MAT 2410-R11 Written Midterm, Spring, 2026 (5 pts) Name _____
125 Points Covers Chapters 1 – 3

Do your own work. Show all your work. Circle final answers.

Turn in *everything*, including your cheat sheet. Submit all exercises, in order. If they are out of order, that's 10% off the top of your test score.

EVERYTHING you do related to a given question should be included with the work on that question, including your scratch work. Do not make a separate scratch sheet.

Do not write on this test sheet, except for printing your name at the top.

Leave a margin on the left of each page of your work. Do not work in more than one column. #2 goes under #1. #4b goes *under* #4a.

1. Evaluate the following limits, if they exist. If one does not exist, explain why.

   a. (5 pts) $\lim\limits_{x\to 5^-} \dfrac{x^2-2x-15}{|x-5|}$

   b. (5 pts) $\lim\limits_{x\to 5^+} \dfrac{x^2-2x-15}{|x-5|}$

   c. (5 pts) $\lim\limits_{x\to 5} \dfrac{x^2-2x-15}{|x-5|}$

2. Consider the piecewise-defined function $f(x)=\begin{cases} x^2+2x-15 & \text{if } x<4 \\ 10x-31 & \text{if } x\geq 4 \end{cases}$.

   a. (5 pts) Sketch the graph of $f(x)$. Label the *x*- and *y*-intercepts, the suture point(s), and the vertex of the quadratic piece, if it's in the picture. When I say "Label," I mean an ordered pair, like (0, 5), next to the point.

   b. (5 pts) Is $f(x)$ continuous? Explain.

   c. (5 pts) Is $f(x)$ differentiable? Explain.

3. (5 pts) Simplify the limit of the difference quotient $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-5x+8$. You may use $\lim\limits_{x\to c}\dfrac{f(x)-f(c)}{x-c}$ to compute $f'(c)$, if you prefer.

4. The point $P(5,-14)$ lies on the graph of $f(x)=x^2-5x-14$.

   a. (5 pts) Write the equation of the tangent line $L_5(x)$ to $f(x)$ at $x = 5$.

   b. (5 pts) Sketch a graph of $f(x)$ and $L_5(x)$ on the same set of coordinate axes. Include all x- and y-intercepts, and the vertex of $f(x)$.

5. (5 pts) Prove that $\lim_{x\to 5}(-3x+6)=-9$, using the $\varepsilon-\delta$ definition of limit.

6. (5 pts) Prove that the equation $f(x)=x^4-9x^3+21x^2+x-30$ has a root in the interval $(4,6)$, but *do not solve*!

7. Differentiate the following with respect to the indicated independent variable. **Do not simplify!**

   a. (5 pts) $f(x)=\sqrt[8]{x^5}-5x^{-\frac{4}{7}}-\dfrac{11}{x^{-2/3}}$; x.

   b. (5 pts) $g(x)=\sec(4x)\sin(4x)$; x.

   c. (5 pts) $h(\theta)=\dfrac{\cot(\theta)}{\cos(\theta)}$; θ.

   d. (5 pts) $r(w)=\left(w^2+11w\right)^{-5}\left(w^3+\dfrac{1}{w}\right)^{13}$; w.

   e. (5 pts) $G(\tau)=\sqrt{\sin\left(\dfrac{\pi}{6}\tau^3\right)}$; τ; w

8. Consider the relation $\sin\left(x^2+y^2\right)=\sqrt{\dfrac{2}{\pi}}x+2y$.

   a. (5 pts) Use implicit differentiation to find $y'=\dfrac{dy}{dx}$

   b. (5 pts) Find an equation of the tangent line to the curve at the point $\left(\sqrt{\dfrac{\pi}{2}},0\right)$.

9. (10 pts) If the second hand of a clock has length 5 inches, find the rate at which it sweeps out area, in square meters per hour.

10. The Montgomery, Alabama capitol building has a hemispherical domed roof that is 50 feet in diameter that needs to be painted.

    a. (5 pts) Use a differential to estimate how many gallons of paint it will take to give the dome a coat of paint 0.005 inches thick. Round your answer to 4 decimal places.

    Hints: Volume of a sphere is $\frac{4}{3}\pi r^3$ and there are approximately 7.48052 gallons per cubic foot. Round final answer to 4 decimal places.

    b. (5 pts) Make a direct calculation to answer part a *without* a differential. Round final answer to 4 decimal places.

BONUS:

1. (5 pts) Prove that $\lim_{x\to 3}\left(3x^2-2x-1\right)=20$, using the $\varepsilon-\delta$ definition of limit.

2. (5 pts) Compute the derivative of $f\left(x\right)=x^{\frac{3}{3}}$ by the limit definition of the derivative.

3. (5 pts) See if you can *squeeze* out a *convincing* argument to support the statement

$$f\left(x\right)=\begin{cases} x^2\sin\left(\dfrac{\pi}{x}\right) & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$$ is differentiable on $\left(-\infty,\infty\right)$. You may assume that it is continuous.

There's really only one point to worry about, for differentiability.