Area bdd by $y = x-1$ & $y^2 = 2x+6$

Save for possible test question.

$x = y+1$

$y^2 - 6 = 2x$

$x = \frac{y^2-6}{2} = \frac{1}{2}y^2 - 3 = x$

$y^2 = 2x+6$

$y = \pm\sqrt{2x+6} = \pm\sqrt{2(x+3)} = \pm\sqrt{2}\sqrt{x+3}$

$\frac{1}{2}y^2 - 3 = y+1$

$\frac{1}{2}y^2 - y - 4 = 0$

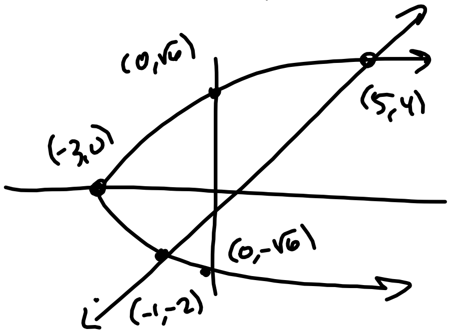
$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y \in \{-2, 4\} \Rightarrow$

$x = y+1 \Rightarrow$

$-1 = x = -2+1,\ 4+1 = -1, 5 = x$



$(-1,-2), (5,4)$

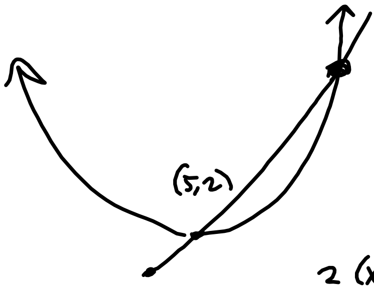
METHOD 1 : $\int dx$

$$\int_{-3}^{-1} \left(\sqrt{2x+6} - \left(-\sqrt{2x+6}\right)\right) dx$$

$$+ \int_{-1}^{5} \left(\sqrt{2x+6} - (x-1)\right) dx$$

Method 2: $\int dy$

$$\int_{-2}^{4} \left((y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right) dy$$



$2(x-5)^2+2$

$(x-4)^2+1$

$2(x-5)^2+2 = (x-4)^2+1$

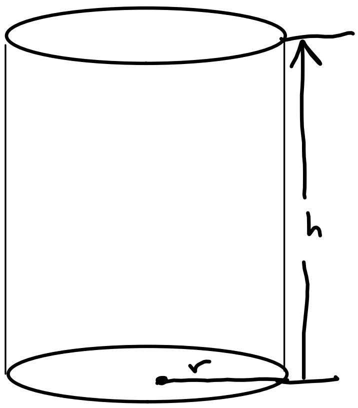
$2(x^2-10x+25)+2 = x^2-8x+16+1$

$2x^2-20x+52 = x^2-8x+17$

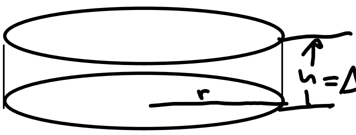
$x^2-12x+34=0$

$x^2-12x+36-36+34 = (x-6)^2+2$

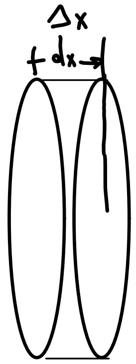
Volume of a cylinder is area of base times the height

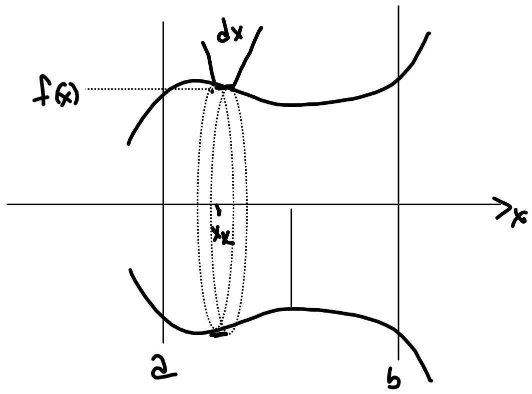


$V = \pi r^2 h$



$h = \Delta y \approx dy$

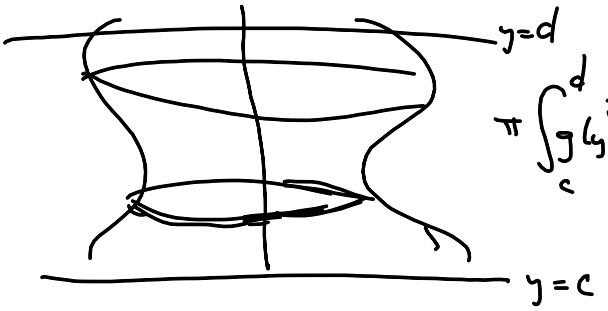




volume $= \pi r^2 h = \pi f(x_k) \Delta x$

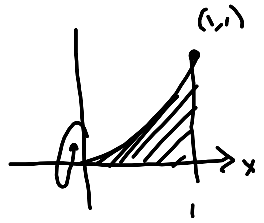
$\pi \sum_{k=1}^{n} f(x_k)^2 \Delta x$

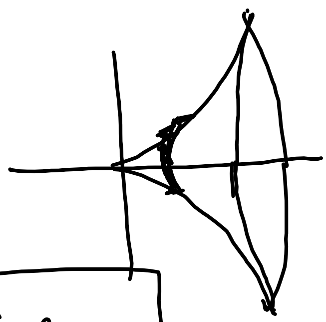
$\xrightarrow{n \to \infty} \pi \int_a^b f(x)^2 dx$ = Volume of the solid of revolution when f(x) is revolved around the x-axis.



