Section 6.3 I

Solve $\ln(x) + \ln(x+2) = 0$

$e^{\ln(x^2+2x)} = e^0$

$x^2+2x = 1$

$x^2+2x-1 = 0$

$x^2+2x + 1^2 - 1 - 1 = 0$

$(x+1)^2 = 2$

$x = -1 \pm \sqrt{2}$

Solve $2^{3x-2} = K$

Solve

$10(e^{-x}+1) = 3$

Solve $e^{2x} - 5e^{x} - 14 = 0$

Let $u = e^{x}$. Then

$u^2 - 5u - 14 = 0$

$(u-7)(u+2) = 0$

$u = 7$ or $u = -2$

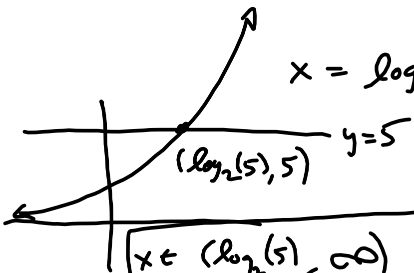
Section 6.3 II

Find Domain of $f(x) = \ln(2^x - 5)$

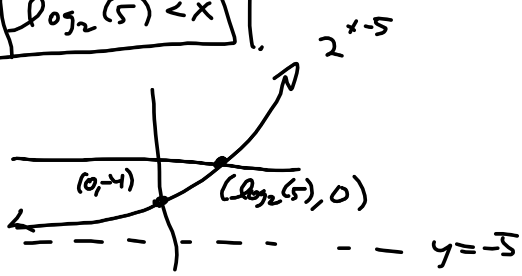
Need $2^x - 5 > 0$

$2^x > 5$

$x = \log_2(2^x) > \boxed{\log_2(5) < x}$



$\boxed{x \in (\log_2(5), \infty)}$



Let $f(x) = (\ln(x))^3$. Find $f^{-1}(x)$.

$(\ln(y))^3 = x$

$\boxed{\underline{\text{Not}}\ \ln(x^3)}$

$\ln(y) = \sqrt[3]{x}$

$y = e^{\ln(y)} = \boxed{e^{\sqrt[3]{x}} = y = f^{-1}(x)}$

$2^x = 5$

$\ln(2^x) = \ln(5)$

$x\ln(2) = \ln(5)$

$x = \ln(5) / \ln(2)$

$\log_2(2^x) = \log_2(5)$

$x = \log_2(5) = \dfrac{\ln(5)}{\ln(2)}$

Good for calculator.

$f(x) = \dfrac{e^x}{2-e^x}$ Find f^{-1}

$$\frac{e^y}{2-e^y} = x$$

$$e^y = (2-e^y)x = 2x - e^y x$$

$$e^{-y} + xe^y = 2x$$

$$e^{-y}(1+x) = 2x$$

$$e^{-y} = \frac{2x}{x+1}$$

$$\ln(\quad) = -y = \ln\left(\frac{2x}{x+1}\right)$$

$$y = -\ln\left(\frac{2x}{x+1}\right) = \ln\left(\left(\frac{2x}{x+1}\right)^{-1}\right)$$

$$= \boxed{\ln\left(\frac{x+1}{2x}\right) = \ln(x+1) - \ln(2x) = f^{-1}(x)}$$

either one

Differentiate the following

$$y = \log_3\left(\frac{x^2+5x}{\sin(x)}\right) = \frac{1}{\ln(3)} \ln\left(\frac{x^2+5x}{\sin(x)}\right)$$

$$= \frac{1}{\ln(3)} \ln\left(\frac{x(x+5)}{\sin(x)}\right) = \frac{1}{\ln 3}\left[\ln(x) + \ln(x+5) - \ln(\sin(x))\right]$$

$$\Rightarrow \boxed{y' = \frac{1}{\ln(3)}\left[\frac{1}{x} + \frac{1}{x+5} - \frac{\cos(x)}{\sin(x)}\right]}$$

Logarithmic Differentiation

$$\frac{d}{dx}\left[(x^2+2x)^5(\sin(x))(2x-3)^{-7}\right]$$

$$y = (x^2+2x)^5(\sin(x))(2x-3)^{-7} \Rightarrow$$

$$\ln(y) = 5\ln(x^2+2x) + \ln(\sin(x)) - 7\ln(2x-3) \Rightarrow$$

$$\frac{y'}{y} = 5\left(\frac{2x+2}{x^2+2x}\right) + \frac{\cos(x)}{\sin(x)} - 7\left(\frac{2}{2x-3}\right)$$

$$\Rightarrow \boxed{y' = \left[5\left(\frac{2x+2}{x^2+2x}\right) + \frac{\cos(x)}{\sin(x)} - 7\left(\frac{2}{2x-3}\right)\right](x^2+2x)^5(\sin(x))(2x-3)^{-7}}$$

Differentiating $f(x)^{g(x)}$

$$y = (\sin(x))^{x^2-11x}$$

$$\ln y = \ln\left(\sin(x)^{x^2-11x}\right) = (x^2-11x)\ln(\sin(x)) = uv$$

$$\cancel{\Rightarrow \frac{y'}{y} =}$$

$$u = x^2-11x \Rightarrow u' = 2x-11$$

$$v = \ln(\sin(x)) \Rightarrow v' = \frac{\cos(x)}{\sin(x)}$$

$$\Rightarrow \frac{y'}{y} = u'v + uv' = (2x-11)\ln(\sin(x)) + (x^2-11x)\frac{\cos(x)}{\sin(x)}$$

$$\Rightarrow y' = \left[(2x-11)\ln(\sin(x)) + (x^2-11x)\frac{\cos(x)}{\sin(x)}\right]\left((\sin(x))^{x^2-11x}\right)$$