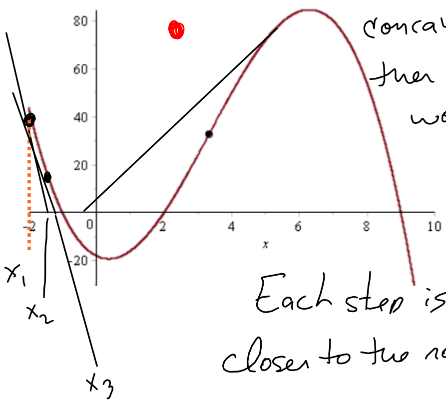
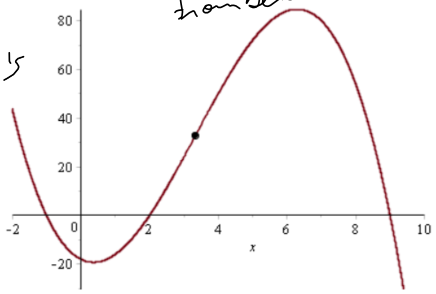
Section 3.8 Newton's Method. The General Idea.

Find x-intercept!

ABOVE & concave up, then Newton's works.

Concave Down from below works





Each step is closer to the root.

x_2 = x-coord of where tangent line at $x=x_1$ hits the x-axis.

$$y = m(x-x_1)+y_1 \qquad y = f'(x_1)(x-x_1)+f(x_1) \overset{SET}{=} 0$$

$$f'(x_1)(x-x_1) = -f(x_1)$$

$$f'(x_1)\underbrace{x}_{x_2} - f'(x_1)x_1 = -f(x_1)$$

$$f'(x_1)x_2 = f'(x_1)x_1 - f(x_1)$$

$$x_2 = \frac{f'(x_1)x_1 - f(x_1)}{f'(x_1)} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

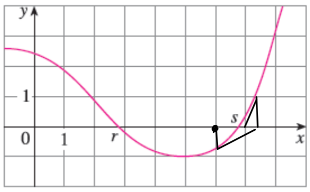
The recursion

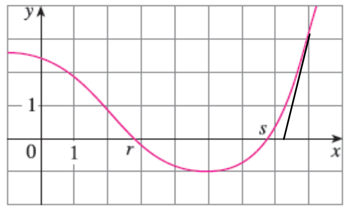
$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

The Newton Recursion.

1. Question Details SCalc8 3.8.001. [3353785]

The figure shows the graph of a function f. Suppose that Newton's method is used to approximate the root s of the equation $f(x) = 0$ with initial approximation $x_1 = 6$.





(a) Draw the tangent lines that are used to find x_2 and x_3, and estimate the numerical values of x_2 and x_3. (Round your answers to one decimal place.)

$x_2 \approx 7.3$, $x_3 \approx 7$

(b) Would $x_1 = 8$ be a better first approximation? Explain.

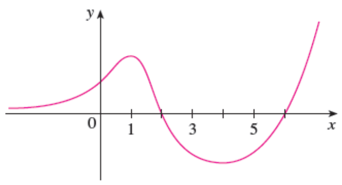
$x_1 = 8$ [---Select---] be a better first approximation because the tangent line at $x = 8$ intersects the x-axis [---Select---] s than does the first approximation $x_1 = 6$.

Tough to tell, but $x_1 = 8$ is sorta better because we keep getting closer, without going past.

$x_1 = 6$, we jumped clear past the root to the other side. Want to get closer, without going over.

2. Question Details SCalc8 3.8.004. [3353836]

For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown.



