\sqrt{2x+1}}\,dx$ $\qquad u = 2x+1 \longrightarrow u-1 = 2x \Rightarrow$

$du = 2dx \qquad \frac{(u-1)}{2} = x$

$= \frac{1}{2}\int \frac{x}{\sqrt{2x+1}} \cdot 2dx = \frac{1}{2}\int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\,du$

$= \frac{1}{4}\int \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du = \frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right) + C$

$= \boxed{\frac{1}{6}\sqrt{2x+1}^{\,3} - \frac{1}{2}\sqrt{2x+1} + C}$

(6) $\int \frac{\sin x}{\cos^2 x}\,dx = \int \sec x \tan x\,dx = \boxed{\sec x + C}$

(B4) $\frac{1}{x}$ is decreasing on $[1,3]$

$m = \frac{1}{3},\ M = 1 \Longrightarrow$

$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$

$\frac{1}{3}(3-1) \le \int_1^3 \frac{dx}{x} \le 1(3-1)$

$\boxed{\frac{2}{3} \le \int_1^3 \frac{dx}{x} \le 2}$

(7) (a) $\int_0^5 v(t)dt$ = NET DISPLACEMENT / CHANGE.

(b) $\int_0^5 |v(t)|dt$ = Total displacement / distance.

201 TEST 3

B5 $\displaystyle\int_0^6 |-t^2+4t+5|\,dt$

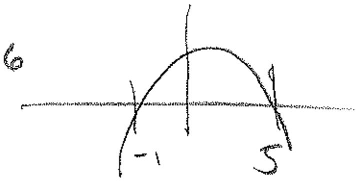
$-t^2+4t+5=0$

$\Rightarrow t^2-4t-5=0$

$\Rightarrow (t-5)(t+1)=0$

$t=-1,5$

$$= \int_0^5 (-t^2+4t+5)\,dt + \int_5^6 (t^2-4t-5)\,dt$$



$$= \left[-\tfrac{1}{3}t^3+2t^2+5t\right]_0^5 + \left[\tfrac{1}{3}t^3-2t^2-5t\right]_5^6$$

$$= \left[-\tfrac{1}{3}(5)^3+2(5)^2+5(5)-(0+0+0)\right]$$
$$+\left[\tfrac{1}{3}(6)^3-2(6)^2-5(6)-\left(\tfrac{1}{3}(5)^3-2(5)^2-5(5)\right)\right]$$

$$= \left[-\tfrac{125}{3}+50+25\right] + \left[\tfrac{216}{3}-72-30-\left(\tfrac{125}{3}-50-25\right)\right]$$

$$= \frac{-125+216-125}{3} + 75 - 102 + 75$$

$$= \frac{-34}{3} + 48 = \frac{-34+144}{3} = \boxed{\frac{110}{3}} = \boxed{36.\overline{6}}$$

36.667234

201 TEST 3 ⒸY

8 (a) $\frac{d}{dx}\int_{\frac{\pi}{6}}^{x} \frac{t^2-3t}{\csc^3 t}\,dt = \frac{x^2-3x}{\csc^3 x}$

(b) $\frac{d}{dx}\int_{\frac{\pi}{6}}^{x^3} \frac{t^2-3t}{\csc^3 t}\,dt = \frac{x^6-3x^3}{\csc^3(x^3)} \cdot 3x^2$