Final Test Notes:

I found a couple of problems that were poorly posed, I'm afraid.

#4b is an antiderivative you can't work with the tools you have.

#8 is an epsilon-delta question that's also in the bonus. It's fine in the bonus, but I don't test over higher-degree polynomials or rather, I make them bonus. So I'm throwing that one out.

> **Know going in that #4b and #8 are getting thrown out. I'm pretty sure that will make it a 150-point test, rather than a 160-point test. You may want to make a small note of this on your cheat sheet.**

Last bit of new knowledge:

Why $\int \frac{dx}{x} = \ln|x| + C$?

Because $\frac{d}{dx}\left[\ln|x|\right] = \begin{cases} \frac{d}{dx}[\ln x] & \text{if } x>0 \\ \frac{d}{dx}[\ln(-x)] & \text{if } x<0 \end{cases}$

and it $\nexists$ @ $x=0$.

$= \begin{cases} \frac{1}{x} & \text{if } x>0 \\ \frac{-1}{-x} = \frac{1}{x} & \text{if } x<0 \end{cases} \longrightarrow \int \frac{1}{x}\,dx = \ln|x| + C$

since $\frac{d}{dx}\left[\ln|x|\right] = \frac{1}{x}$!

Finally,

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$$

$u = \cos(x)$

$du = -\sin(x)\,dx \implies dx = \frac{du}{-\sin(x)}$

$$= \int \frac{\sin(x)}{u} \cdot \frac{du}{-\sin(x)} = -\int \frac{du}{u} = -\ln|u| + C = -\ln|\cos(x)| + C$$

$$= \ln\left(\frac{1}{|\cos(x)|}\right) + C = \ln|\sec(x)| + C$$

$$\int \sec(x)\,dx = \int (\sec(x))\left(\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\right)dx$$

$$= \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{du}{u} = \ln|u| + C$$

$u = \sec(x) + \tan(x)$

$du = (\sec(x)\tan(x) + \sec^2(x))\,dx$

$$\boxed{= \ln|\sec(x)+\tan(x)| + C = \int \sec(x)\,dx}$$

$$\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C$$

$$= \ln\left(\frac{1}{|\csc(x)+\cot(x)|}\right) + C$$

Show all work. **Circle Final Answers.** Scratch work goes with the problem, and not on a separate sheet. **Do your own work**. Leave at least ½-inch margins around each sheet. If you're taking the test in Horizon Hall, this has already been done for you.

Supporting work comes *before* the final answer.

Work as few or as many bonus as you like. Bonus problems tend to be higher difficulty and more time-consuming. If you work all of them, you're likely to run out of time.

If you skip a problem, to come back to, later, start the next problem on a fresh sheet of paper.

Deductions taken off the top: (Bad things).
-10%: Work is cramped and there's no room for comments or your work is hard to follow.
-10%: Problems are submitted in the wrong order.

Partial credit for each question:
A typical 5-pointer is broken down as follows:
2 pts – Setup
2 pts – Supporting work
1 pt - Final Answer

Leave plenty of space. If it's too cramped to efficiently award partial credit, there will be no partial credit. If you get the answer right, but I can't understand what you did, you will get ½-credit, at most. I need to see the support. I need to see all the scratch work for each problem WITH that problem, not on a separate sheet.

Turn in your test sheets, your work, and your cheat sheet. Test sheets on top. Your work, in order, next, and Cheat Sheet at the bottom of the pile.

2. Consider the region bounded by $y = x^2 + 4x$, the x-axis, $x = -2$, and $x = 2$. We will find the area of this region in two ways.

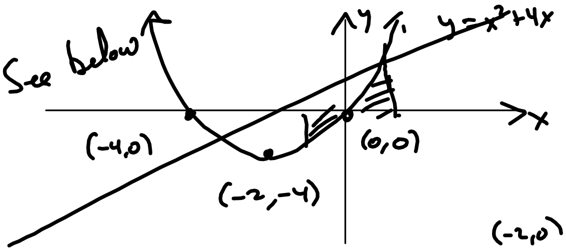
   a. (5 pts) Sketch the region.