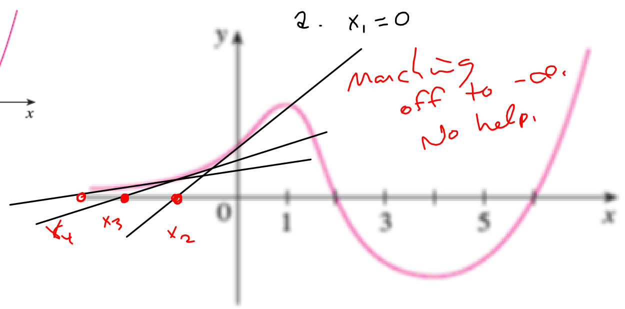
(a) $x_1 = 0$

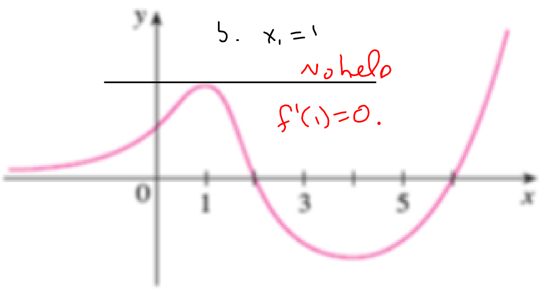
(b) $x_1 = 1$

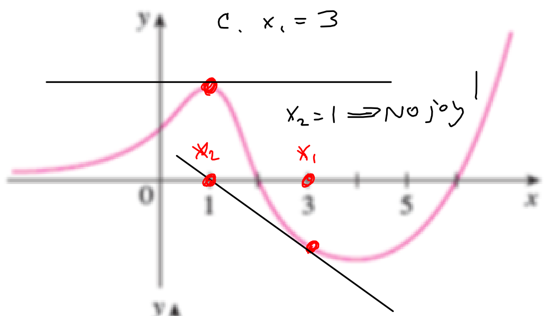
(c) $x_1 = 3$

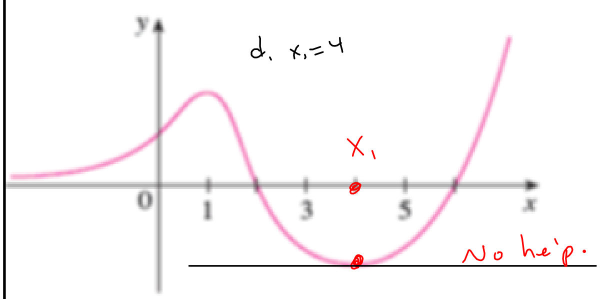
(d) $x_1 = 4$

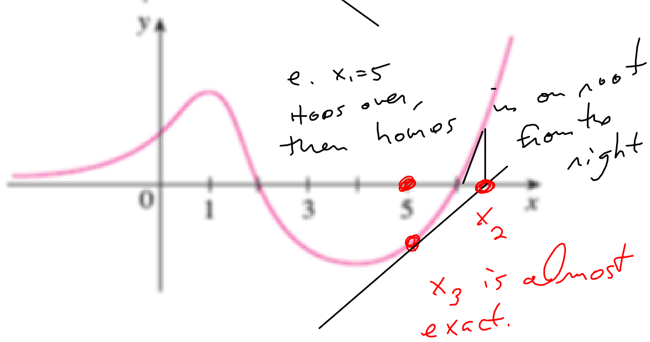
(e) $x_1 = 5$











3. Question Details SCalc8 3.8.006. [3353711]

Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation. (Round your answer to four decimal places.)

$2x^3 - 3x^2 + 2 = 0, \quad x_1 = -1$

$f(x) = 2x^3 - 3x^2 + 2$

$f'(x) = 6x^2 - 6x$

$x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n-1})}$

-0.68254 $\approx \boxed{-.6825}$ → Not x_3, but my best answer for the root.

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$\#3.\ f(x) = 2x^3 - 3x^2 + 2,\ x_1 = -1$

k	x_k	$f(x_k)$	$f'(x_k)$	$x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$
1	-1	-3	12	-0.75
2	-0.75	-0.5313	7.875	-0.682539683
3	-0.68254	-0.0335	6.8904006	-0.677675278
4	-0.67768	-0.0002	6.8215144	-0.677650699
5	-0.67765	-4E-09	6.821167	-0.677650699
6	-0.67765	0	6.821167	-0.677650699
7	-0.67765	0	6.821167	-0.677650699

4. Question Details SCalc8 3.8.007. [3353928]

Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation. (Round your answer to four decimal places.)

