

1. Question Details

SCalc8 2.4.001. [?]

Differentiate.

$$f(x) = x^2 \sin(x) \Rightarrow f'(x) = 2x \sin x + x^2 \cos x$$

2. Question Details

SCalc8 2.4.005. [?]

Differentiate.

$$y = \sec(\theta) \tan(\theta) \Rightarrow y' = \sec \theta \tan \theta \tan \theta + \sec \theta \sec^2 \theta$$

$$= \sec \theta \tan^2 \theta + \sec^3 \theta$$

3. Question Details

iCalc8 2.4.009.MI. [?]

Differentiate.

$$y = \frac{8x}{9 - \tan(x)} \Rightarrow y' = \frac{8(9 - \tan x) - 8x \sec^2 x}{(9 - \tan x)^2}$$

4. Question Details

SCalc8 2.4.015. [?]

Differentiate.

$$f(\theta) = \theta \cos(\theta) \sin(\theta) = \theta \cos \theta \sin \theta$$

$$(fgh)' = f'gh + fg'h + fgh'$$

Lemma

$$\Rightarrow f'(\theta) = 1 \cos \theta \sin \theta + \theta [\sin^2 \theta - \cos^2 \theta]$$

$$= 1 \cos \theta \sin \theta + \theta \sin^2 \theta - \theta \cos^2 \theta$$

5. Question Details

SCalc8 2.4.016.

Differentiate.

$$y = 2x^2 \cos(x) \cot(x) \Rightarrow y' = 4x \cos x \cot x + 2x^2 \sin x \cot x$$

$$- 2x^2 \cos x \csc^2 x$$

6. Question Details

SCalc8 2.4.017. [

Prove that $\frac{d}{dx}(\csc(x)) = -\csc(x) \cot(x)$

See 00b-video in §2.3

$$\begin{aligned}\frac{d}{dx}[\csc x] &= \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{0 \sin x - 1 \cos x}{\sin^2 x} = -\frac{\cos x}{\sin^2 x} \\ &= -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x\end{aligned}$$

7. Question Details

SCalc8 2.4.027. [

If $f(x) = 7 \sec(x) - 6x$, find $f'(x)$.

$$f'(x) = 7 \sec x \tan x - 6$$

8. Question Details

SCalc8 2.4.029. [

If $H(\theta) = \theta \cos(\theta)$, find $H'(\theta)$ and $H''(\theta)$.

$$\Rightarrow H'(\theta) = \cos \theta - \theta \sin \theta$$

$$\Rightarrow H''(\theta) = -\sin \theta - \sin \theta - \theta \cos \theta$$

9. Question Details

SCalc8 2.4.031. [

(a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan(x) - 1}{\sec(x)}$$

(b) Simplify the expression for $f(x)$ by writing it in terms of $\sin(x)$ and $\cos(x)$, and then find $f'(x)$.

(c) Are your answers to parts (a) and (b) equivalent?

$$(a) f'(x) = \frac{(\sec^2 x)(\sec x) - (\tan x - 1)(\sec x \tan x)}{\sec^2 x}$$

$$\begin{aligned}(b) &= \frac{\sec^3 x - [\sec x \tan^2 x - \sec x \tan x]}{\sec^2 x} \\ &= \frac{\sec^3 x - \sec x \tan^2 x + \sec x \tan x}{\sec^2 x} = \frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} \\ &= \frac{\sec^2 x - (\sec^2 x - 1) + \tan x}{\sec x} = \frac{\sec^2 x - \sec^2 x + 1 + \tan x}{\sec x} \\ &= \frac{\tan x + 1}{\sec x} = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}{\frac{1}{\cos x}} = \left(\frac{\sin x + \cos x}{\cos x}\right) \left(\frac{\cos x}{1}\right) = \sin x + \cos x\end{aligned}$$

Yes, they're equivalent, except for their domains.
 $D(\text{last version}) = \mathbb{R}!$

10. Question Details

SCalc8 2.4.032.

