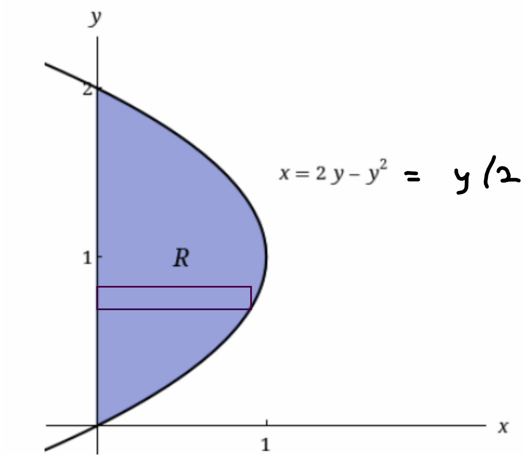
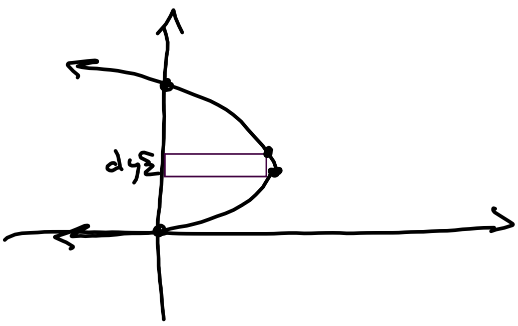
§4.4 #15

The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 2y - y^2$ (the shaded region in the figure) is given by the integral $\int_0^2 (2y - y^2)\,dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 2y - y^2$ from $y = 0$ to $y = 2$.)



$= y(2-y) \overset{\text{SET}}{=} 0 \Rightarrow y = 0, 2$



$$\int_0^2 (2y - y^2)\,dy$$

Section 4.5 - *u*-Substitution

Chain Rule:

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$$\frac{d}{dx}\left[\sin(x^3)\right] = (\cos(x^3))(3x^2)$$

The differential of $\sin(x^3)$ is

$d\sin(x^3) = 3x^2\cos(x^3)dx$

Let $y = \sin(x^3)$. Find dy

$= d(\sin(x^3)) = \cos(x^3)\cdot(3x^2)dx$

$$\int d\sin(x^3) = \sin(x^3) + C = \int 3x^2\cos(x^3)dx$$

$\int du = u + c$, $\int d☺ = ☺ + c$

$\int d(\text{ANYTHING}) = \text{ANYTHING} + C$

Evaluate

$$\frac{1}{3}\int 3x^2\cos(x^3)dx$$

The messy thing/inside.

$u = x^3$

Need a du, here

$du = 3x^2\,dx$

$\int du = u + c$

$\int dx = x + c$

$d(\tan(x)) = \tan(x) + C$

$$\int \cos(u)\,du = \sin(u) + C$$

$u = x^3$

$du = 3x^2dx \implies dx = \frac{du}{3x^2}$

$$\int x^2\cos(x^3)\,dx = \int \cancel{x^2}\cos(u)\cdot\frac{du}{3\cancel{x^2}} = \frac{1}{3}\int\cos(u)\,du$$

$$= \frac{1}{3}\sin(u) + C = \frac{1}{3}\sin(x^3) + C$$

$$\int f(g(x))\boxed{g'(x)dx}$$

$$\int f(u)du$$

It's all about spotting the $g'(x)\,dx$.

$\int dx \neq x^2\int x^2dx$

Can't do this with variables!

But $\int 3f(x)dx$

$= 3\int f(x)dx$

$$\int\left(\sin^3(x) + \sin^2(x) - \sin(x)\right)\cos(x)\,dx$$

$$= \int \sin^3(x)\cos(x)\,dx + \int\sin^2(x)\cos(x)\,dx - \int\sin(x)\cos(x)\,dx$$

$\int(\sin(x))^3\cos(x)\,dx$

The challenge is to determine "u" & building "du"

$u = \sin(x) \implies du = \cos(x)dx$

$f(u) = u^3$

$u(x) = \sin(x)$

$du(x) = \cos(x)\,dx =$

$d(\sin(x))$

Evaluate the indefinite integral.

1 $\int \sqrt{2t+1}\,dt = \frac{1}{2}\int (2t+1)^{\frac{1}{2}}(2\,dt) = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]+C$

$u=2t+1$

$du=2dt \Rightarrow dt=\frac{du}{2}$ STUDENT WAY

$\boxed{=\frac{1}{3}(2t+1)^{\frac{3}{2}}+C}$

$\Rightarrow \int(\sqrt{u})\frac{du}{2} = \frac{1}{2}\int u^{\frac{1}{2}}du$, etc

2 $\frac{1}{4}\int \frac{4x^3}{(x^4-5)^2}\,dx = \frac{1}{4}\int u^{-2}du = \frac{1}{4}\left[-u^{-1}\right] = \boxed{-\frac{1}{4}(x^4-5)^{-1}+C}$ Fine 4 Test.

$= -\frac{1}{4(x^4-5)}+C$

Scratch

$\left(\begin{array}{l} u = x^4-5 \longrightarrow \\ du = 4x^3dx \Rightarrow dx = \frac{du}{4x^3} \end{array} \right)$

$= \int \frac{\cancel{x^3}}{(u)^2}\cdot\frac{du}{4\cancel{x^3}} = \frac{1}{4}\int u^{-2}du$, etc.