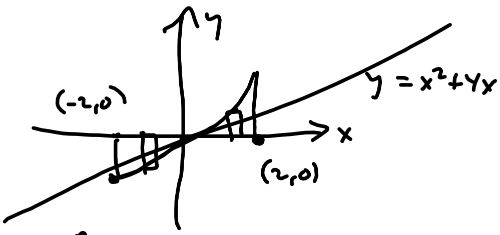
$x^2 + 4x = x(x+4) = 0 \longrightarrow$

$x = -4, 0$

$f(-2) = (-2)^2 + 4(-2) = 4 - 8 = -4$

See below



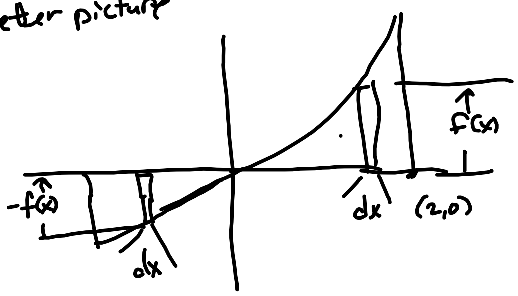


$$\text{Area} = \int_{-2}^{2} |f(x)|\,dx = -\int_{-2}^{0} (x^2+4x)\,dx + \int_{0}^{2} (x^2+4x)\,dx$$

$$= -\left[\frac{x^3}{3} + \frac{4x^2}{2}\right]_{-2}^{0} + \left[\frac{x^3}{3} + 2x^2\right]_{0}^{2} \text{, etc.}$$

Better picture

=



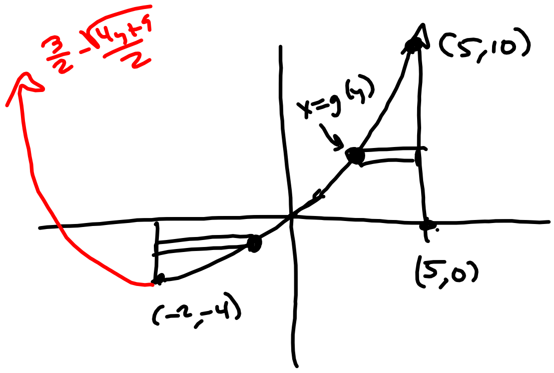
ⓒ evaluate the integral (I went too far on part b.)

I think you can do that.

Give each problem 2-4 minutes before moving on

If you move on, start a fresh page.

Answer the same as #1b?



$y = f(x) = x^2 - 3x$

Need $x = g(y)$. This is equivalent to finding $f^{-1}(x)$.

$g(y) = f^{-1}(y)$

$$\text{Area} = \int_{-4}^{0} (\text{right} - \text{left})\,dy + \int_{0}^{10} (\text{right} - \text{left})\,dy$$

$$\boxed{= \int_{-4}^{0} (g(y) - (-2))\,dy + \int_{0}^{10} (5 - g(y))\,dy}$$

Done writing the integral.

$x^2 + 4x = y$

$x^2 + 4x + 2^2 = y + 4$

$(x+2)^2 = y + 4$

$x = -2 \pm \sqrt{y+4}$

$x = \boxed{-2 + \sqrt{y+4} = g(y)}$

Quadratic formula.