Suppose $f(\pi/3) = 4$ and $f'(\pi/3) = -3$, and let $g(x) = f(x) \sin(x)$ and $h(x) = \cos(x)/f(x)$. Find the following.

(a) $g'(\pi/3)$

(b) $h'(\pi/3)$



$$g(x) = f(x) \sin x \Rightarrow$$

$$g'(x) = f'(x) \sin x + f(x) \cos x$$

$$\Rightarrow g'(\pi/3) = f'(\pi/3) \sin \pi/3 + f(\pi/3) \cos \pi/3$$

$$= -3 \cdot \frac{\sqrt{3}}{2} + 4 \cdot \frac{1}{2} = -\frac{3\sqrt{3}}{2} + 2$$

$$h(x) = \frac{\cos x}{f(x)} \Rightarrow h'(x) = \frac{(-\sin x)(f(x)) - (\cos x)f'(x)}{(f(x))^2}$$

$$\Rightarrow h'(\pi/3) = \frac{(-\sin \pi/3)f(\pi/3) - (\cos \pi/3)(-3)}{4^2} = \frac{(-\frac{\sqrt{3}}{2})(4) - (\frac{1}{2})(-3)}{16}$$

$$= \frac{-2\sqrt{3} + \frac{3}{2}}{16} = -\frac{\sqrt{3}}{8} + \frac{3}{32} = \frac{-4\sqrt{3} + 3}{32}$$

12. Question Details

SCalc8 2.4.034. [3354]

Find the values of x on the curve $y = \frac{\cos x}{2 + \sin x}$ at which the tangent is horizontal. (Let n be an integer. Enter your answers as a comma-separated list.)

$$\text{Algebra: } \frac{A}{B} = 0 \text{ iff } A = 0$$

$$y' = \frac{(-\sin x)(\sin x + 2) - (\cos x)(\cos x)}{(\sin x + 2)^2} = \frac{-\sin^2 x - 2\sin x - \cos^2 x}{(\sin x + 2)^2}$$

$$\text{Note } -\sin^2 x - \cos^2 x = -(\sin^2 x + \cos^2 x) = -1$$

$$\frac{-1 - 2\sin x}{(\sin x + 2)^2} \stackrel{\text{SET } 0}{=} 0 \Rightarrow -1 - 2\sin x = 0 \Rightarrow -2\sin x = 1$$

$$180^\circ + 30^\circ$$

$$\pi + \frac{\pi}{6}$$



$$\Rightarrow \sin x = -\frac{1}{2}$$

$$x = -\frac{\pi}{6} + 2n\pi, \frac{7\pi}{6} + 2n\pi, \forall n \in \mathbb{Z}$$

$$\text{Sol'n set:}$$

$$\left\{ -\frac{\pi}{6} + 2n\pi \text{ or } \frac{7\pi}{6} + 2n\pi \mid n \in \mathbb{Z} \right\}$$

11. Question Details

SCalc8 2.4.033. [335]

For what values of x does the graph of f have a horizontal tangent? (Use n as your integer variable. Enter your answers as a comma-separated list.)

$$f(x) = x - 2 \sin(x) \Rightarrow f'(x) = 1 - 2 \cos(x) \stackrel{\text{SET } 0}{=} 0$$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

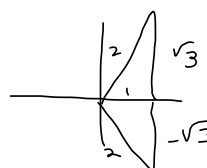
$$60^\circ$$

$$\frac{\pi}{3}$$

$$300^\circ$$

$$\frac{5\pi}{3}$$

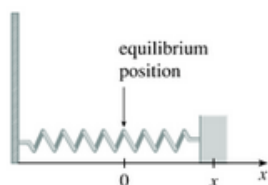
$$\left\{ \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi \mid n \in \mathbb{Z} \right\}$$



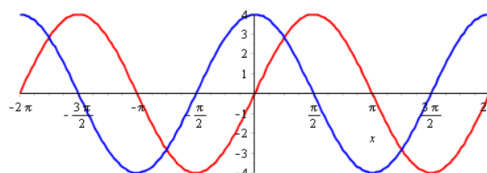
13. Question Details

SCalc8 2.4.035.MI [33]

A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is $x(t) = 4 \sin(t)$, where t is in seconds and x is in centimeters.