3 $\int x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int(u^{\frac{1}{2}})(-2x)dx = -\frac{1}{2}\int u^{\frac{1}{2}}du$, etc.

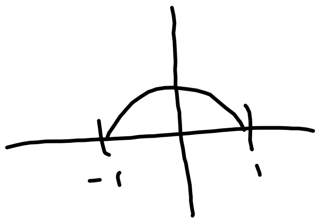
$u=1-x^2$

$du=-2x\,dx$

A related question to #3 that requires an understanding of analytic geometry.

$$\int_{-6}^{6}(x-3)\sqrt{1-x^2}\,dx$$

$$= \int_{-6}^{6}x\sqrt{1-x^2}\,dx - 3\int_{-6}^{6}\sqrt{1-x^2}\,dx$$



Poorly Posed

$$\int_{-5}^{5}(x-3)\sqrt{25-x^2}\,dx$$

$$= \int_{-5}^{5}x\sqrt{25-x^2}\,dx - 3\int_{-5}^{5}\sqrt{25-x^2}\,dx$$

$= -\frac{1}{5}\int_{-5}^{5}(\sqrt{25-x^2})(-5x\,dx) - 3$ (Area of top $\frac{1}{2}$ of circle!)

$\frac{1}{2}\pi(5^2)$

$$\int x(4x+1)^8\,dx$$

$$\left(\begin{array}{ll} u=4x+1 \longrightarrow & \\ du=4\,dx \longrightarrow & \text{Scratch} \\ dx=\frac{du}{4} \longrightarrow & \end{array}\right)$$

$$= \int x(4x+1)^8\left(\frac{du}{4}\right) = \frac{1}{4}\int x\cdot u^8\,du$$

$$\left(\begin{array}{ll} u=4x+1 \longrightarrow & \\ 4x=u-1 \longrightarrow & \text{Scratch} \\ x=\frac{u-1}{4} & \end{array}\right)$$

$$= \frac{1}{4}\int\left(\frac{u-1}{4}\right)u^8\,du = \frac{1}{16}\int(u^9-u^8)\,du = \frac{1}{16}\left[\frac{u^{10}}{10}-\frac{u^9}{9}+\hat{C}\right]$$

$$= \boxed{\frac{1}{16}\left[\frac{(4x+1)^{10}}{10}-\frac{(4x+1)^9}{9}\right]+C}$$

Stewart's 8th Edition.

#64 f cnt's on $[0,\pi] \Rightarrow$

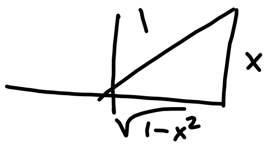
$$\int_0^\pi x\, f(\sin(x))\, dx = \frac{\pi}{2}\int_0^\pi f(\sin(x))\, dx$$

use the substitution $u = \pi - x$ in your proof.

$u = \sin(x) \Rightarrow \quad x = \arcsin(u) \Rightarrow$

$\boxed{du = \cos(x)\, dx}$ $\quad dx = \frac{d}{dx}[\arcsin(u)]\, du = \frac{1}{\sqrt{1-u^2}}\, du$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$



$$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\cos(\arcsin(x))}$$

$$= \frac{1}{\sqrt{1-x^2}} \; !$$

Not sure what the $\pi - x$ sub is doing for me.

$$\int \frac{\arcsin(u)\, f(u)}{\sqrt{1-u^2}}\, du$$

Fresh Look

$u = \pi - x \longrightarrow x = \pi - u$

$x = 0 \Longrightarrow u = \pi$

$x = \pi \Longrightarrow u = 0$

$$\int_0^\pi x\, f(\sin(x))\, dx$$

$du = -dx \longrightarrow dx = -du$

$$= \int_\pi^0 (\pi - u)\, f(\sin(\pi - u))(-du) = \int_0^\pi (\pi - u)\, f(\sin(\pi - u))\, du$$

$$= -\int_0^\pi (u - \pi)\, f(\sin(\pi - u))\, du$$

$$= -\int_0^\pi u\, f(\sin(\pi - u))\, du + \pi \int_0^\pi f(\sin(\pi - u))\, du = \int_0^\pi x\, f(\sin(x))\, dx$$

/ Scratch:

$\sin(\pi - u) =$

$\sin\pi \cos(-u) + \sin(-u)\cos(\pi)$

$= (-\sin(u))(-1) = \sin(u)$!

$$= -\int_0^\pi u\, f(\sin(u))\, du + \pi \int_0^\pi f(\sin(u))\, du = \int_0^\pi x\, f(\sin(x))\, dx$$

Re-label the dummy variable, inside.

$$\Longrightarrow -\int_0^\pi x\, f(\sin(x))\, dx + \pi \int_0^\pi f(\sin(x))\, dx = \int_0^\pi x\, f(\sin(x))\, dx$$

$$\Longrightarrow \pi \int_0^\pi f(\sin(x))\, dx = 2\int_0^\pi x\, f(\sin(x))\, dx$$

$$\Longrightarrow \frac{\pi}{2} \int_0^\pi f(\sin(x))\, dx = \int_0^\pi x\, f(\sin(x))\, dx \quad \blacksquare$$