§ 3.9 Antiderivatives

F is an antiderivative of f if $F'(x) = f(x)$
$\forall\, x \in I$ (depending on $D(f)$)

$\frac{d}{dx}\left[x^2\right] = 2x \longrightarrow$

x^2 is an antiderivative of $2x$.
so is $x^2 - 50$
so is $x^2 + C$ for any $C \in \mathbb{R}$.

We represent an antiderivative as a family
$x^2 + C$.

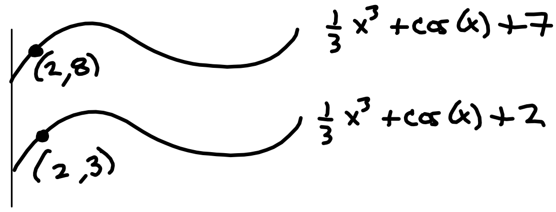
Antiderivative notation:

$$\int 2x\,dx = x^2 + C$$

$$\int \sin(x)\,dx = -\cos(x) + C, \quad \text{b/c} \ \frac{d}{dx}\left[\cos(x)\right] = \sin(x)$$

Suppose $f'(x) = x^2 - \sin(x)$

Then $f(x) = \frac{1}{3}x^3 + \cos(x) + C, \ \forall\, C \in \mathbb{R}$



Two members of the family

You only have to know $f(x_1)$ at some x_1.
Just one.

$f'(x) = x^2 + 5x + 1$ and $f(1) = 1 \longrightarrow$

$$\boxed{f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + x + C}$$

$$f(1) = \frac{1}{3} + \frac{5}{2} + 1 + C$$

$$= \frac{2 + 15 + 6}{6} + C = \frac{23}{6} + C = 1 \longrightarrow$$

$$\longrightarrow C = 1 - \frac{23}{6} = \frac{6 - 23}{6} = \boxed{-\frac{17}{6} = C},$$

$$\boxed{\text{i.e., } f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + x - \frac{17}{6}}$$

is the unique function with $f'(x) = x^2 + 5x + 1$ and

$f(1) = 1$

S = position (height) of a falling body.

$g = 9.8 \frac{m}{sec^2}$ = acceleration due to gravity

t = time in sec.

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

v_0 = initial velocity in $\frac{m}{sec}$ (up or down)

s_0 = initial height

$s'(t) = -gt + v_0 = v(t)$ = velocity

$s''(t) = -g$ = acceleration.

$s'''(t) = 0$ = jerk !

$s''''(t) = 0$ = snap !

Volume of sphere : $\frac{4}{3}\pi r^3$

Surface area of sphere : $4\pi r^2$

$f(x)$	$F(x)$
$\sin(x)$	$-\cos(x)+C$
$\cos(x)$	$\sin(x)+C$
$\tan(x)$	$\ln\lvert\sec(x)\rvert \quad +C$ Not Ready for yet.

$$\int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{-\sin(x)\,dx}{\cos(x)} = -\int \frac{du}{u} = -\ln\lvert\cos(x)\rvert + C$$

$= \ln\lvert u\rvert + C$, where $u = \cos(x)$ & $du = -\sin(x)dx$

$$\ln\left(\lvert\cos(x)\rvert^{-1}\right) + C = \ln\lvert(\cos(x))^{-1}\rvert + C = \ln\lvert\sec(x)\rvert + C$$

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$$

$$\frac{d}{dx}\left(e^x\right) = e^x \qquad \int e^x\,dx = e^x + C$$

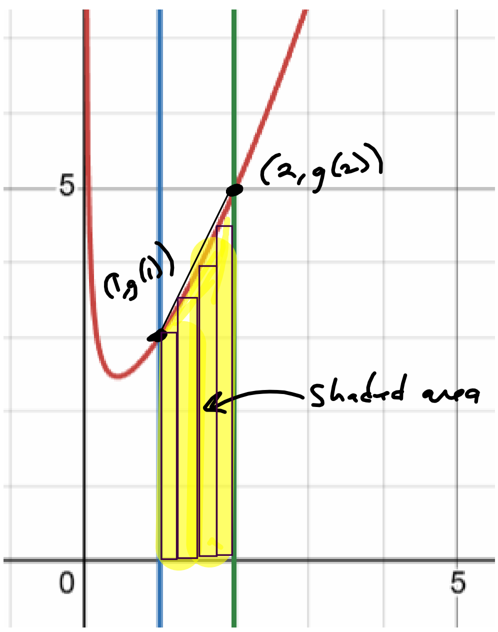
$$g(t) = \frac{t^2+t+1}{\sqrt{t}} = t^{\frac{3}{2}} + t^{\frac{1}{2}} + t^{-\frac{1}{2}}$$

$$= t^{-\frac{1}{2}}(t^2+t+1) = t^{2-\frac{1}{2}} + t^{1-\frac{1}{2}} + t^{-\frac{1}{2}}$$

$$\rightarrow G(t) = \frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}} + 2t^{\frac{1}{2}} + C$$

$$(G'(t) = g(t))$$

$$\int_1^2 g(t)\,dt = G(2) - G(1)$$ 2^{nd} Fundamental Theorem of Calculus.



Shaded area is $\int_1^2 g(t)\,dt \overset{\text{FTC II}}{=} G(2) - G(1)$

≈ 3.91012024104