Simple harmonic motion.



(a) Find the velocity and acceleration at time t .

$$x'(t) = 4 \cos(t) = v(t)$$

$$x''(t) = -4 \sin(t) = a(t)$$

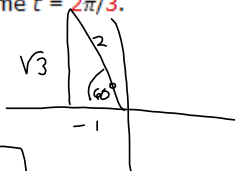
(b) Find the position, velocity, and acceleration of the mass at time $t = 2\pi/3$.

In what direction is it moving at that time?

$$x\left(\frac{2\pi}{3}\right) = 4 \sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2} = 2\sqrt{3} \text{ cm}$$

$$x'\left(\frac{2\pi}{3}\right) = 4 \cos\left(\frac{2\pi}{3}\right) = 4\left(-\frac{1}{2}\right) = -2 \frac{\text{cm}}{\text{s}}$$

$$x''(t) = -4 \sin\left(\frac{2\pi}{3}\right) = -2\sqrt{3} \frac{\text{cm}}{\text{s}^2}$$



14. Question Details

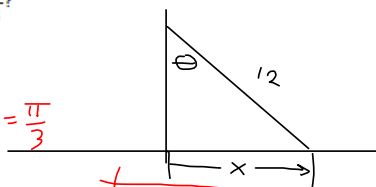
SCalc8 2.4.037. [3354450]

A ladder 12 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \frac{\pi}{3}$?

$$\frac{x}{12} = \sin \theta$$

$$x = 12 \sin \theta$$

$$\left. \frac{dx}{d\theta} = 12 \cos \theta \right|_{\theta = \frac{\pi}{3}} = 12 \cos \frac{\pi}{3} = 12 \left(\frac{1}{2} \right) = 6 \frac{\text{ft}}{\text{radian}}$$



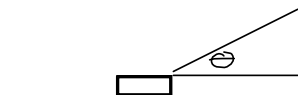
5. Question Details

SCalc8 2.4.038. [3354403]

An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin(\theta) + \cos(\theta)}$$

where μ is a constant called the coefficient of friction.



(a) Find the rate of change of F with respect to θ .

(b) When is this rate of change equal to 0?

(c) If $W = 40$ lb and $\mu = 0.8$, draw the graph of F as a function of θ .

$$(a) \frac{dF}{d\theta} = \frac{0 \cdot \text{Bottom} - \mu W (\mu \cos \theta - \sin \theta)}{(\mu \sin \theta + \cos \theta)^2}$$

$$(b) \stackrel{SET}{=} 0 \Rightarrow -\mu W \mu \cos \theta + \mu W \sin \theta = 0$$

$$\Rightarrow -\mu \cos \theta + \sin \theta = 0$$

$$\Rightarrow -\mu \cos \theta = -\sin \theta$$

$$\mu \cos \theta = \sin \theta$$

$$\mu = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\boxed{\text{When } \theta = \arctan \mu} \quad \left(0 \leq \theta < \frac{\pi}{2} \right)$$

$$(c) W = 40 \text{ lb}, \mu = 0.8$$

16. Question Details

SCalc8 2.4.039.

Find the limit.

$$\lim_{x \rightarrow 0} \frac{\sin(8x)}{9x} = \lim_{x \rightarrow 0} \frac{8}{9} \cdot \frac{\sin(8x)}{8x} = \frac{8}{9}$$

OO - videos

$$OO2 - \frac{d}{dx} \sin x$$

$$OO6 - \frac{d}{dx} \text{ other trigs}$$

in 2.4 directory.

