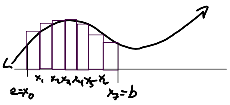


Right-Endpoint Riemann Sum

$x_k = a + k\Delta x = a + \frac{b-a}{n}k$

$\Delta x = x_1 - x_0 = x_2 - x_1 = \dots = \frac{b-a}{n} = \frac{b-a}{7}$

$$\text{Area} \approx \sum_{k=1}^{n} f(x_k)\Delta x = \sum_{k=1}^{n} f(x_k)\frac{b-a}{n} = \frac{b-a}{n}\sum_{k=1}^{n} f(x_k)$$

$$\sum_{k=1}^{n} 1 = 1+1+\dots+1 = n$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} = \frac{n^2+\dots}{2}$$

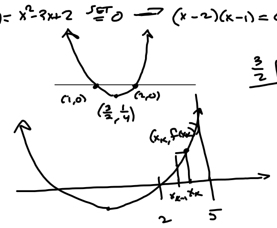
The "$+\dots$" stuff will always vanish in the limit.

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+\dots}{6} = \frac{n^3+\dots}{3}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2 = \frac{n^4+\dots}{4}$$

Find the area under the curve x^2-3x+2 over $[a,b]=[2,5]$

$f(x) = x^2-3x+2 \overset{\text{SET}}{=} 0 \longrightarrow (x-2)(x-1)=0$



$\frac{3}{2}\Big|\ 1 \quad -3 \quad 2$
$\quad\quad\ \ \frac{3}{2} \quad -\frac{9}{4}$
$\quad 1 \quad -\frac{3}{2} \quad \frac{1}{4} = f(\frac{3}{2}) =$

$-\frac{9}{4}+\frac{8}{4} = -\frac{1}{4}$

$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$

$x_k = a + k\Delta x = 2 + (k)(\frac{3}{n}) = \frac{3k}{n}+2 = \frac{3k+2n}{n}$

$$\longrightarrow \Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n}\sum_{k=1}^{n}(x_k^2 - 3x_k + 2)$$

$(a+b)^2 = a^2+2ab+b^2$

$(3k+2n)^2 = (3k)^2 + 2(3k)(2n) + (2n)^2$

$= 9k^2 + \overset{\to 12Kn}{12K} + 4n^2$

So $\left(\frac{3k+2n}{n}\right)^2 = \frac{9k^2 + \overset{\to 12Kn}{12K} + 4n^2}{n^2}$

$$= \frac{3}{n}\sum_{k=1}^{n}\left[\left(\frac{3k+2n}{n}\right)^2 - 3\left(\frac{3k+2n}{n}\right) + 2\right]$$

$$= \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2 + \overset{\to 12Kn}{12K} + 4n^2}{n^2} - 3\left(\frac{3k+2n}{n}\right) + 2\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2 + \overset{\to 12Kn}{12K} + 4n^2}{n^2} - \frac{9k}{n} - \frac{6n}{n} + 2\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2} + \frac{\overset{\to 12Kn}{12K}}{n^2} + \overset{\to 4}{\frac{4n^2}{n^2}} - \frac{9k}{n} - 6 + 2\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2} - \frac{12k}{n} - \frac{9k}{n}\right) = \frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2}{n^2} - \frac{21k}{n}\right) \to + \frac{3k}{n}$$

$$= \frac{3}{n}\left(\sum_{k=1}^{n}\frac{9k^2}{n^2} + \sum_{k=1}^{n}\frac{\overset{3K}{21k}}{n}\right)$$

$$= \frac{3}{n}\sum_{k=1}^{n}\frac{9k^2}{n^2} \overset{\to +}{-} \frac{3}{n}\sum_{k=1}^{n}\frac{21k \to 3K}{n}$$

$$= \frac{27}{n^3}\sum_{k=1}^{n} k^2 + \frac{3}{n^2}\sum_{k=1}^{n} k$$

$$= \frac{27}{n^3}\left(\frac{n^3+\dots}{3}\right) - \frac{63}{n^2}\left(\frac{n^2+\dots}{2}\right) \to \frac{3}{2n}$$

$$= 9\left(\frac{n^3+\dots}{n^3}\right) \overset{\to +}{-} \frac{63}{2} \overset{\to \frac{3}{2}}{} \left(\frac{n^2+\dots}{n^2}\right) \xrightarrow{n\to\infty} 9(1) \overset{\to +}{-} \frac{63}{2} \overset{\to \frac{3}{2}}{} (1)$$

