§4.4 Indefinite Integrals & the Net Change Theorem

Recall $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$ is a number

$\int_a^x f(t)\,dt$ is a function of x.

Different way of writing $\int_a^b f(x)\,dx$ is $\int f(x)\,dx \Big]_a^b$

which by FTC II is $F(b) - F(a)$ for any antiderivative F of f.

Table of Indefinite Integrals. I wrote these out about 10 time, at least, until I didn't have to look to remember them all.

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$\int dx = x + C$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

NOTE: No $\int \sec x\,dx$, $\int \csc x\,dx$, $\int \tan x\,dx$, $\int \cot x\,dx$ Need LOGS!

Notice one that's not on the list, because it's not true:

$$\int f(x)g(x)\,dx = \int f(x)\,dx \int g(x)\,dx$$

Some students are tempted to say this, but it all goes back to the fact that $\sum ab \neq \left(\sum a\right)\left(\sum b\right)$

$a_k = k \qquad k = 1, 2, \ldots$

$b_k = 2k-1 \qquad k = 1, 2, \ldots \qquad b_1, b_2, b_3 = 1, 3, 5$

$$\sum_{k=1}^{3} a_k b_k = (1)(2(1)-1) + (2)(2(2)-1) + 3(2(3)-1)$$

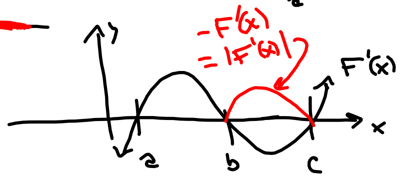
$$= 1 + (2)(3) + (3)(5) = 1 + 6 + 15 = 22$$

$$\sum_{k=1}^{3} a_k \sum_{k=1}^{3} b_k = (1+2+3)(1+3+5) = (6)(9) = 54$$

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)\,dx = F(b) - F(a) = \text{NET CHANGE}$$

$$\text{TOTAL CHANGE} = \int_a^b |F'(x)|\,dx = \underline{\text{TOTAL}} \ \text{CHANGE}$$



$F'(x) \geq 0$ on $[a,b]$ (or (a,b))*

& $F'(x) \leq 0$ on $[b,c]$ (or (b,c))*

$$\text{NET CHANGE} = \int_a^c F'(x)\,dx$$

*Throwing out one point does not change the definite integral.

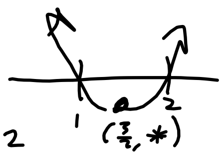
$$\text{TOTAL CHANGE} = \int_a^c |F'(x)|\,dx = \int_a^b |F'(x)|\,dx + \int_b^c |F'(x)|\,dx$$

$$= \int_a^b F'(x)\,dx + \int_b^c -F'(x)\,dx = \int_a^b F'(x)\,dx - \int_b^c F'(x)\,dx$$

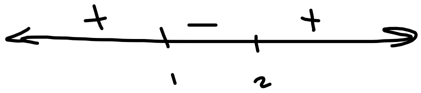
Suppose $F'(x) = x^2 - 3x + 2$ for $x \in [\frac{3}{2}, 3]$ (or $= \{x \mid \frac{3}{2} \leq x \leq 3\}$

Evaluate the total change in F from $x = \frac{3}{2}$ to $x = 3$

$$\int_{\frac{3}{2}}^3 |F'(x)|\,dx$$



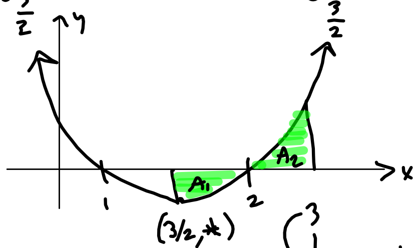
$x^2 - 3x + 2 = 0 \longrightarrow (x-2)(x-1) = 0 \longrightarrow x = 1, 2$



$x \leq 1$ OR $2 \leq x$

Then $|F'(x)| = \begin{cases} x^2 - 3x + 2 & \text{if } x \in (-\infty, 1] \cup [2, \infty) \\ -(x^2 - 3x + 2 & \text{if } x \in [1, 2] \end{cases}$

So $\displaystyle\int_{\frac{3}{2}}^3 |F'(x)|\,dx = -\int_{\frac{3}{2}}^2 (x^2 - 3x + 2)\,dx + \int_2^3 (x^2 - 3x + 2)\,dx$



$$\int_{\frac{3}{2}}^3 |F'(x)|\,dx = -A_1 + A_2$$

Section 4.5 - *u*-Substitution

Chain Rule:

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$$\frac{d}{dx}\left[\sin(x^3)\right] = (\cos(x^3))(3x^2)$$

The differential of $\sin(x^3)$ is

$$d\sin(x^3) = 3x^2\cos(x^3)dx$$

$$\int d\sin(x^3) = \sin(x^3) + C = \int 3x^2\cos(x^3)dx$$

$$\int d(\text{ANYTHING}) = \text{ANYTHING} + C$$

$$\int du = u + C$$

$$\int dx = x + C$$

$$d(\tan(x)) = \tan(x) + C$$

Evaluate

$$\int x^2\cos(x^3)dx$$

$$\int \cos(u)du = \sin(u) + C$$

$u = x^3$

$du = 3x^2dx \Longrightarrow dx = \frac{du}{3x^2}$

$$\int x^2\cos(x^3)dx = \int x^2\cos(u)\cdot\frac{du}{3x^2} = \frac{1}{3}\int\cos(u)du$$

$$= \frac{1}{3}\cos(u) + C = \frac{1}{3}\cos(x^3) + C$$

$$\int f(g(x))\boxed{g'(x)dx}$$

$$\int f(u)du$$

It's all about spotting the $g'(x)\,dx$.

$$\int\left(\sin^3(x) + \sin^2(x) - \sin(x)\right)\cos(x)\,dx$$

$$= \int\sin^3(x)\cos(x)\,dx + \int\sin^2(x)\cos(x)\,dx - \int\sin(x)\cos(x)\,dx$$

The challenge is to determine "u" & building "du"

$u = \sin(x) \Longrightarrow du = \cos(x)dx$

$f(u) = u^3$

$u(x) = \sin(x)$

$du(x) = \cos(x)\,dx$

#64 f cnt^{s} on $[0,\pi]$ $\Longrightarrow$

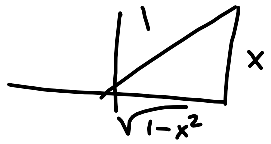
$$\int_0^{\pi} x\, f(\sin(x))\, dx = \frac{\pi}{2}\int_0^{\pi} f(\sin(x))\, dx$$

use the substitution $u=\pi-x$ in your proof.

$u=\sin(x) \Longrightarrow x=\arcsin(u) \Longrightarrow$

$\boxed{du=\cos(x)\,dx}$ $\qquad dx=\frac{d}{dx}\left[\arcsin(u)\right] du = \frac{1}{\sqrt{1-u^2}}\,du$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$



$$\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\cos(\arcsin(x))}$$

$$= \frac{1}{\sqrt{1-x^2}} \;!$$

$$\int \frac{\arcsin(u)\, f(u)}{\sqrt{1-u^2}}\, du$$

Not sure what the $\pi-x$ sub is doing for me.