17. Question Details

Find the limit.

$$\lim_{t \rightarrow 0} \frac{\tan(8t)}{\sin(2t)} \Rightarrow \frac{\frac{\sin(8t)}{\cos(8t)}}{\frac{\sin(2t)}{\cos(2t)}} = \frac{\sin(8t)}{\sin(2t)} \cdot \frac{\cos(2t)}{\cos(8t)} = \frac{8}{2} \cdot \frac{2t}{\sin(2t)} \cdot \frac{\sin(2t)}{2t} \cdot \frac{1}{\cos(8t)}$$

$$t \rightarrow 0 \Rightarrow 4 \cdot 1 \cdot 1 \cdot 1 = 4$$

18. Question Details

SCalc8 2.4.042.

Find the limit.

$$\lim_{\theta \rightarrow 0} \frac{\cos(5\theta) - 1}{\sin(6\theta)}$$

$$\frac{\cos(5\theta) - 1}{5\theta} \cdot \frac{6\theta}{\sin(6\theta)} \cdot \frac{5}{6} \xrightarrow{\theta \rightarrow 0} 0 \cdot 1 \cdot \frac{5}{6} = 0$$

19. Question Details

SCalc8 2.4.045.

Find the limit.

$$\lim_{\theta \rightarrow 0} \frac{\sin(8\theta)}{\theta + \tan(9\theta)}$$

$$\frac{\sin(8\theta)}{\theta + \tan(9\theta)} = \frac{\sin 8\theta}{\theta(1 + \frac{\sin(9\theta)}{\theta} \cdot \cos(9\theta))}$$

$$= \frac{8 \sin(8\theta)}{8\theta (1 + \frac{\sin 9\theta}{9\theta} \cos 9\theta)} = \frac{8}{10} = \frac{4}{5}$$

will do L'Hôpital's version

$$\frac{\tan \theta}{\theta} = \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\theta}{1}} = \frac{\sin \theta}{\theta \cos \theta} = \frac{\sin \theta}{\theta} \cos \theta$$

L'Hôpital observed that if $\frac{x}{y} \rightarrow \frac{0}{0}$, then how FAST $x \rightarrow 0$ & $y \rightarrow 0$

will decide, so if

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0}, \text{ then } \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)} \quad \text{Also works for } \frac{\infty}{\infty}, 0 \cdot \infty, \dots$$

$$\frac{\sin(8\theta)}{\theta + \tan(9\theta)} \xrightarrow{\theta \rightarrow 0} \frac{8 \cos \theta}{1 + 9 \sec^2 \theta} \xrightarrow{\theta \rightarrow 0} \frac{8}{1+9} = \frac{8}{10} = \frac{4}{5}$$

20. Question Details

SCalc8 2.4.048.

Find the limit.

$$\lim_{x \rightarrow 0} \frac{\sin(x^9)}{x}$$

$$\frac{x^9}{x^9} \cdot \frac{\sin(x^9)}{x} = \frac{x^9 \sin(x^9)}{x^9} \xrightarrow{x \rightarrow 0} 0 \cdot 1 = 0$$

$\rightarrow 1$ as $x \rightarrow 0$

21. Question Details

SCalc8 2.4.050.

Find the limit.

$$\lim_{x \rightarrow 3} \frac{\sin(x-3)}{x^2 + 6x - 27} = \frac{\sin(x-3)}{x^2 + 6x - 27} = \frac{\sin(x-3)}{(x-3)(x+9)} = \frac{\sin(x-3)}{x-3} \cdot \frac{1}{x+9}$$

Let $u = x-3$. Then $x \rightarrow 3$ in $x-3$ means $x-3 \rightarrow 0$
 $u \rightarrow 0$

$$= \frac{\sin u}{u} \cdot \frac{1}{x+9} \xrightarrow{u \rightarrow 0, x \rightarrow 3} 1 \cdot \frac{1}{-7+9} = \boxed{\frac{1}{6}}$$

22. Question Details

SCalc8 2.4.053.