~~$= \frac{18-63}{2} = -\frac{45}{2}$?!~~ ~~$\frac{n(n+1)}{n^2} = \frac{n^2+n}{n^2} = 1 + \frac{1}{n} \xrightarrow{n\to\infty} 1$~~

But $f(x)$ ~~≥ 0 on $[2,5]$~~ !

$$= \frac{18-3}{2} = \boxed{\frac{15}{2} = \text{AREA}}$$

FTC II Area $= \int_2^5 (x^2-3x+2)\,dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_2^5$

$$= \frac{5^3}{3} - \frac{3(5)^2}{2} + 2(5) - \left[\frac{2^3}{3} - \frac{3(2)^2}{2} + 2(2)\right]$$

$$= \frac{125}{3}\cdot\frac{2}{2} - \frac{75}{2}\cdot\frac{3}{3} + 10\cdot\frac{6}{6} - \frac{8}{3}\cdot\frac{2}{2} + \frac{12}{2}\cdot\frac{3}{3} - 4\cdot\frac{6}{6}$$

$$= \frac{250 - 225 + 60 - 16 + 36 - 24}{6}$$

$$\frac{85+20-24}{6} = \frac{81}{6} = \frac{27}{2}$$

x^2-3x+2 over $[2,5]$

$\frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n} = \Delta x$

$x_k = a + k\Delta x = 2 + \frac{3k}{n} = \frac{3k+2n}{n}$

$\Delta x \sum_{k=1}^{n} f(x_k) = \frac{3}{n} \sum_{k=1}^{n} \left[\frac{9k^2+12kn+4n^2}{n^2} - 3\left(\frac{3k+2n}{n}\right) + 2 \right]$

$= \frac{3}{n} \sum_{k=1}^{n} \left[\frac{9k^2}{n^2} + \frac{12kn}{n^2} + \frac{4n^2}{n^2} - \frac{9k}{n} - \frac{6n}{n} + 2 \right]$

$\cancel{= \frac{3}{n} \sum_{k=1}^{n} \frac{9k^2}{n^2} + \frac{3}{n} \sum_{k=1}^{n} \frac{12k}{n} + \frac{3}{n} \sum_{k=1}^{n} 4 - \frac{3}{n} \sum - \frac{9k}{n} - 6 + 2}$

$= \frac{3}{n} \sum_{k=1}^{n} \left[\frac{9k^2}{n^2} + \frac{12k}{n} + 4 - \frac{9k}{n} - 6 + 2 \right]$

$= \frac{3}{n} \sum_{k=1}^{n} \left[\frac{9k^2}{n^2} + \frac{3}{n} \sum_{k=1}^{n} \frac{3k}{n} \right]$

$= \frac{27}{n^3} \sum_{k=1}^{n} k^2 + \frac{9}{n^2} \sum_{k=1}^{n} k = \frac{27}{n^3}\left[\frac{n^3+\dots}{3}\right] + \frac{9}{n^2}\left[\frac{n^2+\dots}{2}\right]$

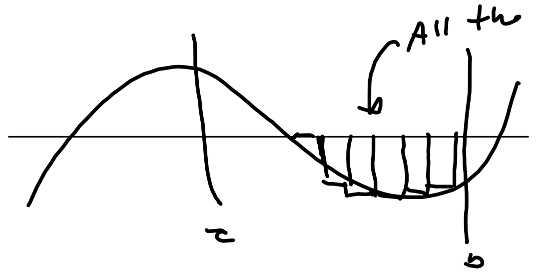
$\xrightarrow{n\to\infty} \quad 9 + \frac{9}{2} = \frac{18+9}{2} = \frac{27}{2}$

The definite integral of $f(x)$ over $[a,b]$ is written $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$.

It's SIGNED area!

To this point, only for $f(x) \geq 0$ on $[a,b]$

All the rectangles have height $f(x_k) < 0$!



$a < b < c$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

$$\int_a^c f - \int_a^b f = \int_a^b f + \int_b^c f - \int_a^b = \int_b^c$$

$$\int_a^{x+h} f - \int_a^x f = \int_x^{x+h} f$$