$\dfrac{2}{x} - x^2 + 1 = 0, \quad x_1 = 2$

$f(x) = 2x^{-1} - x^2 + 1$

$f'(x) = -2x^{-2} - 2x$

$\boxed{x_3 \approx 1.5125}$

$\#4.\ f(x) = \dfrac{2}{x} - x^2 + 1$

k	x_k	$f(x_k)$	$f'(x_k)$	$x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$
1	2	-2	-4.5	1.55556
2	1.55556	-0.134	-3.93764	1.52152
3	1.52152	-0.0005	-3.90696	1.52138
4	1.52138	-8E-09	-3.90684	1.52138
5	1.52138	0	-3.90684	1.52138
6	1.52138	0	-3.90684	1.52138

5. Question Details SCalc8 3.8.011.

Use Newton's method to approximate the given number correct to eight decimal places.

$x = \sqrt[4]{72}$

$x^4 = 72$

$f(x) = x^4 - 72 = 0$

$\Rightarrow f'(x) = 4x^3$

#5. $f(x) = x^4 - 72$ $f'(x) = 4x^3$

k	x_k	$f(x_k)$	$f'(x_k)$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$
1	2	-56	32	3.75
2	3.75	125.754	210.9375	3.153833333
3	3.153833333	26.9361	125.48049	2.939169377
4	2.939169377	2.62742	101.56261	2.913299374
5	2.913299374	0.03449	98.904337	2.912950693
6	2.912950693	6.2E-06	98.868829	2.91295063
7	2.91295063	2.3E-13	98.868823	2.91295063
8	2.91295063	0	98.868823	2.91295063
9	2.91295063	0	98.868823	2.91295063

$\boxed{x \approx 2.91295063}$

6. Question Details SCalc8 3.8.013.

Consider the following equation.

$f(x) = 3x^4 - 8x^3 + 6 = 0, \quad [2, 3]$

(a) Explain how we know that the given equation must have a root in the given interval.

(b) Use Newton's method to approximate the root correct to six decimal places.

a) $f(2) = -10 < 0 \Rightarrow \exists\, c \in (2,3) \ni f(c) = 0$

$f(3) = 33 > 0$

b) $f'(x) = 12x^3 - 24x^2$

2.5453933 $\approx \boxed{2.545393}$

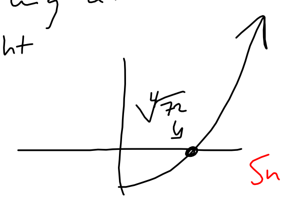
#6. $f(x) = 3x^4 - 8x^3 + 6.$

k	x_k	$f(x_k)$	$f'(x_k)$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$
1	2.5	-1.8125	209.375	2.50866
2	2.50865672	-1.4839	212.0805	2.51565
3	2.5156537	-1.2125	214.2837	2.52131
4	2.52131226	-0.9892	216.0761	2.52589
5	2.52589042	-0.806	217.5333	2.5296
6	2.52959576	-0.6561	218.7173	2.5326
7	2.53259553	-0.5336	219.6789	2.53502
8	2.53502463	-0.4337	220.4596	2.53699
66	2.54539329	-2E-06	223.8118	2.54539
67	2.5453933	-2E-06	223.8118	2.54539

Snip

$x \approx 2.545393$ after 66 iterations !?

The issue is maybe not coming at it from the right



START WITH $x_1 = 2.6$ & see what happens!

7. Question Details SC

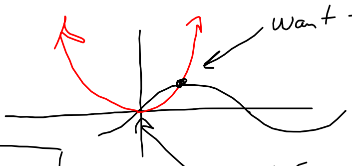
Use Newton's method to approximate the indicated root of the equation correct to six decimal places.

The positive root of $3 \sin x = x^2$

$f(x) = x^2 - 3\sin x$

$f'(x) = 2x - 3\cos x$

$x \approx 1.722125112 \approx \boxed{1.722125 \approx x}$



#7. $f(x) = x^2 - 3\sin(x)$. Find positive root to 6 places.

k	x_k	$f(x_k)$	$f'(x_k)$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$
1	3	8.57664	8.9699775	2.043850446
2	2.043850446	1.50678	5.4545223	1.76760595
3	1.76760595	0.18234	4.1218365	1.723367287
4	1.723367287	0.00484	3.9026738	1.722126094
5	1.722126094	3.8E-06	3.8965107	1.722125112
6	1.722125112	2.4E-12	3.8965058	1.722125112
7	1.722125112	0	3.8965058	1.722125112
8	1.722125112	0	3.8965058	1.722125112

8. Question Details SCalc8 3.8.018. [3353992]

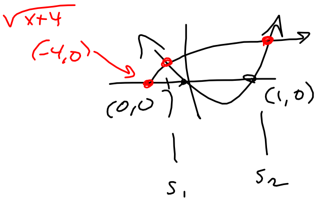
Use Newton's method to find all solutions of the equation correct to six decimal places. (Enter your answers as a comma-separated list.)