$f(x) = x^2 + 4x$. Find $f^{-1}(x)$

$y^2 + 4y = x$

$y^2 + 4y - x = 0 \longrightarrow a = 1, b = 4, c = -x$

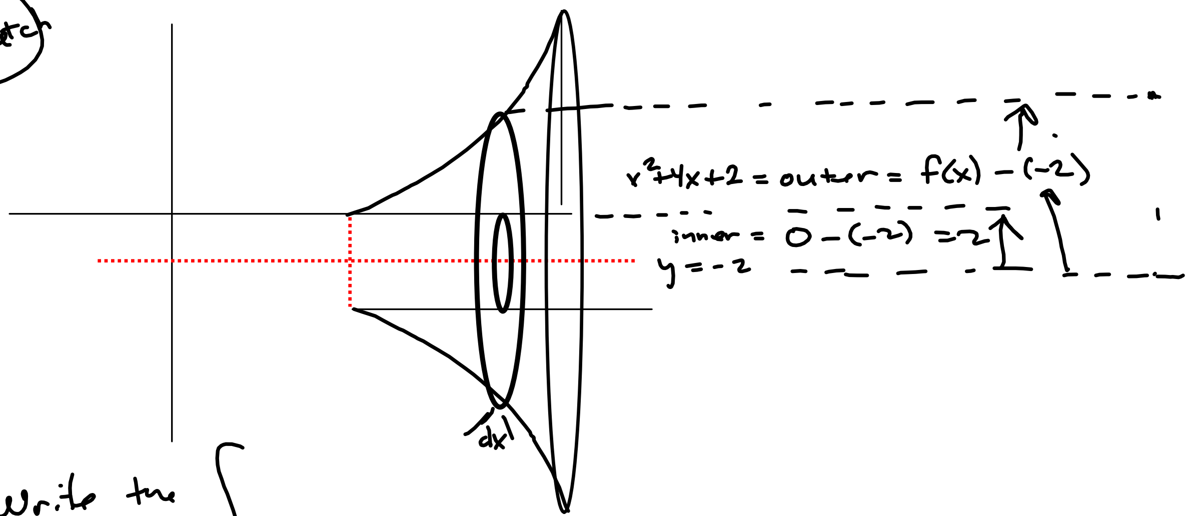
$b^2 - 4ac = 4^2 - 4(1)(-x) = 4x + 16$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4x+16}}{2} = \frac{-4 \pm 2\sqrt{x+4}}{2}$$

$f^{-1}(x) = -2 + \sqrt{x+4}$

32) $f(x) = x^2+4x$ from $x=0$ to $x=2$, revolved about $y=-2$

Sketch

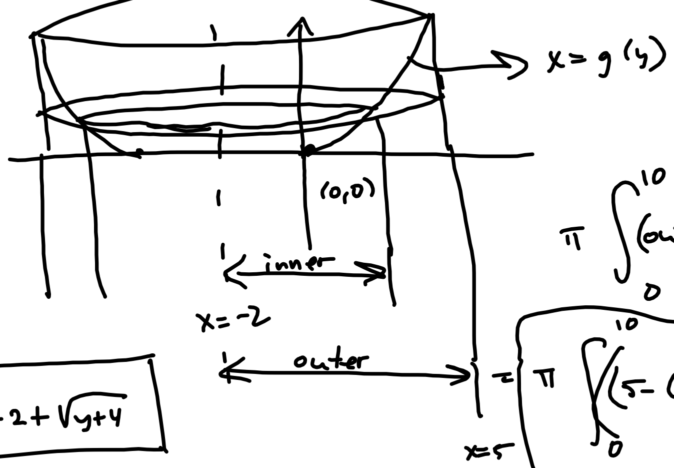


b) Write the $\int$

$$\text{Volume} = \pi \int_0^2 (\text{outer}^2 - \text{inner}^2)\,dx = \boxed{\pi \int_0^2 \left((x^2+4x+2)^2 - (2^2)\right) dx}$$

$$\boxed{\pi \int_0^2 \left((f(x)+2)^2 - 2^2\right) dx \qquad f(x) = x^2+4x}$$

3d Revolve same region around $x = -2$



$$\pi \int_0^{10} (\text{outer}^2 - \text{inner}^2)\, dy$$

$$= \boxed{\pi \int_0^{10} \left((5-(-2))^2 - (g(y)-(-2))^2 \right) dy}$$

$$\boxed{g(y) = -2 + \sqrt{y+4}}$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$