$\pi \int_c^d [g(y)]^2 dy$

Find volume of the solid obtained when the region bounded by $y = x^2$, $y=0$, and $x=1$ is revolved about the x-axis.





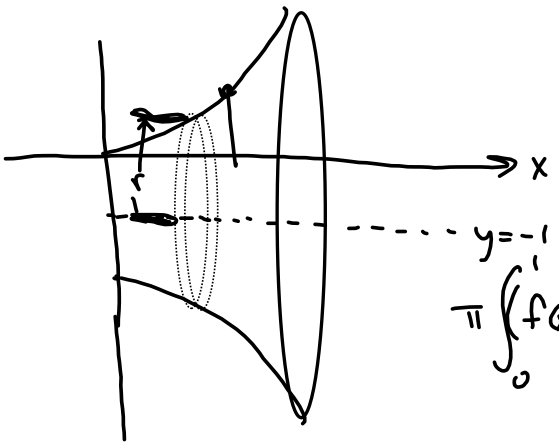
$\pi r^2 h$

$$\pi \int_a^b f(x)^2\,dx = \boxed{\pi \int_0^1 (x^2)^2\,dx}$$

$$= \pi \int_0^1 x^4\,dx = \pi \left[\tfrac{1}{5} x^5 \right]_0^1 = \pi \left[\tfrac{1}{5} [1]^5 \right] = \boxed{\tfrac{\pi}{5} = \text{Volume}}$$

Revolving around another horizontal axis:

$y = -1$



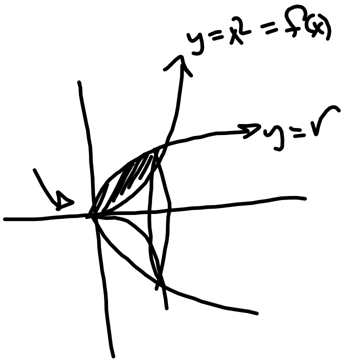
$$\pi \int_0^1 (f(x) - (-1))^2 dx$$

About $y = -5$

$$\pi \int_0^1 (x^2 - (-5)) dx$$

Washer Method

Revolve the region bounded by $y=x^2$ & $y=\sqrt{x}$ about the x-axis.



The idea for washer method is to subtract the inner solid (smaller radius) from the outer solid (larger radius)

$$\pi\int_a^b \text{outer}^2\,dx - \pi\int_a^b \text{inner}^2\,dx$$

$$= \pi\int_a^b (\text{outer}^2 - \text{inner}^2)\,dx$$

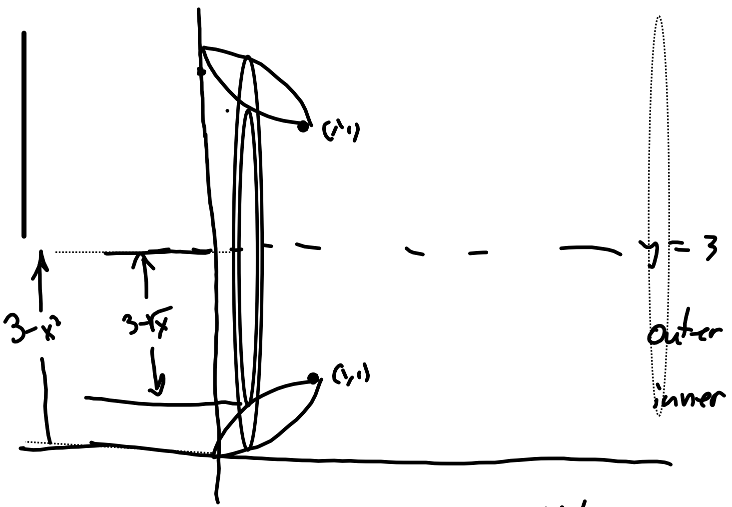
outer $= f(x) = \sqrt{x}$

inner $= g(x) = x^2$

So, Volume $= \int_0^1 (f(x)^2 - g(x)^2)\,dx = \int_0^1 ((\sqrt{x})^2 - (x^2)^2)\,dx$

$= \int_0^1 (x - x^4)\,dx = \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \frac{1}{2} - \frac{1}{5} = \frac{5-2}{10} = \boxed{\frac{3}{10} = \text{Volume}}$

Revolve same region about $y = 3$.



outer $= 3 - x^2$

inner $= 3 - \sqrt{x}$

Volume $= \pi \int_0^1 ((3-x^2)^2 - (3-\sqrt{x})^2)\,dx$

$= \pi \int_0^1 ((x^2-3)^2 - (\sqrt{x}-3)^2)\,dx$