$\sqrt{x+4} = x^2 - x \Longrightarrow$

$\sqrt{x+4} - x^2 + x = 0$

$x^2 - x - \sqrt{x+4} = 0$

$= f(x) \Longrightarrow f'(x) = \frac{1}{2}(x+4)^{-\frac{1}{2}} - 2x + 1$



$\boxed{x \approx -0.916345232, \; 2.152392728}$

#8. $f(x) = \sqrt{x+4} - x^2 + x$

k	x_k	$f(x_k)$	$f'(x_k)$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$
1	-2	-4.58579	5.70710678	-1.196477898
2	-1.196477898	-0.95367	4.23014187	-0.971032715
3	-0.971032715	-0.17354	3.81226186	-0.92551003
4	-0.92551003	-0.02866	3.72773124	-0.91782265
5	-0.91782265	-0.00461	3.71345185	-0.916581764
6	-0.916581764	-0.00074	3.71114676	-0.916383058
7	-0.916383058	-0.00012	3.71077764	-0.91635128
8	-0.91635128	-1.9E-05	3.71071861	-0.916346199
9	-0.916346199	-3E-06	3.71070917	-0.916345387
10	-0.916345387	-4.8E-07	3.71070766	-0.916345257
11	-0.916345257	-7.7E-08	3.71070742	-0.916345236
12	-0.916345236	-1.2E-08	3.71070738	-0.916345233
13	-0.916345233	-2E-09	3.71070737	-0.916345232
14	-0.916345232	-3.1E-10	3.71070737	-0.916345232
15	-0.916345232	-5E-11	3.71070737	-0.916345232

#8. $f(x) = \sqrt{x+4} - x^2 + x$

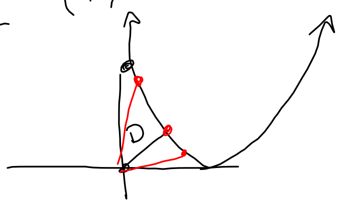
k	x_k	$f(x_k)$	$f'(x_k)$	$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$
1	2	0.44949	-1.7752551	2.253197265
2	2.253197265	-0.32306	-2.2560748	2.110001094
3	2.110001094	0.129738	-1.9840814	2.175390608
4	2.175390608	-0.0719	-2.1082646	2.141286554
5	2.141286554	0.03434	-2.0434921	2.158091306
6	2.158091306	-0.01772	-2.0754075	2.149554864
7	2.149554864	0.008798	-2.0591949	2.15382758
30	2.152392728	-1.2E-09	-2.0645846	2.152392728
31	2.152392728	6.1E-10	-2.0645846	2.152392728
32	2.152392728	-3.1E-10	-2.0645846	2.152392728

9. Question Details SCalc8 3.8.037.MI. [3353588]

Use Newton's method to find the coordinates, correct to six decimal places, of the point on the parabola $y = (x - 9)^2$ that is closest to the origin.

Distance from $(x, f(x))$ to $(0,0)$ is $\sqrt{(x-0)^2 + (f(x)-0)^2}$ to minimize √

minimize



$$f(x) = D^2 = x^2 + \left((x-9)^2 - 0\right)^2$$

$$f(x) = x^2 + (x-9)^4$$

$g(x) = f'(x) = 2x + 4(x-9)^3 \overset{SET}{=} 0$ & apply Newton's.

$$g'(x) = 2 + 12(x-9)^2 = 12(x-9)^2 + 2$$

Newton's reveals

$x \approx 7.449854051$

$\approx \boxed{7.449854 \approx x}$

Oct 27-11:10 AM