Find constants A and B such that the function $y = A \sin(x) + B \cos(x)$ satisfies the differential equation $y'' + y' - 3y = \sin(x)$.

$$y' = A \cos x - B \sin x$$

$$y'' = -A \sin x - B \cos x$$

$$-A \sin x - B \cos x + A \cos x - B \sin x - 3(A \sin x + B \cos x) = \sin x$$

$$-A \sin x - B \cos x + A \cos x - B \sin x - 3A \sin x - 3B \cos x = \sin x$$

$$-A \sin x - B \sin x - 3A \sin x = \sin x$$

$$-A - B - 3A = 1$$

$$-4A - B = 1$$

$$\boxed{B = -4A - 1}$$

$$-B \cos x + A \cos x - 3B \cos x = 0$$

$$(\cos x)[-B + A - 3B] = 0$$

$$A - 4B = 0$$

$$A - 4[-4A - 1] = A + 16A + 4 = 0$$

$$17A + 4 = 0$$

$$\boxed{A = -\frac{4}{17}}$$

$$\Rightarrow B = -4\left(-\frac{4}{17}\right) - 1$$

$$= \frac{16}{17} - \frac{17}{17} = -\frac{1}{17}$$

$$\boxed{B = -\frac{1}{17}}$$

23. Question Details

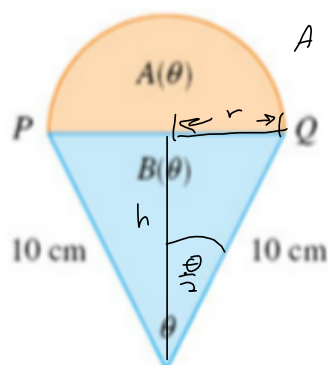
SCalc8 2.4.056. [3354197]

A semicircle with diameter PQ sits on an isosceles triangle PQR to form a region shaped like a two-dimensional ice-cream cone, as shown in the figure. If $A(\theta)$ is the area of the semicircle and $B(\theta)$ is the area of the triangle, find

$$\lim_{\theta \rightarrow 0^+} \frac{A(\theta)}{B(\theta)}.$$

$$\frac{h}{10} = \cos \frac{\theta}{2}$$

$$h = 10 \cos \frac{\theta}{2}$$



$$A = \frac{1}{2} \pi r^2$$

$$\frac{r}{10} = \sin \frac{\theta}{2}$$

$$r = 10 \sin \frac{\theta}{2}$$

$$B = \frac{1}{2} \cdot 2r \cdot h$$

$$\frac{A}{B} = \frac{\frac{1}{2} \pi r^2}{r h} = \frac{1}{2} \pi \frac{r}{h}$$

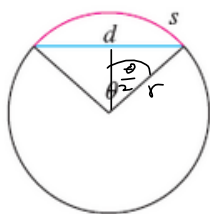
$$= \frac{1}{2} \pi \cdot \frac{10 \sin(\frac{\theta}{2})}{10 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2})} = \frac{1}{2} \pi \tan(\frac{\theta}{2}) \xrightarrow{\theta \rightarrow 0} \frac{1}{2} \pi \cdot 0 = 0!$$

24. Question Details

SCalc8 2.4.057. [3]

The figure shows a circular arc of length s and a chord of length d , both subtended by a central angle θ . Find

$$\lim_{\theta \rightarrow 0^+} \frac{s}{d}.$$



Remember this one, when we later use arc length & straight line segments interchangeably when lengths are small, relative to the radius.

$$s = r\theta$$

$$\frac{d/2}{r} = \sin\left(\frac{\theta}{2}\right)$$

$$d = 2r \sin \frac{\theta}{2}$$

$$\theta \rightarrow 0 \rightarrow 1!$$

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{s}{d} &= \frac{r\theta}{2r \sin(\frac{\theta}{2})} \\ &= \frac{2r \cdot \frac{\theta}{2}}{2r \sin \frac{\theta}{2}} = \frac{\frac{\theta}{2}}{\sin \frac{\theta}{2}} \end{aligned}$$